FIRST TERM: E-LEARNING NOTES

SCHEME FIRST TERM

S S S 3 SCHEME FOR 2ND TERM

WEEK	TOPIC	CONTENT
1	GRAPHS OF TRIGONOMETRIC RATIOS	(a) Graphs of: (i) Sine $0 \leq x \leq 360$ (ii) Cosine $0 \leq x \leq 360$. (b) Graphical solution of simultaneous linear and trigonometric equations.
2	DIFFERENTIATION OF ALGEBRAIC FUNCTIONS 1	(a) Meaning of differentiation/derived function. (b) Differentiation from the first principle. (c) Standard derivatives of some basic functions.
3	DIFFERENTIATION OF ALGEBRAIC FUNCTIONS 2	(a) Rules of differentiation such as: (i) sum and difference (ii) chain rule (iii) product rule (iv) quotient rule. (b) Application to real life situation such as maxima and minima, velocity, acceleration and rate of change etc.
4	INTEGRATION OF SIMPLE ALGEBRAIC FUNCTIONS	(a) Integration and evaluation of definite simple Algebraic functions. (b) Application of integration in calculating area under the curve. (c) Use of Simpson's rule to find area under the curve.
5	REVISION	
6	REVISION	
7	REVISION	
8	REVISION	
9	REVISION	
10	REVISION	

WEEK 1:
Date:……………….
Subject: Mathematics
Class: SS 3
TOPIC: Trigonometry Graphs of Trigonometric Ratios
Content:

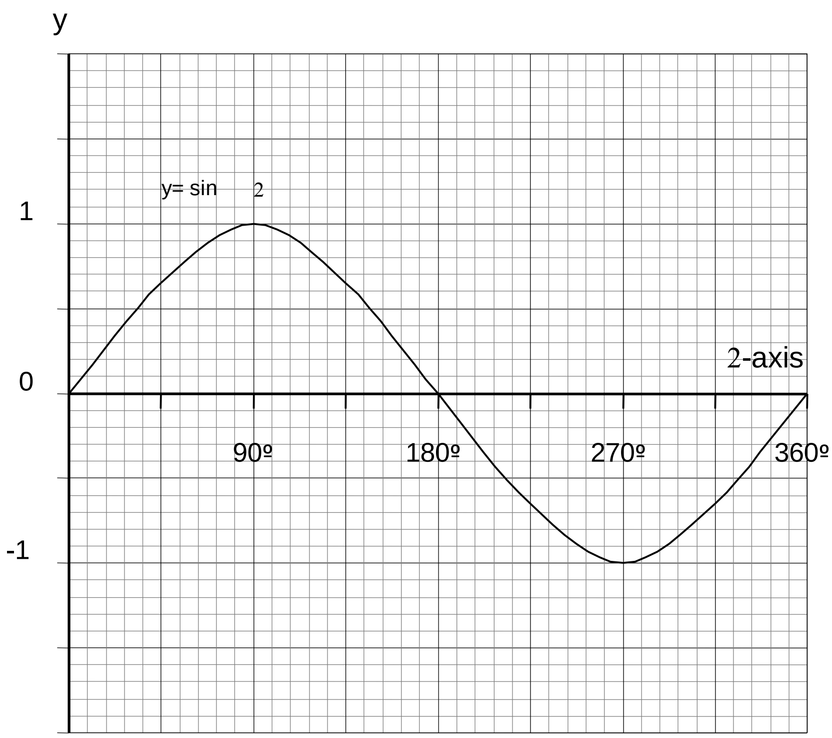
- Graphs of: (i) Sine $0^0 \leq x \leq 360^0$ (ii) Cosine $0^0 \leq x \leq 360^0$

- Graphical solution of simultaneous linear and trigonometric equations.

THE GRAPH OF Y = SINθ FOR $0^0 < \theta < 360^0$

The graph of y = sinθ is drawn by considering the table of values for sin θ from θ = 0^0 to θ = 360^0 at intervals of 90^0 as shown in the table below.

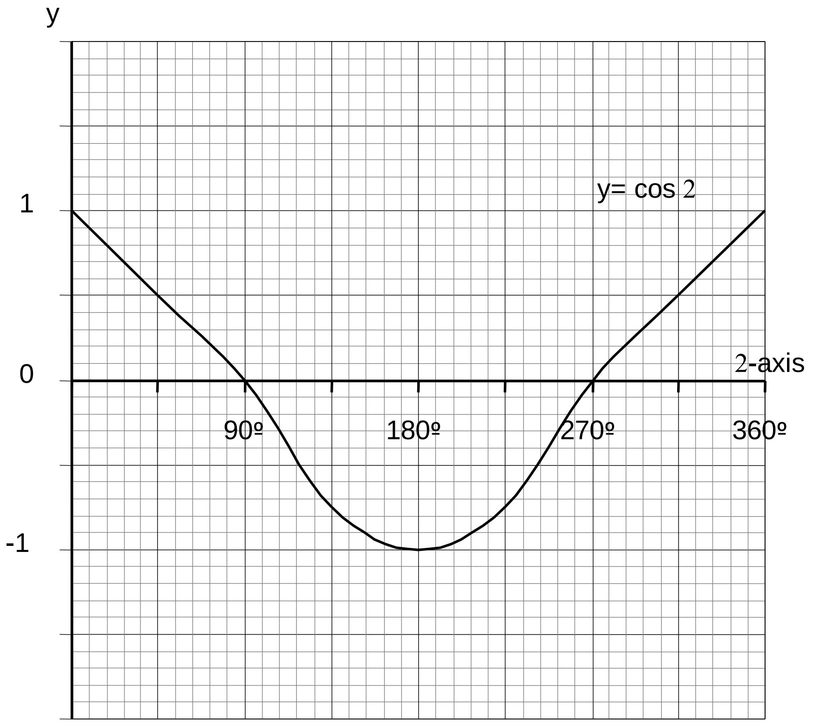
θ	0^0	90^0	180^0	270^0	360^0
y = sin θ	0	1	0	-1	0



THE GRAPH OF Y= cos θ for $0^0 < \theta^0 < 360^0$

The graph of y = cos θ is also drawn by considering the table of values for cos θ from θ = 0^0 to θ = 360^0 at intervals of 90^0 as shown in the table below.

θ	0^0	90^0	180^0	270^0	360^0
Y = cos θ	1	0	-1	0	1

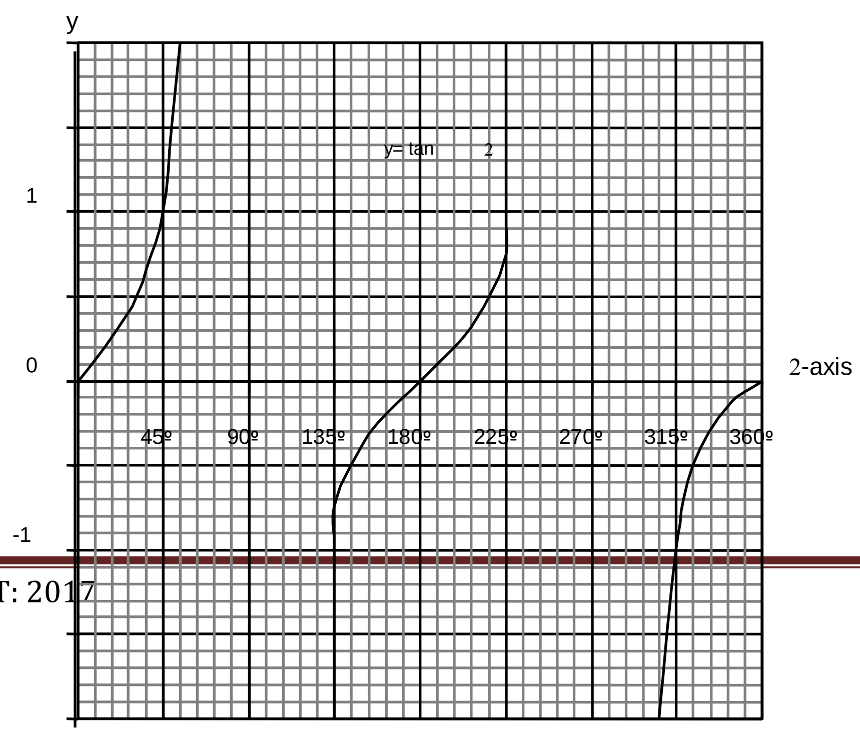


THE GRAPH OF Y = tanθ for $0^0 < \theta < 360^0$

The graph of y = tanθ is drawn by considering the table of values for tanθ from $\theta = 0^0$ to $\theta = 360^0$ at intervals of 45^0 as shown below.

θ	0^0	45^0	90^0	135^0	180^0	225^0	270^0	315^0	360^0
y=tanθ	0	1	udf	-1	0	1	udf	-1	0

udf. ⇒ Undefined



CLASS ACTIVITY

1. Draw the graph of each of the following functions
   (a) Y= - sinx (b) y = - cosx
2. Draw the graph of each of the following functions
   (c)y = -tanx (d) y = sinx + cosx (e) y = 1+ sinx

GRAPHICAL SOLUTION OF SIMULTANEOUS LINEAR AND TRIGONOMETRIC GRAPH

Example 1:

(a) Copy and complete the table of values for the function y = 2 cos2x - 1

X	0^0	30^0	60^0	90^0	120^0	150^0	180^0
y=2cos2x-1	1.0	0.0					1.0

(b) Using a scale of 2cm to 30^0 on the x-axis and 2cm to 1 unit on the y-axis draw the graph of y = 2 cos 2x – 1 for $0^0 \leq x \leq 180^0$

(c) On the same axes draw the graph of

$$y = \frac{1}{180}(x - 360^\circ)$$

(d) Use your graph to find the

(i) Values of x for which 2 cos 2x + ½ = 0

(ii) Roots of the equation

$$2\cos 2x - \frac{x}{180} + 1 = 0$$ ***(WAEC).***

Solution:

y = 2cos2x – 1

x	0^0	30^0	60^0	90^0	120^0	150^0	180^0
y=2cos2x-1	1.0	0.0	-2.0	-3.0	-2.0	0.0	1.0

For x = 60^0

$y = 2\cos 2 \times 60^0 - 1$

$= 2\cos 120 - 1$

$= -2\cos(180 - 120) - 1$

$= -2\cos 60^0 - 1$

$= -2 \times 0.5 - 1$

$= -1 - 1$

$= -2$

For x = 90

$y = 2\cos 2 \times 90^0 - 1$

$= 2\cos 180 - 1$

$= -2 - 1$

$= -3$

For x = 120

$y = 2\cos 2 \times 120^0 - 1$

$= 2\cos 240 - 1$

$= -2\cos(240 - 180) - 1$

$= -2\cos 60 - 1$

$= -2 \times 0.5 - 1$

$= -1 - 1$

$= -2$

For x = 150^0

$y = 2\cos 2x - 1$

$= 2\cos 2 \times 150 - 1$

$= 2\cos 300 - 1$

$= 2\cos(360 - 300) - 1$

$= 2\cos 60 - 1$

$= 2 \times 0.5 - 1$

$= 1 - 1$

$= 0$

(b) **Turn to the next page for graph**

(c) To draw the graph of $y = \frac{1}{180}(x - 360°)$

select any three values from the x-axis of the table above

x	0^0	90^0	180^0
y	-2.0	-1.5	-1.0

(d) (i)

$2\cos 2x + ½ = 0$

$2\cos 2x = -\frac{1}{2}$

$2\cos 2x - 1 = -\frac{1}{2} - 1$

$2\cos 2x - 1 = -1\frac{1}{2}$

The values of x for which $2\cos 2x + \frac{1}{2} = 0$ can be obtained at the point where $y = -1\frac{1}{2}$ (point A and B on the graph)

i.e. $x = 52^0$ or $x = 129^0$

(ii) The roots of $2\cos 2x - \frac{x}{180} + 1 = 0$

$$2\cos 2x - \frac{x}{180} + 1 = 0$$

$$2\cos 2x = \frac{x}{180} - 1$$

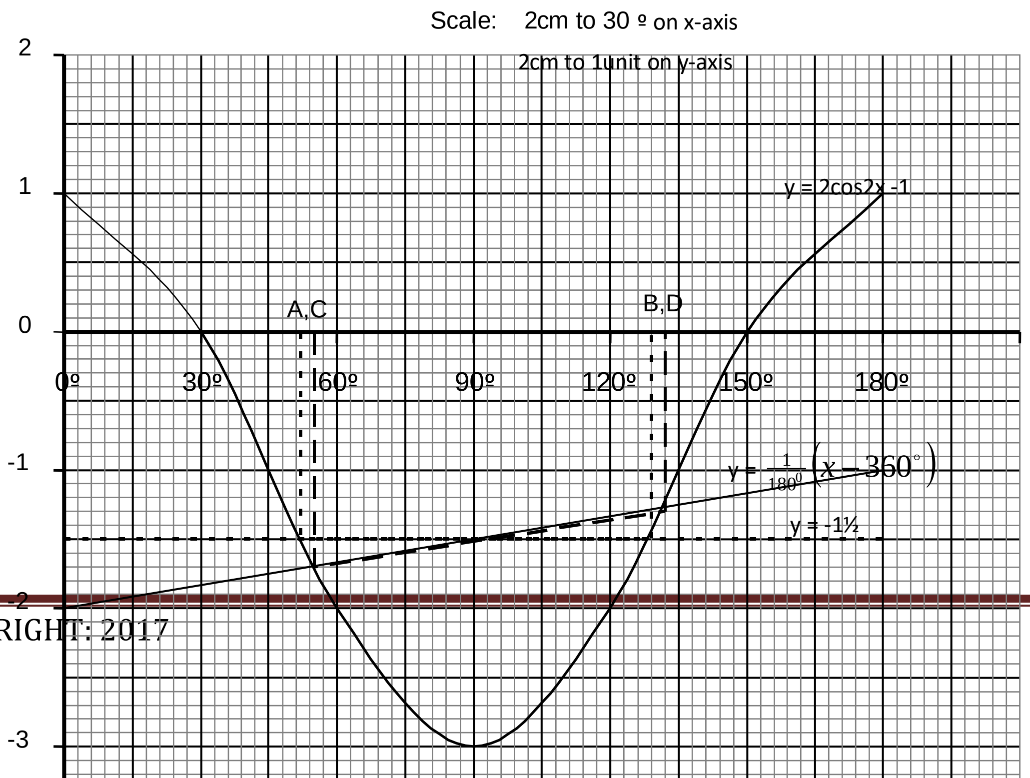
$$2\cos 2x - 1 = \frac{x}{180} - 1 - 1$$

$$2\cos 2x - 1 = \frac{x}{180} - 2$$

{where $\frac{x}{180} - 2 = \frac{1}{180}(x - 360^0)$*}*

The roots are found at the point where the two graphs $y = 2\cos 2x - 1$ and $y = {}^1/_{180}(x - 360^0)$ meet.(points C and D on the graph)

i.e. $x = 55^0$ or $x = 132^0$



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Example 2:

(a) Copy and complete the table of values for the function y = 2 cos 2θ+ sin θ

x	-120^0	-90^0	-60^0	-30^0	30^0	60^0	90^0	120^0	0^0
y			-1.87			-0.13	-1	0.13	2

(b) Using a scale of 2cm to 30^0 on θ-axis and 2cm to 1 unit on y-axis draw the graph of y = 2 cos 2θ+ sin θ for $-120^0 \leq \theta \leq 120^0$

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(c) Using the same scale and axes draw the graph of $y = \frac{7\theta}{410} - 1$

(d) From your graph, find the roots of the following equations

(i) 2 cos 2θ+ sin θ = 0

(ii) 2 cos 2θ+ sin θ+ ½ = 0

(iii) $2\cos 2\theta + \sin\theta = \frac{7\theta}{410} - 1$

Solution:

y = 2cos 2θ + sin θ

x	-120^0	-90^0	-60^0	-30^0	30^0	60^0	90^0	120^0
y	-1.87	-3.00	-1.87	0.50	1.50	-0.13	-1.00	-0.13

For θ = -120^0

$-120^0 \equiv 240^0$

y = 2 cos 2 x 240 + sin 240

= 2 cos 480 + sin 240

= 2 (-0.5) + (-0.8660)

= -1 –0.8660

= -1.87

For θ = -30

$-30^0 \equiv 330^0$

y = 2 cos 2 x 330^0 + sin 330^0

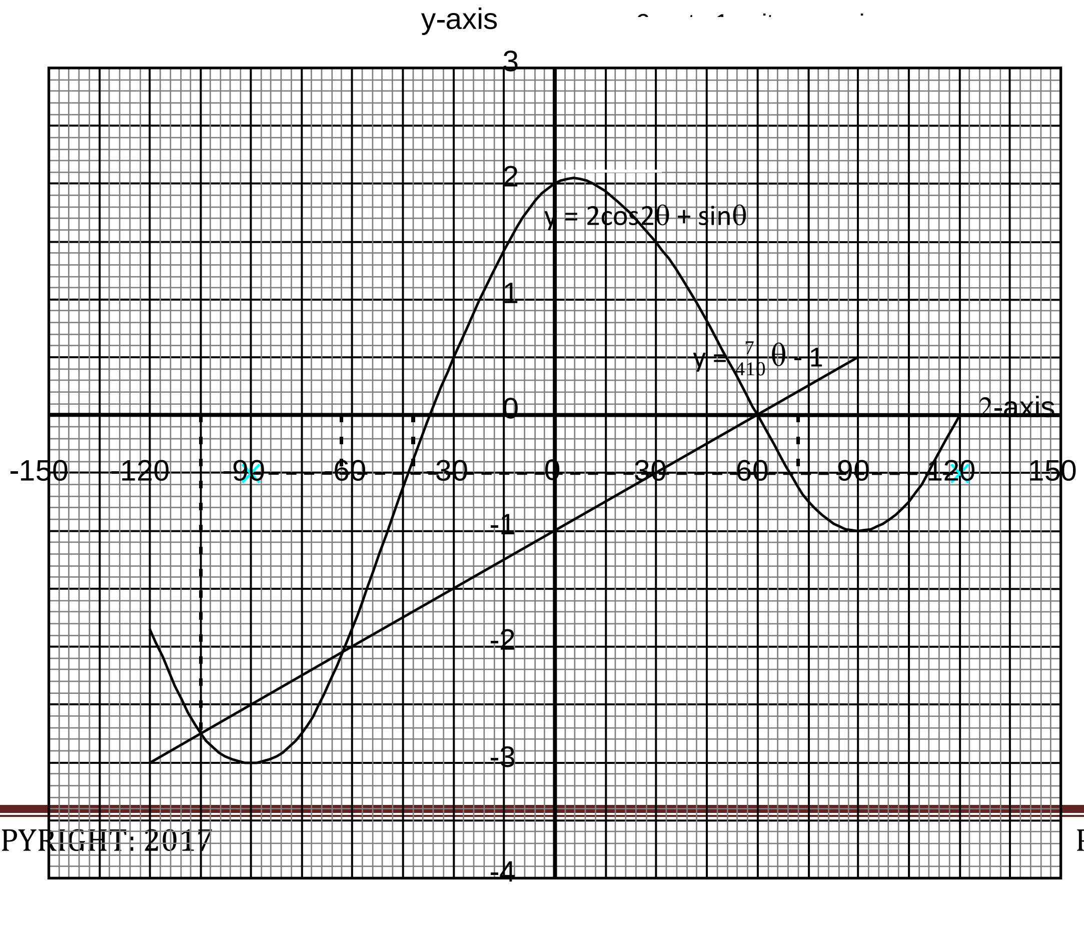
= 2 cos 660^0 + sin 330^0

= 2 (0.5) + (-0.5)

= 1 – 0.5

= 0.5

Scale: 2cm to 30º on 2-axis



For $\theta = -90$

$-90^0 \equiv 270^0$

$y = 2 \cos 2 \times 270^0 + \sin 270^0$

$= 2 \cos 540 + \sin 270$

$= 2(-1) + (-1)$

$= -2 - 1$

$= -3$

For $\theta = 30$

$y = 2 \cos 2 \times 30 + \sin 30^0$

$= 2 \cos 60^0 + \sin 30^0$

$= 2(0.5) + (0.5)$

$= 1 + 0.5$

$= 1.5$

(c) $y = \frac{7\theta}{410} - 1$

θ	-120^0	0^0	9^0
y	-3.05	-1	0.54

For θ = -120

$y = \frac{-7 \times 120}{410} - 1$

$y = -2.05 - 1$

$y = -3.05$

For $\theta = 0$

$y = \frac{7 \times 0}{410} - 1$

$y = -1$

For θ = 90^0

$y = \frac{7 \times 90}{410} - 1$

$y = \frac{630}{410} - 1$

$y = 1.54 - 1$
$y = 0.54$

(d) (i) The roots of 2 cos θ + sin θ = 0 is at the points where y = 0 i.e. where the graph crosses the θ-axis.
$\theta = -35^0$ or $\theta = 59^0$

(ii) $2\cos 2\theta + \sin\theta + ½ = 0$
$2\cos 2\theta + \sin\theta = {}^{-1}/_2$
The roots are the values of θ where $y = {}^{-1}/_2$ i.e. $\theta = -42^0$ or $\theta = 69^0$ or $\theta = 112^0$

(iii) $2\cos 2\theta + \sin\theta = \frac{7\theta}{410} - 1$
The roots are at the points where the two graphs y = 2 cos 2θ + sin θ and $y = \frac{7\theta}{410} - 1$ meet. i.e. $\theta = -103^0$ or $\theta = -64^0$ or $\theta = 60^0$

CLASS ACTIVITY

(1) (a) Copy and complete the following table of values for the function
$y = \sin\theta - \cos 2\theta$.

θ	0^0	30^0	60^0	90^0	120^0	150^0	180^0
y	-1.00	0.00			1.37		-1.00

(b) Using a scale of 2cm to 30^0 on θ-axis and 2cm to 1 unit on y-axis draw the graph of $y = \sin\theta - \cos 2\theta$ for $0^0 \le \theta \le 180^0$

(c) Using the same scale and axes draw the graph of $y = \frac{2\theta + 1}{270}$

(d) From your graph, find the roots of the following equation.
(i) $\sin\theta - \cos 2\theta = 0$
(ii) $\sin\theta - \cos 2\theta = 1$
(iii) $\sin\theta - \cos 2\theta = \frac{2\theta + 1}{270}$

(2) (a) Copy and complete the following table of values for the function
$y = 1 - 3\sin 2x$

x	0^0	15^0	30^0	45^0	60^0	90^0	105^0	120^0	135^0	150^0	165^0	180^0	75^0
y	1	-0.5			-1.6	1	2.5			3.6	2.5	1	-0.5

(b) Using a scale of 1cm to 15^0 on the x-axis and 2cm to 1 unit on the y-axis draw the graph of $y = 1 - 3\sin 2x$ for $0^0 \le x \le 180^0$

(c) Using the same scale and axes, draw the graph of $y = \frac{x-40}{50}$

(d) From your graph, find the roots of the following equation

(i) 1 – 3 sin 2x = 0

(ii) 1 – 3 sin 2x = 2

(iii) 1 – 3 sin 2x = $\frac{x-40}{50}$

PRACTICE EXERCISE

(1) (a) Copy and complete the following table of values for the function

y = 2 sin 2x + 1

x	-45^0	-30^0	-15^0	0^0	15^0	30^0	45^0	60^0	75^0	90^0	105^0	120^0
y		-0.7	0.0	1.0				2.7	2.0		0.0	-0.7

(b) Using a scale of 1cm to 15^0 on the x-axis and 2cm to 1 unit on the y-axis draw the graph of y = 2 sin 2x + 1 for $-45 \leq x \leq 120^0$

(c) From your graph, find the roots of the equations

(i) 2 sin 2x + 1 = 0

(ii) 2 sin 2x – 1 = 0

(2) (a) Copy and complete the following table of values for the function

y = cos 2x + 2 sin x

x	0^0	30^0	60^0	90^0	120^0	150^0	180^0	210^0	240^0	270^0	300^0	330^0	360^0
y	1	1.5		1	1.24		1	-0.5					1

(b) Using a scale of 1cm to 30^0 on the x-axis and 2cm to 1 unit on y-axis draw the graph of

y = cos 2x + 2 sin x for $0^0 \leq x \leq 360^0$

(c) From your graph, find the roots of the following equations

(i) cos 2x + 2sin x = 0

(ii) cos2x + 2sin x + 2 = 0

(3)(a) Copy and complete the following table of values for y = 3 sin 2θ – Cos θ

θ	0^o	30^o	60^o	90^o	120^o	150^o	180^o
Y	-1.0			0.0			1.0

(b) Using a scale of 2cm to 30^o on the θ axis and 2cm to 1 unit on the y axis, draw the graph of y=3sin 2θ-cosθ for $0^o \leq \theta \leq 180^o$

(c) Use your graph to find the:

(i) solution of the equation 3 sin2θ - cosθ=0, correct to the nearest degree;

(ii) maximum value of y, correct to one decimal place.

(4) (a)Copy and complete the following table of values for y = 9 cos x + 5 sin x to one decimal place.

x	0°	30°	60°	90°	120°	150°	180°	210°
Y		10.3			-0.2	-5.3		10.3

(b) Using a scale of 2cm to 30o on the x-axis and 2cm to 1 unit on the y-axis, draw the graph of *y* = 9 cos x + 5 sin x, for $0 \leq x \leq 210^{o}$

(c) Use your graph to solve the equation;

(i) 9 cos x + 5 sin x = 0

(ii) 9 cos x + 5 sin x = 3.5, correct to the nearest degree

(d) Find the maximum value of *y*, correct to one decimal place.

ASSIGNMENT

1. (a)Copy and complete the table of values for $y = 3 \sin x + 2 \cos x$ for $0^{o} \leq x \leq 360^{o}$.

x	0°	60°	120°	180°	240°	300°	360°
y	2.00	-	-	-	-	-	2.00

(b) Using a scale of 2cm to 60o on x-axis and 2cm to 1 unit on y-axis, draw the graph of $y = 3 \sin x + 2 \cos x$ for $0^{o} \leq x \leq 360^{o}$.

(c) Use your graph to solve the equation $3 \sin x + 2 \cos x = 1.5$.

(d) Find the range of values of x for which $3 \sin x + 2 \cos x < -1$.

2. (a) Copy and complete the table of values for $y = \sin x + 2\cos x$, correct to one decimal place.

x	0^o	30^o	60^o	90^o	120^o	150^o	180^o	210^o	240^o
y		2.2				-1.2	-2.0		-1.9

(b) Using a scale of 2cm to 30^o on the x-axis and 2cm to 0.5 units on the y-axis, draw the graph of $y = \sin x + 2\cos x$ for $0^o \leq x \leq 240^o$

(c) Use your graph to solve the equation:

(i) $\sin x + 2\cos x = 0$;

(ii) $\sin x - 2.1 - 2\cos x$.

(d) From the graph, find y when $x = 171^o$.

3. (a) Copy and complete the table of the relation y – 2sin x – cos 2x

x	0^o	30^o	60^o	90^o	120^o	150^o	180^o
y		0.5					-1.0

Using a scale of 2 cm to 30^o on the x-axis and 2cm to 0.5 unit on the y-axis, draw the graph of $y = 2\sin x - \cos 2x$, for $0^o \leq x \leq 180^o$.

(b) Using the same axes, draw the graph of y = 1.25

(c) Use your graphs to find the:

(i) values of x for which $2\sin x - \cos 2x = 0$.

(ii) roots of the equations $2\sin x - \cos 2x = 1.25$.

4. A. Copy and complete the table of values for $y = 1 - 4\cos x$

x	0^0	30^0	60^0	90^0	120^0	150^0	180^0	210^0	240^0	270^0	300^0

y	-3.0			1.0				4.5			-1.0

(B) using a scale of 2cm to 30^0 on the x-axis and 2cm to 1 unit on the y-axis , draw the graph of y= 1- 4cosx for $0^0 \leq x \leq 300^0$.

(C)use the graph to:

(i)solve the equation 1- 4cosx=0;

(ii) find the value of y when x=105^0

(iii)find x when y=1.5

5. (a) Copy and complete the table for the relation *y* =2 cos 2*x* – 1.

x	0°	30°	60°	90°	120°	150°	180°
Y=2cos 2x-1	1.0	0.0					1.0

(b) Using a scale of 2cm=30o on the x -axis and 2cm =1 unit on the y –axis draw the graph of y = 2 cos 2x +$^1/_2$ = 0

(c) On the same axis draw the graph of y =$^1/_{180}$ (x – 360).

(d) Use your graphs to find the:

(i) values of x for which 2 cos 2x+ ½ = 0

(ii) roots of equation 2 cos 2x – $^x/_{180}$ + 1 = 0

KEYWORDS: roots, y-axis, x-axis, coordinate, graph, table of values, scale etc.

WEEK 2:
Date:……………….
Subject: Mathematics
Class: SS 3
TOPIC: Differentiation of Algebraic functions 1
Content:

- Meaning of differentiation/derived function.
- Differentiation from the first principle.
- Standard derivatives of some basic functions.

MEANING OF DIFFERENTIATION/DERIVED FUNCTION#

The process of finding the differential coefficient of a function is called differentiation.Differentiation deals with the measure of the rate of change in a particular function when some quantities in the function is either increased or decreased. For example, given the function y = f(x), a change in x will produce a corresponding change in y. When y is increased, x is bound to increase in proportion and vice versa. Note: The reverse of differentiation is integration.

DIFFERENTIATION FROM THE FIRST PRINCIPLE

The method of finding the derivative of a function from definition is called differentiation from the first principle. Note: A change in x to $x + \Delta x$ produces a corresponding change in y to $y + \Delta y$.

Example 1:

Differentiate the following from the first principle

$A. y = 2x + 5$

$B. y = x^2$

SOLUTION

$A. y = 2x + 5$

Take increment in both x and y

$$y + \Delta y = 2(x + \Delta x\ ) + 5$$

$$\Delta y = 2(x + \Delta x) + 5 - y$$

$$\Delta y = 2(x + \Delta x) + 5 - (2x + 5)$$

$$\Delta y = 2x + 2\Delta x + 5 - 2x - 5$$

$$\Delta y = 2\Delta x$$

Divide both sides by Δx

$$\frac{\Delta y}{\Delta x} = \frac{2\Delta x}{\Delta x}$$

Take limits of both sides as $\Delta x\ tends\ to\ zero$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} 2$$

$$\therefore\ \frac{dy}{dx} = 2$$

$B. y = x^2$

Take increment in both x and y

$$y + \Delta y = (x + \Delta x)^2$$

$$\Delta y = (x + \Delta x)^2 - y$$

$$\Delta y = (x + \Delta x)^2 - x^2$$
$$\Delta y = (x + \Delta x)(x + \Delta x) - x^2$$
$$\Delta y = x^2 + x\Delta x + x\Delta x + (\Delta x)^2 - x^2$$
$$\Delta y = 2x\Delta x + (\Delta x)^2$$
$$divide\ both\ sides\ by\ \Delta x$$
$$\frac{\Delta y}{\Delta x} = \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$
$$\frac{\Delta \mathrm{y}}{\Delta \mathrm{x}} = \frac{2\mathrm{x}\Delta \mathrm{x}}{\Delta \mathrm{x}} + \frac{(\Delta \mathrm{x})^2}{\Delta \mathrm{x}}$$
$$\frac{\Delta y}{\Delta x} = 2x + \Delta x$$
$$\text{take limits of both sides as } \Delta \mathrm{x} \rightarrow 0$$
$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x)$$
$$\therefore \frac{dy}{dx} = 2x + 0$$
$$\Rightarrow \frac{dy}{dx} = 2x$$

EXAMPLE 2:

Differentiate the following from the first principle

$$y = \frac{1}{x}$$

SOLUTION

$$y = \frac{1}{x}$$

Take increment in both x and y

$$y + \Delta y = \frac{1}{x + \Delta x}$$
$$\Delta y = \frac{1}{x + \Delta x} - y$$
$$\Delta y = \frac{1}{x + \Delta x} - \frac{1}{x}$$
$$\Delta y = \frac{x - (x + \Delta x)}{x(x + \Delta x)}$$

$$\Delta y = \frac{x - x - \Delta x}{x(x + \Delta x)}$$

$$\Delta y = \frac{-\Delta x}{x(x + \Delta x)}$$

$$divide\ both\ sides\ by\ \Delta x$$

$$\frac{\Delta y}{\Delta x} = \frac{-\Delta x}{x(x + \Delta x)} \div \Delta x$$

$$\frac{\Delta \mathrm{y}}{\Delta \mathrm{x}} = \frac{-\Delta x}{x(x + \Delta x)} \times \frac{1}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{-1}{x(x + \Delta x)}$$

take limits of both sides as $\Delta \mathrm{x} \rightarrow 0$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \left(\frac{-1}{x(x + \Delta x)} \right)$$

$$\frac{dy}{dx} = \frac{-1}{x(x + 0)}$$

$$\frac{dy}{dx} = \frac{-1}{x \times x}$$

$$\therefore \ \frac{dy}{dx} = \frac{-1}{x^2}$$

CLASS ACTIVITY

Differentiate the following from the first principle

(1) $y = 1 - 4x$

(2) $y = 2x^2 - 5x$

***STANDARD DERIVATIVES OF SOME BASIC FUNCTIONS**

(1) if y = a , where 'a' is constant , then $\frac{dy}{dx} = 0$

(2) if y = ax , then $\frac{dy}{dx} = a$

(3) if y = x^n , then $\frac{dy}{dx} = nx^{n-1}$

(4) if y = ax^n , then $\frac{dy}{dx} = nax^{n-1}$

(5) if y = $(ax + b)^n$, then $\frac{dy}{dx} = na(ax + b)^{n-1}$

(6) if y = sin ax , then $\frac{dy}{dx} = a \cos ax$

(7) if y = cos ax , then $\frac{dy}{dx} = -a\sin ax$

(8) if y = e^{ax} , then $\frac{dy}{dx} = ae^{ax}$

EXAMPLE 1:

Use the standard derivatives given above to find $\frac{dy}{dx}$ of the following functions

A. $y = 4$

$$\frac{dy}{dx} = 0$$

B. $y = 7x$

$$\frac{dy}{dx} = 7$$

C. $y = x^3$

$$\frac{dy}{dx} = 3x^2$$

D. $y = 3x^5$

$$\frac{dy}{dx} = 3 \times 5 \times x^4$$

$$\frac{dy}{dx} = 15x^4$$

EXAMPLE 2:

Differentiate the following functions with respect to x

A. $y = x^{-4}$

$$\frac{dy}{dx} = -4x^{-5}$$

B. $y = 7x - 2x^{-3}$

$$\frac{dy}{dx} = 7 - (-3)2x^{-4}$$

$$\frac{dy}{dx} = 7 + 6x^{-4}$$

C. $y = \sin 2x$

$$\frac{dy}{dx} = 2cos2x$$

D. $y = \cos x$

$\frac{dy}{dx} = -sinx$

E. $y = e^{-4x}$

$\frac{dy}{dx} = -4e^{-4x}$

F. $y = (4x-1)^{\frac{-1}{2}}$

$\frac{dy}{dx} = -\frac{1}{2} \times 4(4x-1)^{-3/2}$

$\frac{dy}{dx} = -2(4x-1)^{-3/2}$

CLASS ACTIVITY

Find the derivative of the following function

1. $y = x^3$
2. $y = \frac{2}{x}$
3. $y = 3x^2 - \frac{7}{x^2}$

PRACTICE EXERCISE

1. Find from first principle, the derivative, with respect to x of $2x^2 + \frac{1}{x}$.
2. By first principle , $find\ \frac{dy}{dx}\ of\ y = 5x - \frac{x^2}{x}$.
3. Find the derivative of the following functions:
   a. $y = x^{-7}$ b. $y = x^{1/2}$ c. $y = 1/x^4$
4. Differentiate the following functions with respect to x.
   A. $2x^2-x-1$
   B. $6+5x-x^2$
5. $Find\ \frac{dy}{dx}\ of\ the\ functions$
   A. $X - \frac{1}{x}$
   B. $-\frac{3}{x^2}$

ASSIGNMENT

1. For each of the following functions , use the method of differentiation from the first principles to find $\frac{dy}{dx}$

a. $3x^2$

b. $-5x^2$

2. Use the standard derivatives to find the derivatives of the following functions

   i. $2x^2-3x+3$
   ii. $y=2e^x$
3. Differentiate the following:
   a. $y=-\cos x$
   b. $y=2\sin x$
4. Differentiate with respect to x.
   a. $8\sqrt{x}$
   b. $5x^{-2/3}$
5. Differentiate with respect to x.

   $a.\ \frac{1}{\sqrt[3]{x}}$

   b. $\frac{1}{\sqrt{x}}$

   KEYWORDS: derivative, differentiate , rate of change ,increase, increment, first principle, derived function etc.

WEEK 3:

Date:……………….

Subject: Mathematics

Class: SS 3

TOPIC: Differentiation of Algebraic functions 2

Content:

- Rules of differentiation such as: (i) sum and difference (ii) chain rule (iii) product rule (iv)quotient rule.
- Application to real life situation such as maxima and minima, velocity, acceleration and rate of change etc

RULES OF DIFFERENTIATION

SUM AND DIFFERENCE RULE

(a) If $y = u + v$

$$\frac{dy}{dx} = \frac{d(u+v)}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}} \qquad (\textbf{\textit{sum rule}})$$

(b) $If\ y = u - v$

$$\frac{dy}{dx} = \frac{d(u-v)}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}} \qquad (\textbf{\textit{difference rule}})$$

$\boldsymbol{EXAMPLE 1}$:

$Differentiate\ the\ following\ with\ respect\ to\ x$

(a) Y = $x^4 + 4x^3 - 2x\ 1$

(b) Y = $6x^3 - x^2 - 4x + 1$

SOLUTION

(a) Y = $x^4 + 4x^3 - 2x + 1$

$$\frac{dy}{dx} = 4x^3 + 12x^2 - 2$$

$$\therefore \frac{dy}{dx} = 4x^3 + 12x^2 - 2$$

$(b)\quad y = 6x^3 - x^2 - 4x + 1$

$$\frac{dy}{dx} = 18x^2 - 2x - 4$$

$$\therefore \frac{dy}{dx} = 18x^2 - 2x - 4$$

EXAMPLE 2:

$find \frac{dy}{dx}\ of\ the\ equation\ of\ the\ curve\ x^3 + 3x^2 - 9x + 5$

SOLUTION

$$y = x^3 + 3x^2 - 9x + 5$$

$$\frac{dy}{dx} = 3x^2 + 6x - 9$$

$$\therefore \frac{dy}{dx} = 3x^2 + 6x - 9$$

CHAIN RULE(function of a function)

The chain rule is used to find the derivatives of functions that have powers.

For example, $(x-3)^5, (2x-5)^3$ *etc.*

The chain rule formula is:

$$\boldsymbol{\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}}$$

Example 1:

Differentiate

(a) $y = (2x+8)^4$

(b) $y = \sqrt{x^2 - 1}$

solution

$(a)\ y = (2x+8)^4$

$let\ \ u = 2x + 8$

$\rightarrow y = u^4$

$$\frac{du}{dx} = 2$$

$$\frac{dy}{du} = 4u^3$$

$$\frac{dy}{dx} = 4u^3 \times 2$$

$$= 8u^3$$

$$\therefore \frac{dy}{dx} = 8(2x + 8)^3$$

$(b)\ y = \sqrt{x^2 - 1}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$let\ y = \sqrt{u}\ and\ u = x^2 - 1$$

$$y = u^{1/2}$$

$$\frac{dy}{du} = \frac{1}{2}u^{-1/2}$$

$$\frac{du}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{1}{2}u^{-1/2} \times 2x$$

$$= \frac{1}{2u^{1/2}} \times 2x$$

$$= \frac{x}{u^{1/2}}\ , but\ u = x^2 - 1$$

$$\therefore \frac{dy}{dx} = \frac{x}{(x^2 - 1)^{1/2}}$$

$$\frac{dy}{dx} = \frac{x}{\sqrt{x^2 - 1}}$$

EXAMPLE 2:

Differentiate $(2x^2 + 1)^4$ *with respect to* x.

Solution

$$Let\ y = (2x^2 + 1)^4$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$y = (2x^2 + 1)^4$$

$$let\ u = 2x^2 + 1$$

$$y = u^4$$

$$\frac{dy}{du} = 4u^3$$

$$\frac{du}{dx} = 4x$$

$$\frac{dy}{dx} = 4u^3 \times 4x$$

$$= 4(2x^2 + 1)^3 \times 4x$$

$$\frac{dy}{dx} = 16x(2x^2 + 1)^3$$

CLASS ACTIVITY

Differentiate the following with respect to x.

1. $y = (x^2 - 1)^4$
2. $y = (3x + 8)^{-3}$
3. $(2x - 1)^3$

PRODUCT AND QUOTIENT RULES

If $y = uv$, *where u and v are separate functions of x, then the product rule states that:*

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Similarly, if $y = \frac{v}{u}$, *then the quotient rule states that:*

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Example 1:

a. Differentiate the following with respect to x

$$y = (3 + 2x)(1 - x)$$

Solution

$$y = (3 + 2x)(1 - x)$$

Let $u = 3 + 2x \quad v = 1 - x$

$$\frac{du}{dx} = 2 \text{ and } \frac{dv}{dx} = -1$$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$
$$\frac{dy}{dx} = (1-x)2 + (3+2x)(-1)$$
$$\frac{dy}{dx} = 2 - 2x - 3 - 2x$$
$$\frac{dy}{dx} = -4x - 1$$

b. *Find the derivative of the function*: $(2x-1)^3 \ (x^2 - 1)^2$

Let $u = (2x-1)^3$ and $v = (x^2 - 1)^2$

$\frac{du}{dx} = 3 \times 2(2x-1)^2$ $\frac{dv}{dx} = 2 \times 2x(x^2 - 1)^1$

$\frac{du}{dx} = 6(2x-1)^2$ $\frac{dv}{dx} = 4x(x^2 - 1)$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$
$$\frac{dy}{dx} = (x^2 - 1)^2 \times 6(2x-1)^2 + (2x-1)^3 \times 4x(x^2-1)$$
$$\frac{dy}{dx} = 2(x^2-1)(2x-1)^2[3(x^2-1) + 2x(2x-1)]$$
$$\frac{dy}{dx} = 2(x^2-1)(2x-1)^2[3x^2 - 3 + 4x^2 - 2x]$$
$$\frac{dy}{dx} = 2(x^2-1)(2x-1)^2(7x^2 - 2x - 3)$$

EXAMPLE 2:

Find the derivative of the function: $\frac{1-x^2}{1+x^2}$

SOLUTION

Let $u = 1 - x^2 \ \ and \ \ v = 1 + x^2$

$$\frac{du}{dx} = -2x \quad and \quad \frac{dv}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{(1+x^2)(-2x) - (1-x^2)2x}{(1+x^2)^2}$$

$$\frac{dy}{dx} = \frac{-2x - 2x^3 - 2x + 2x^3}{(1+x^2)^2}$$

$$\boldsymbol{\frac{dy}{dx} = \frac{-4x}{(1+x^2)^2}}$$

CLASS ACTIVITY

1. a. Differentiate with respect to x: $\frac{1+x}{1-x^2}$

   b.Find the derivative of the following function: $\frac{x^2-1}{2x^2+1}$

2. $find\,\frac{dy}{dx}$
   a.$(2x+1)^3(x^2+1)$
   b. $x^3(2x^2-1)$

HIGHER DERIVATIVE

If $y = x^n$

Then $\frac{dy}{dx} = nx^{n-1}$.

Also, $\frac{d^2x}{dy^2} = n(n-1)x^{n-2} \quad (second\ derivative)$

$\frac{dx^3}{dy^3} = n(n-1)(n-2)x^{n-3}(third\ derivative)$ etc.

EXAMPLE 1:

Find the second derivative of $y=3x^3-5x^2$

SOLUTION

$Y=3x^3-5x^2$

$$\frac{dy}{dx} = 9x^2 - 10x$$

$$\frac{d^2y}{dx^2} = 18x - 10$$

EXAMPLE 2:

Find the $\frac{d^3y}{dx^3}$ $of\ y = x^4 - 6x^3 + 5$

SOLUTION

$$\frac{dy}{dx} = 4x^3 - 18x^2$$

$$\frac{d^2y}{dx^2} = 12x^2 - 36x$$

$$\frac{d^3y}{dx^3} = 24x - 36$$

CLASS ACTIVITY

Find the third derivative of the functions:

1. Y = $x^4 + 4x^3 - 2x\ 1$
2. Y = $6x^5 - \ x^3 - 4x + 1$

APPLICATION TO REAL LIFE SITUATION SUCH AS MAXIMA AND MINIMA, VELOCITY, ACCELERATION AND RATE OF CHANGE ETC.

GRADIENT

If y or f is a function of x , then the first derivative $\frac{dy}{dx}$ or f ′(x) is called the gradient function. The gradient of a curve at any point P(x_1,y_1) is obtained by substituting the values of x_1 and y_1 into the expression for $\frac{dy}{dx}$. This is the same as the gradient of the tangent at that point.

EXAMPLE 1:

Find the gradient of the curve y=x^2+7x-2 at the point (2,16)

SOLUTION

y=x^2+7x-2

$$\frac{dy}{dx} = 2x + 7$$

Thus at (2,16)

$$\frac{dy}{dx} = 2(2) + 7$$

$$\frac{dy}{dx} = 11$$

EXAMPLE 2:

If $f(x)=(x^2+3)^3$, find the gradient of f(x) at x= ½

SOLUTION

Let $y=(x^2+3)^2$

$$\frac{dy}{dx} = 2 \times 2x(x^2 + 3)$$

$$\frac{dy}{dx} = 4x(x^2 + 3)$$

Therefore, the gradient at x= ½ is

$$\frac{dy}{dx} = 4 \times \frac{1}{2}(\frac{1}{2}^2 + 3)$$

$$\frac{dy}{dx} = 2\left(\frac{1}{4} + 3\right)$$

$$\frac{dy}{dx} = 0.5 + 6$$

$$\frac{dy}{dx} = 6.5$$

CLASS ACTIVITY

1. Find the gradient of the curve $y=x^2+3x-2$ at the point x=3.
2. Find the coordinate of the point on the given curve, $y=x^2-x+3$ whose gradient is 1.

VELOCITY AND ACCELERATION

Suppose that a particle's distance, s metres, after t seconds is given by $s=t^2+3t+5$

The velocity is the rate of change of s compared with t, i.e.$\frac{ds}{dt}$. Since s= t^2+3t+5, then $\frac{ds}{dt} = 2t + 3$

Hence the velocity after t seconds is given by 2t+3.

Acceleration is the rate of change of velocity compared with time. If velocity is vm/s then the acceleration is $given\ by\ \frac{dv}{dt}$.

EXAMPLE 1:

A particle moves in a straight line specified by the equation $x=3t^2-4t^3$. Find the velocity and acceleration after 2 seconds.

SOLUTION

$X=3t^2-4t^3$

$$v = \frac{dx}{dt} = 6t - 12t^2$$

At t=2, we have $6(2)-12(2)^2$

= 12- 48

=-36m/s

$$a = \frac{d^2x}{dt^2} = 6 - 24t$$

$$at\ t = 2, \quad a = 6 - 24(2)$$

$$a = 6 - 48$$

$$a = -42m/s^2$$

EXAMPLE 2:

An object projected vertically upwards satisfies the relation $h=27t-3t^2$, where hm is the height after t seconds.

a. Find the time it takes to reach the highest point.
b. How high does it go?

SOLUTION

a. $h=27t-3t^2$

$$v = \frac{dh}{dt} = 27 - 6t$$

$$when\ the\ object\ reaches\ the\ highest\ point, the\ velocity, v = 0$$

i.e. 27-6t=0

6t=27

t=27/6=4.5seconds

b.to find the highest point, substitute t=4.5s into the expression for h.
$h=27(4.5)-3(4.5)^2$

h=60.75m

CLASS ACTIVITY

1. A particle moves along a straight line in such a way that after t seconds it has gone s metres, where $s=t^2+2t$.
   Find the velocity of the particle after
   a. 1 second
   b. 3 seconds

2. A stone is thrown vertically into the air , and its height is s metres after t seconds, where $s =29.4t-4.9t^2$
a. after how many seconds does it reach its greatest height ?
b. what is the greatest height?
c. What is its initial velocity?

INCREASING AND DECREASING FUNCTIONS

A function y is increasing if $\frac{dy}{dx} > 0$ while a function is decreasing if $\frac{dy}{dx} < 0$.

Example1:

Find the range of values of x for which x^2-x is increasing.

SOLUTION

Let $y=x^2-x$

x^2-x is increasing if $\frac{dy}{dx} > 0$

$$\frac{dy}{dx} = 2x - 1 > 0$$

2x>1

x> ½

EXAMPLE 2:

Find the range of values of x for which x^2-x is decreasing?

SOLUTION

Let $y=x^2-2x$

x^2-x is decreasing if $\frac{dy}{dx} < 0$

$$\frac{dy}{dx} = 2x - 2 < 0$$

2x<2

X< 1

CLASS ACTIVITY

1. Find the range of values of x for which x^2-5x is increasing.
2. Find the range of values for which x^2-4x is decreasing.

RATE OF CHANGE

EXAMPLE 1:

1. Find the approximate increase in the area of a circle if the radius increases from 2cm to 2.02cm.

   SOLUTION

   Let A denote the area of the circle of radius r.

   Then, $A=\pi r^2$

   $$\frac{dA}{dr} = 2\pi r$$

   $$now\ ,\delta A = \frac{dA}{dr}\delta r$$

   $$2\pi r(0.02) = 0.04\pi r$$

   $$= \mathbf{0.04 \times \frac{22}{7} \times 2}$$

   =0.2514cm²

EXAMPLE 2:

If the side of a square is increasing by 0.2%, find the approximate percentage increase in the area.

SOLUTION

$A=x^2$

$$\boldsymbol{\frac{dA}{dx} = 2x}$$

$$\delta A = \frac{dA}{dr}\delta r$$

$$\delta A = 2x\frac{0.2}{100}x = \frac{x^2}{250}$$

$$= \boldsymbol{\frac{100x^2}{250 \times 100} = \frac{\frac{2}{5}x^2}{100}}$$

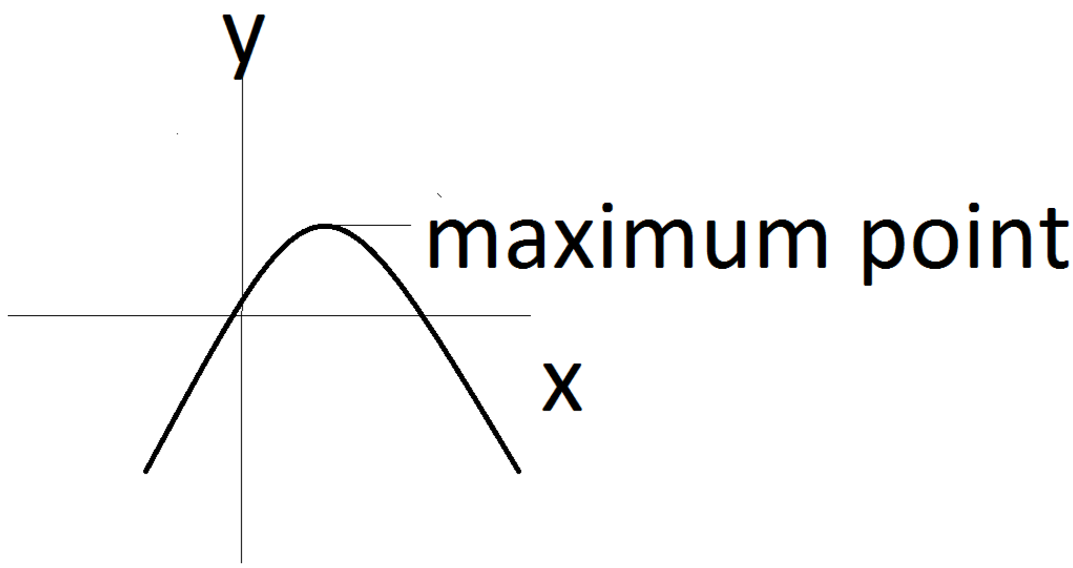
$$=\frac{2}{5}\%\ \boldsymbol{of\ area}$$

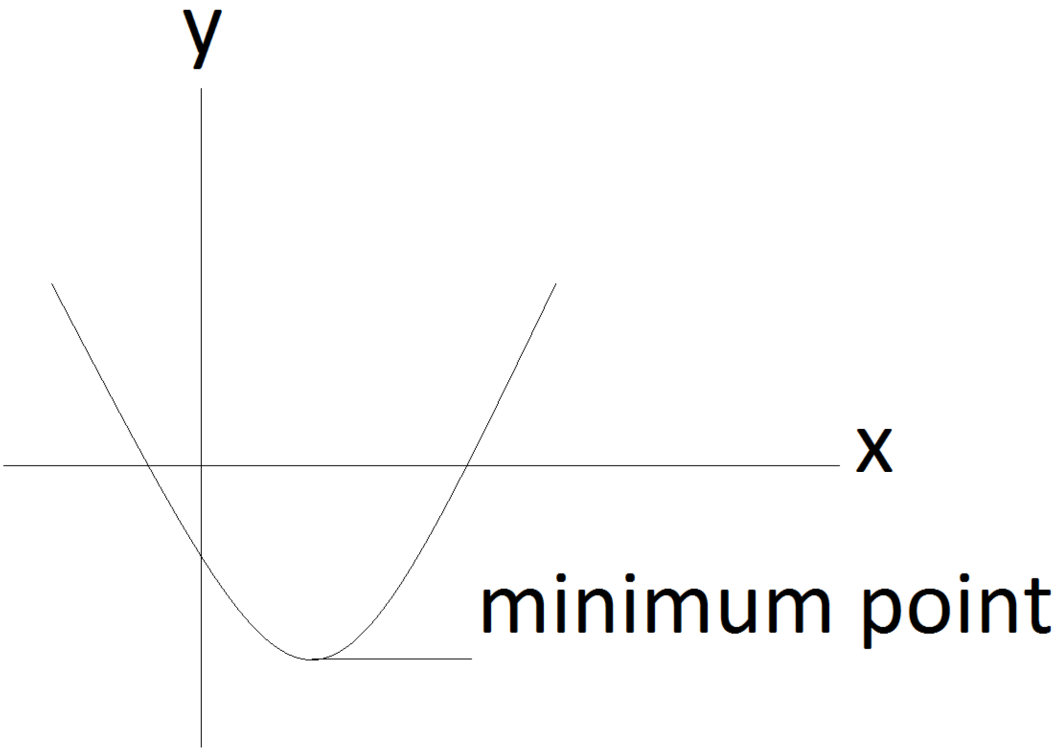
CLASS ACTIVITY

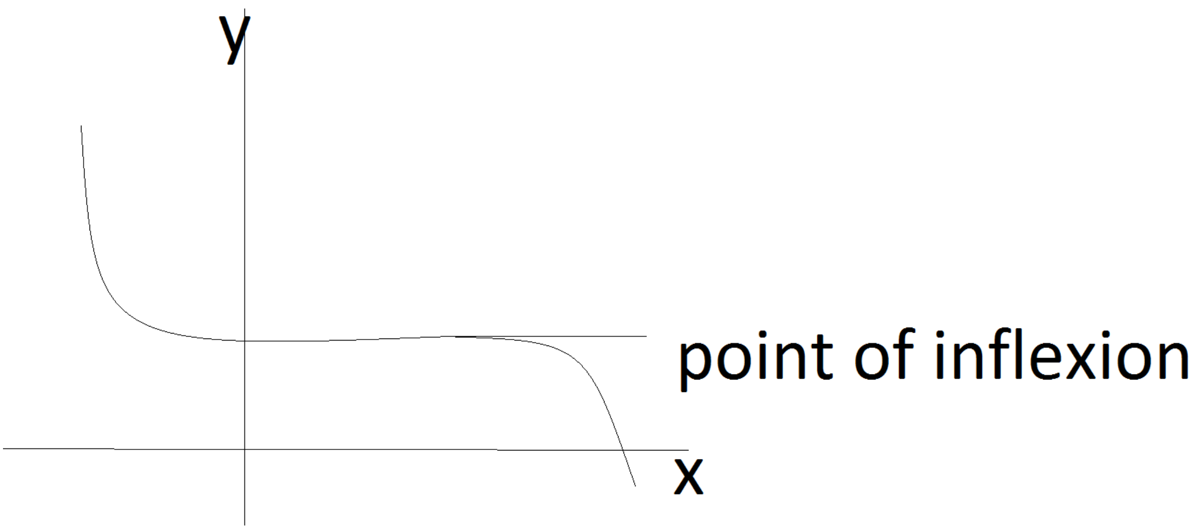
1. **The radius of a circle is increasing at the rate of 0.001m/s. find the rate at which the area is increasing when the radius of the circle is 10cm.**
2. **Find the approximate change in the surface area of a cube of side x metres caused by decreasing its side by 1%.**

MAXIMA AND MINIMA

A turning point/stationary point of a curve is a point at which the gradient is zero. The turning point is either maximum point(highest point) or the minimum point(lowest point) or the point of inflexion.







PROCEDURE FOR TESTING AND DISTINGUISHING BETWEEN STATIONARY POINTS

1. **Given y=f(x),determine $\frac{dy}{dx}$ $and\ \frac{d^2y}{dx^2}$**
2. **Put $\frac{dy}{dx} = 0$ and solve for x**
3. **Substitute x into equation to obtain the y, i.e.(x,y) of turning point.**
   NATURE OF TURNING POINT

Using $\frac{d^2y}{dx^2}$

A. **If** $\frac{d^2y}{dx^2} < 0(i.e. negative) - \ the\ point\ is\ maximum$
B. **If** $\frac{d^2y}{dx^2}$ **>0** $(i.e\ positive) - \ the\ point\ is\ minimum$
C. **If** $\frac{d^2y}{dx^2} = 0 - point\ of\ inflexion.$

EXAMPLE 1:

A curve is defined by the function y=x^3-6x^2-15x-1,find the maximum and minimum point.

SOLUTION

First, we find $\frac{dy}{dx}$ and equate to zero.

$$\frac{dy}{dx} = 3x^2 - 12x - 15 = 0$$

$$x^2 - 4x - 5 = 0$$

$$x^2 - 5x + x - 5 = 0$$

$$x(x-5) + 1(x-5) = 0$$

$$(x-5)(x+1) = 0$$

$$X = 5\ or\ -1$$

To test for maximum or minimum,
We differentiate the second time

$$\frac{dy}{dx} = 3x^2 - 12x - 15$$

$$\frac{d^2y}{dx^2}=6x - 12$$

Put x=5, 6(5)-12=18>0…………..minimum point at x=5
Put x=-1, 6(-1)-12=-6-12=-18<0……………maximum point at x=-1

To find the corresponding y put x=5 to the first equation i.e. **y=x^3-6x^2-15x-1**

Y=125-150-75-1=-101, minimum point (5,-101)

$$Put\ x = -1$$

$y = -1 - 6 + 15 - 1$

$y = 7$

Maximum point(-1,7), minimum point(5,-101)

$\boldsymbol{NOTE}: \quad The\ \ maximum\ value = 7$

$\qquad The\ minimum\ value\ \ = -101$

CLASS ACTIVITY

1. **Find the maximum and minimum points of the curve $y=x^3-3x+5$**
2. **Find the turning point of $y=x^3-6x^2+12x-11$**

EXAMPLE 2:

Find the maximum or minimum value of the curve $y=x^2-6x+5$

SOLUTION

$\frac{dy}{dx} = 2x - 6 = 0$

2x-6=0

2x=6

X=3

$\frac{d^2y}{dx^2} = 2$, 2>0 therefore, it is minimum

To obtain y,

$y=3^2-6(3)+5$

y=9-18+5

y=-4

minimum point is (3,-4)

PRACTICE EXERCISE

1.Find the maximum and minimum values of y for the function

$y=2x^3+3x^2-36x-6$

2 . Differentiate the following with respect to x.

(i) $(4x+9)^3$

(ii) $(3x-2)^3(x^2+4)^2$ NECO

3.If $y= 2x^3-6x^2-15x+19$, find the coordinates of the points on the graph at which the gradient is 3.

4. Differentiate $\frac{\sqrt{x}}{\sqrt{x+1}}$

5. A moving body has gone s metres in t seconds, where $s=3t^2-4t+5$. Find its velocity after 3 seconds. Show that the acceleration is constant, and find its value.

ASSIGNMENT

1. After t seconds a particle has gone s metres where $s=t^3-6t^2+9t-5$. Find the time (in seconds) for its velocity and acceleration to be zero. Calculate also the velocity and acceleration initially and after 5 seconds.
2. Differentiate $\frac{x^{-3}}{3} - 2x^2 - x^{-1}$
3. Find dy/dx of $(3-2x)^{-1/2}$
4. Find the equation of the tangent to the curve $y=x^2-2x+3$ at the point (2,3)
5. If the radius of a circle is increased from 5cm to 5.1cm, find the approximate increase in area.

KEYWORDS: derivative, gradient, differentiate , rate of change ,increase, maximum, minimum, velocity, acceleration, derived function etc.

WEEK 4:
Date:……………….
Subject: Mathematics
Class: SS 3
TOPIC: Integration of Simple Algebraic functions:
Content:

- Integration and evaluation of definite simple Algebraic functions.
- Application of integration in calculating area under the curve.

- Use of Simpson's rule to find area under the curve.

INTEGRATION AND EVALUATION OF DEFINITE SIMPLE ALGEBRAIC FUNCTIONS

Integration is the opposite of Differentiation. It is the process of obtaining a function from its derivative. A function F (x) is an anti derivative of a given function F (x) if d $\frac{F(x)}{dx}$ = f (x).

In general, if F (x) is any anti derivative of f (x), then the most general anti derivative of f(x) is specified by f (x) + c and we write: $\int f(x)dx$ + c

The symbol $\int$ is called an integral sign and $\int f(x)dx$ is called the indefinite integral. The arbitrary constant c is called the constant of integration, and the function f (x) is called the integral.

For example , F(x) = x^4 + c is an anti derivative of f(x) = $4x^3$ because F'(x) = $\frac{dx^4}{dx}$ = $4x^3$ = f(x).

In general, if n $\neq$ -1, then an anti derivative of f(x) = x^n is F (x) = $\frac{x^{n+1}}{n+1}$ + C

To integrate a power of x (apart from power n = - 1, increase the power of x by 1 (one) and divide by the new power.

EXAMPLE 1:

a. $\int x^5 dx$ = $\frac{x^{5+1}}{5+1}$ + c

= $\frac{x^6}{6}$ + c

$\frac{d}{dx}$ ($\frac{x^6}{6}$ + 1) = $\frac{6x^5}{6}$ = x^5

$\frac{d}{dx}$ ($\frac{x^6}{6}$ - 2) = x^5

$\frac{d}{dx}$ ($\frac{x^6}{6}$ + 3) = x^5

- Similarly, $\frac{d}{dx}$ [$\frac{ax^{n+1}}{n+1}$ + C] = ax^n

∴ $\int a\ x^n$ dx = $\frac{ax^{n+1}}{n+1}$ + C ; (n ≠ -1)

b. Integrate the following:

(i). $\int 2x\ dx$ (ii). $\int (x^2 + x - 10)dx$ (iii). $\int x^{-1/2}$ dx

(iv). If $\frac{dy}{dx}$ = 4 and y =2 when x = -1, find y in terms of x.

SOLUTION :

(I). $\int 2x\ dx$ =2$\int x\ dx$

$= 2[\frac{x^{1+1}}{1+1}] + C$

$= \frac{2x^2}{2} + C$

$= x^2 + C$

(ii). $\int(x^2 + x - 10)dx$ This can be done term by term.

$\int(x^2 + x - 10)dx = \int x^2\, dx + \int x dx - \int 10dx$

$= \frac{x^{2+1}}{2+1} + \frac{x^{1+1}}{1+1} - \int 10x^0 dx$

$= \frac{x^3}{3} + \frac{x^2}{2} - 10\,\frac{x^{0+1}}{0+1} + C$

$= \frac{1}{3}\, x^3 + \frac{1}{2}x^2 - 10x + C$

Notice that instead of giving three different constants of integration, the three can be combined and written as one.

(iii). $\int x^{-1/2}dx = x^{-\frac{1}{2}+1} + C$

$= \frac{x^{1/2}}{1/2} + C$

$= 2\sqrt{x} + C$

(iv). $\frac{dy}{dx} = 4$, so dy = 4dx

$\int dy = \int 4dx$ ie y = 4x + C. When y = 2, x = -1

2 = 4(-1) + C

C = 6 Hence, y = 4x + 6

- The integral $\int f(ax + b)dx$

Let u = ax + b

$\frac{du}{dx} = a$,

du = adx , so dx = 1/a du

$\int f(\, ax + b)dx = \int f(u).\frac{1}{a}du$

$= 1/a \int f(u)du.$

EXAMPLE 2 :a. Integrate (i). $\int(3x + 2)^4\, dx$ (ii). $\int \frac{3dx}{(2x-1)^2}$

Solution : For $\int(3x + 2)^4\, dx$

Let u = 3x +2, $\frac{du}{dx} = 3$

So 3dx = du , hence dx = 1/3du

:. $\int (3x + 2)^4\, dx = \int u^4(\frac{1}{3})\, du$

$= \frac{1}{3} \int u^4 du$

$= \frac{1}{3}(\frac{u^5}{5}) + C$

$= \frac{1}{15} u^5 + C$

(ii). $\int \frac{3dx}{(2x-1)^2}$, Let U= 2x – 1

$\frac{du}{dx} = 2$

2dx = du

dx= ½ du

$\int \frac{3dx}{(2x-1)^2} = \int \frac{3(\frac{1}{2})du}{u^2}$

$= \frac{3}{2} \int \frac{du}{u^2}$

$= \frac{3}{2} \int u^{-2}$ du

$= \frac{3}{2} (\frac{u^{-2+1}}{-2+1}) + C$

$= \frac{3}{2}(\frac{u^{-1}}{-1}) + C$

$= \frac{-3}{2} . \frac{1}{U} + C$

$= \frac{-3}{2} . \frac{1}{2X-1} + C$

$= -\frac{3}{4x-2} + c$

b.Integrate (i).$\int (\frac{x^4+3x^3-4}{x^2}) dx$ (ii). $\int 2x(x^2 - 1)^2 dx$

$\int (\frac{x^4+3x^3-4}{x^2})dx = \int (\frac{x^4}{x^2} + \frac{3x^3}{x^2} - \frac{4}{x^2})dx$

$= \int x^2 dx + \int 3xdx - 4 \int x^{-2} dx$

$= \frac{x3}{3} + \frac{3}{2} x^2 + \frac{4}{x} + C$

(ii). $\int 2x (x^2 - 1)dx = \int (2x^3 - 2x)dx$

$= \int 2x^3 dx - \int 2xdx$

$= \frac{1}{2} x^4 - x^2 + C$

CLASS ACTIVITY

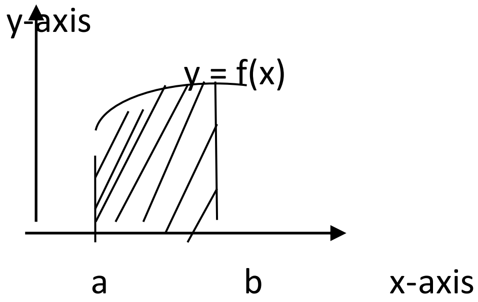
Determine the following integrals:

1) $\int (x^2 + 3x - 2)dx$

2) $\int \sqrt{x} + \frac{2}{\sqrt{x}})$ dx

3). $\int \sqrt{x^3}$ dx

APPLICATION OF INTEGRATION IN CALCULATING AREA UNDER THE CURVE



The integral $\int_a^b f(x)dx$ is called the definite integral of the function f(x) with 'a' and 'b' the lower and upper limits of the integral respectively.

$\int_a^b f(x)dx$ geometrically represents the area bounded by the curve y = f(x), the lines x = a , x =b and the x-axis.

Example 1:

Evaluate $\int_1^4 3x^2 dx$

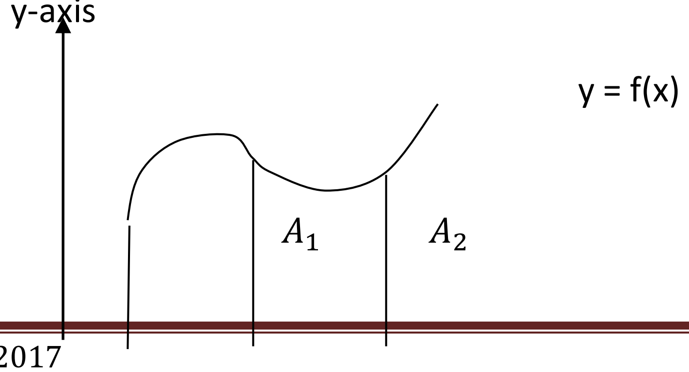
Solution:

$$\int_1^4 3x^2 dx = [x^3 + c]_1^4$$

Now substitute the value of the upper limit for x minus when you substitute the lower limit.

$$= (4^3 + c) - (1^3 + c) = 64 + c - 1 - c = 63$$

Now we shall examine some properties of the definite integral,



$A_1 = \int_a^c f(x)dx$

$A_2 = \int_c^b f(x)dx$

$$\int_a^b f(x)dx = A_1 + A_2$$
$$= \int_a^c f(x)dx + \int_c^b f(x)dx$$

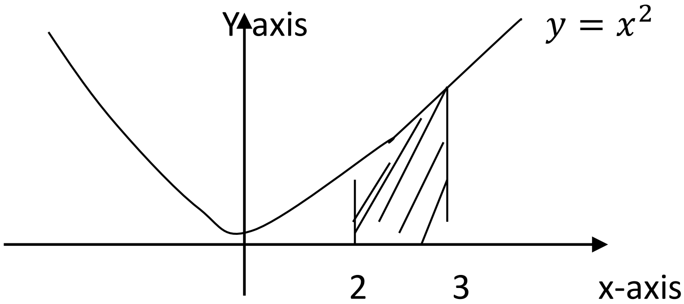
If the area is below the x-axis it will have a negative sign attached to it. Negating such an area will make it positive.

It is very essential to sketch the curve y = f(x) if the definite integral, $\int_a^b f(x)dx$ is to be used in finding the area bounded by the curve y = f(x), the lines x= a , x = b and the x – axis.

Example 2:

Find the area bounded by the curve $y = x^2$ the lines x = 2, x = 3 and the x –axis.

Solution:



Let the shaded area be the required area. i.e $\int_a^b ydx$

The area = $\int_2^3 x^2 dx$

= $\left[\frac{x^3}{3}\right]_2^3$

$= \frac{3^3}{3} - \frac{2^3}{3}$

$= \frac{27-8}{3}$

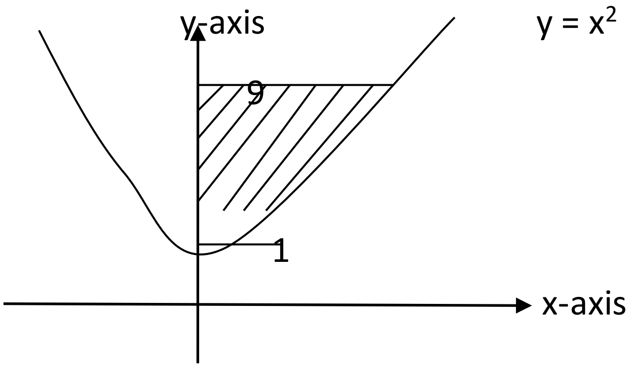
$= \frac{19}{3}$ sq. units

$= 6\frac{1}{3}$ sq. units

Example 3:

Find the area of the finite region bounded by the curve y = x^2 the line y = 1, y = 9 and the y – axis.

Solution:



The area = $\int_1^9 xdy$ as y = x^2 then x = $\sqrt{y}$, so the area = $\int_1^9 \sqrt{y}dy$

$= \left[\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_1^9$

$= \frac{2}{3}\left[9^{\frac{3}{2}} - 1^{\frac{3}{2}}\right]$

$= \frac{2}{3}[3^3 - 1^3]$

$= \frac{2}{3}(26)$

$= 17\frac{1}{3}$ sq. units

CLASS ACTIVITY

1. Find the area of finite region between the axis and the curve
   (i) y = x(x - 2)(x - 3)[ans: $3\frac{1}{12}$ $sq.unit$]   (ii) y = x(x - 1)(x - 3)
2. Find the area bounded by the following curves and lines and the axis
   (i) Y = x^2 for   x = -1 , x = 2

(ii) Y = $x^2 + 1$ for x = 1 , x = 3

(iii) Y = $(x-2)^2$ for x = 2 , x = 4

EQUATION OF CURVE GIVEN GRADIENT

EXAMPLE 1:

A curve passes through the point (0,1) and its gradient at any point P(x,y)=$3x^2$-5. Find the equation of the curve.

SOLUTION

Let $\frac{dy}{dx} = 3x^2 - 5$

dy=($3x^2$-5)dx

$\int dy = \int (3x^2$-5)dx

y=x^3-5x+c

at (0,1)

1=0-5(0)+c

C=1

The equation is y=x^3-5x+1

EXAMPLE 2:

A particle moves in a straight line in such a way that its velocity after t seconds is (3t+4)m/s. find the distance travelled in the first 3 seconds.

SOLUTION

V=3t+4

$$\frac{ds}{dt} = 3t + 4$$

$$ds = (3t + 4)dt$$

$$\int ds = \int_0^3 (3t + 4)dt$$

$$S=\left[\frac{3t^2}{2} + 4t\right]_0^3$$

S=27/2 +12

S=25.5m

VELOCITY AND ACCELERATION

EXAMPLE 1:

A particle moves in a straight line with a constant acceleration of 2cm/s^2. If its velocity after t seconds is vcm/s, find u in terms of t, given that the velocity after 3 seconds is 12cm/s.

SOLUTION

a=$\frac{dv}{dt} = 2$

$dv = \int 2dt$

$v = 2t + c$

When t=3 and v=12

12=2$\times 3 + c$

12=6+c

C=6

$\therefore v = 2t + 6$

EXAMPLE 2:

The velocity, Vms^{-1} of a body after time t seconds is given by V=3t^2-2t-3. Find the distance covered during the 4th second.

SOLUTION

Let S be the distance covered

S=$\int_3^4 (3t^2 - 2t - 3)dt$

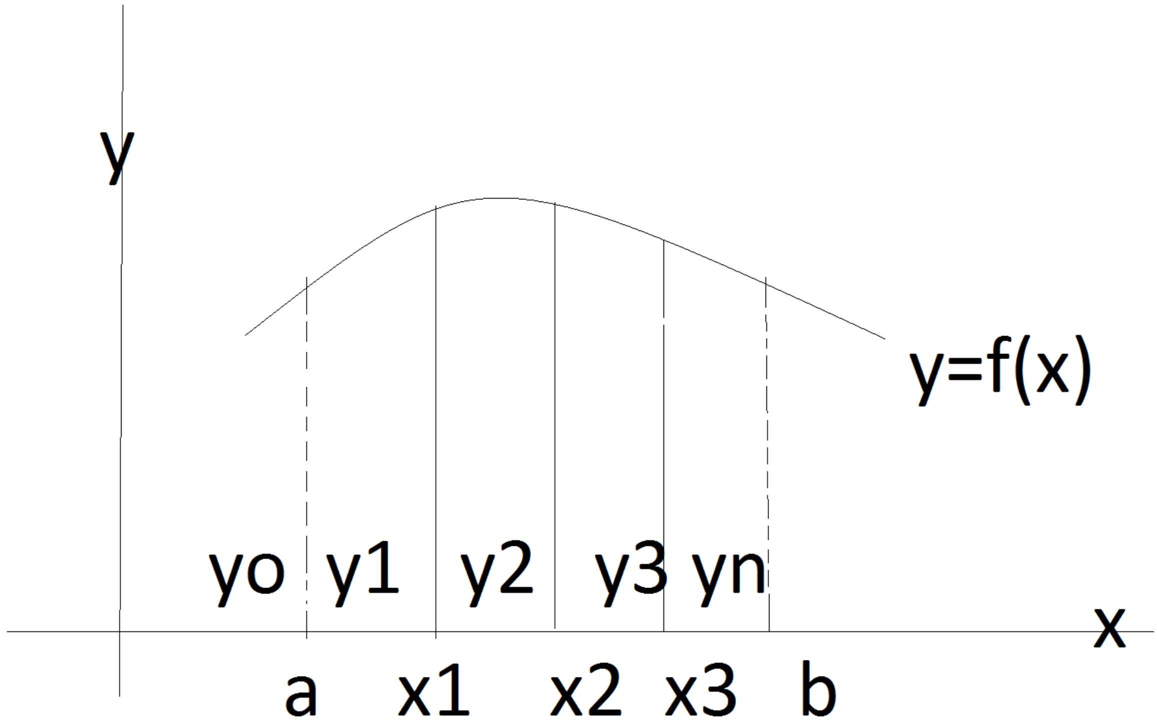
S=$[t^3 - t^2 - 3t]_3^4$

S=(64-16-12)-(27-9-9)

S=27m

SIMPSON'S RULE

Another rule for numerical integration is attributed to Thomas Simpson (1710-1761) an English Mathematician.



By simpson's rule the interval $a \leq x \leq b$ *is divided into an even number* n *of subintervals of length*

$$h = \frac{b-a}{n}$$

with equally spaced points

$a=x_o, x_1, x_2, x_4, \ldots x_n=b$

and their corresponding ordinates at $y_0, y_1, y_2 \ldots, y_n$, Simpson showed that $\int_a^b f(x)dx \simeq \frac{1}{3}h(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_{n-2} + 4y_{n-1} + y_n$

this can also be written as

$$\int_a^b f(x)dx = \frac{1}{3}h[(y_0 + y_n) + 4(y_1 + y_3 + \cdots + y_{n-1} + 2(y_2 + y_4 + y_{n-2})]$$

EXAMPLE:

Using Simpson's rule with 8 strips, evaluate

$$\int_1^5 \frac{1}{x} dx$$

Correct to 2 decimal places.

SOLUTION

$\boldsymbol{The\ integration\ interval = 5 - 1 = 4}$
$\boldsymbol{i.e.\, b - a = 4 \quad n = 8}$
h= $\boldsymbol{\frac{b-a}{8} = \frac{4}{8} = 0.5}$
The working is set in a tabular form as follows:

x	y	First last ordinates	Odd ordinates	Remaining ordinates
1	**Y_0**	**1**		
1.5	**Y_1**		**0.67**	
2.0	**Y_2**			**0.50**
2.5	**Y_3**		**0.40**	
3.0	**Y_4**			**0.33**
3.5	**Y_5**		**0.29**	
4.0	**Y_6**			**0.25**
4.5	**Y_7**		**0.22**	
5.0	**Y_8**	**0.2**		
totals		**1.2**	**1.58**	**1.08**

$$\int_1^5 \frac{1}{x} dx \simeq \frac{1}{3} \times 0.5[1.2 + 4(1.48) + 2(1.08)]$$

$$\int_1^5 \frac{1}{x} dx \simeq \frac{1}{3} \times 0.5[1.2 + 6.32 + 2.16]$$

≃1.613

Hence, $\int_1^5 \frac{1}{x} dx \simeq 1.61 \quad (2d.p)$

EXAMPLE 2:

Using Simpson's rule with 4 strips, evaluate

$$\int_2^6 2^x dx$$

Correct to 2 decimal places.

SOLUTION

$$b - a = 6 - 2 = 4 \quad n = 4$$
$$h = \frac{b-a}{n} = \frac{4}{4} = 1$$

X	Y	First last ordinates	Odd ordinates	Remaining ordinates
2	Y_0	4		
3	Y_1		8	
4	Y_2			16
5	Y_3		32	
6	Y_4	64		
totals		68	40	16

$$\int_2^6 2^x dx = \frac{1}{3} \times 1[(y_0 + y_4) + 4(y_1 + y_3) + 2y_2$$
$$= \frac{1}{3}\big(68 + 4(40) + 2(16)\big)$$
$$= \frac{1}{3} \times 260$$
=86.67 (2 d.p.)

CLASS ACTIVITY

Using Simpson's rule with six strips, evaluate

$$\int_2^5 \frac{1}{x+1} dx$$

PRACTICE EXERCISE

1. Find the area of the finite region bounded by $y = x^2 - 2x - 3$, $x = -1$, $x = 3$ and the x – axis
2. Find the area of the finite region bounded by $y = x^2 - 2x - 3$, $x = 0$, $x = 5$ and the x – axis

3. Find the area of the finite region bounded by the curve $y = 9x^2$ the line $y = 1$, $y = 9$ and the y – axis.

   INTEGRATE THE FOLLOWING

4. $\int(x^3 + 3)2\,x^2$dx
5. $\int\ (2x + 1)^4$ dx

ASSIGNMENT

INTEGRATE THE FOLLOWING

1. $\int\ \frac{7dx}{(5x-4)^3}$
2. $\int\ \frac{6x^4-x^3-1}{x^2}$ dx
3. Find the equation of a curve with gradient given by 2x-3 and passes through the point (3,2)
4. A particle moves along a straight line in such a way that its acceleration after t seconds is (2t-1)cm/s^2.if its velocity after t seconds is vcm/s, find v in terms of t, given that v=9 and t=2.
5. Find the area enclosed by the curve $y=4+3x-x^2$ and the x-axis.
6. Evaluate

   $$\int_0^1 (2x+5)^3\,dx$$

7. **Evaluate**

   $$\int_0^1 \frac{7+\sqrt{x}}{x^2}\,dx$$

KEYWORDS: integrate, integral, tangent, gradient, differentiate , rate of change ,increase, velocity, acceleration, derived function etc.

WEEK 5 REVISION

WEEK 6 EXAMINATION