FIRST TERM: E-LEARNING NOTES
SCHEME FIRST TERM

S S S 3

WEEK	TOPIC	CONTENT
1	**SURDS**	(a) Meaning of rational and irrational numbers leading to the definition of surds. (b) the rules guiding the basic operation with surd i.e $\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$; $\sqrt{a} - \sqrt{b} \neq \sqrt{a-b}$; $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$: $\sqrt{a} \div \sqrt{b} = \sqrt{a/b}$. (c) conjugates of a binomial surd using the idea of the difference of two squares (d) Application to solving triangles involving trigonometric ratios of special angles 30^0, 60^0 , and 45^0 . (e) Evaluation of expressions involving surds.
2	**MATRICES AND DETERMINANT 1**	(a) Definition, order and notation of matrix. (b) Types of matrix. (c) Addition and subtraction of matrix. (d) Scalar multiplication of matrices
3	**MATRICES AND DETERMINANT 2**	(a) Multiplication of matrices. (b) Transpose of a matrix. (c) Determinant of 2x2 and 3x3 matrices. (d) Application to solving simultaneous linear equations in two variables.
4	**LOGARITHM**	(a) Revision of laws of indices. (b) Laws of logarithms. (c) Logarithmic equations.
5	**ARITHMETIC OF FINANCE**	(a) Simple interest (revision). (b) Compound interest. (c) Depreciation. (d) Annuities. (e) Amortization. (f) Further use of logarithm table in problem involving: (i) Bonds and Debentures (ii) shares (iii) Rates (iv) Income tax (v) Value added Tax.
6	**SURFACE AREA AND VOLUME OF SPHERE**	(a) Surface area of sphere. (b) Volume of sphere
7	**MID-TERM BREAK**	
8	**LONGITUDE AND LATITUDE**	(a) Earth as a sphere. (b) Identification of: (i) North and South poles. (ii) Longitudes (iii) Latitudes (iv) Small circles and great circles. (v) Meridian and equator. (vi) Parallel of Latitude. (vii) Radius of parallel of latitude (viii) Radius of Earth.
9	**LONGITUDE AND LATITUDE**	(a) Revision of: arc length of a curve. (b) Calculations of distance between two points on the earth; shortest distance between two points. (d) Nautical rules, time variation.

10	**REVISION**	
11	**EXAMINATION**	

CLASS: S.S 3
TOPIC: SURDS
CONTENT:

- Meaning of rational and irrational numbers leading to the definition of surds.
- the rules guiding the basic operation with surd i.e $\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$; $\sqrt{a} - \sqrt{b} \neq \sqrt{a-b}$; $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$: $\sqrt{a} \div \sqrt{b} = \sqrt{a/b}$.
- conjugates of a binomial surd using the idea of the difference of two squares
- Application to solving triangles involving trigonometric ratios of special angles 30^0, 60^0 , and 45^0 .
- Evaluation of expressions involving surds.

MEANING OF RATIONAL AND IRRATIONAL NUMBERS LEADING TO THE DEFINITION OF SURDS

Rational numbers (Fractions): rational numbers is any number that can be expressed as a ratio of two integers (i.e can be expressed as a fraction in the form $\frac{a}{b}$ where a and b are in integers and where b ≠0. Any integer can be expressed as $\frac{a}{1}$, hence integers are rational numbers such as $\frac{1}{3}, \frac{5}{17}, \frac{7}{10} \frac{-4}{7}, \frac{9}{1}$ etc are rational numbers. Therefore Natural numbers are subsets of Integers while Integers are subset of Rational numbers$N \subset Z \subset Q$.

Examples are:

(i) Proper and improper fractions: $\frac{3}{4,}, \frac{2}{3}\, and\, \frac{14}{9} \frac{17}{10}$

(ii) Mixed numbers: $2\frac{3}{4}$, $5\frac{3}{7}$

(iii) Integers i.e counting numbers : 0 = $\frac{0}{1}$, $6\frac{6}{1}, -9 = \frac{-9}{1}$

(iv) Terminating decimals, eg$0.8 = \frac{8}{10} - 0.13 = \frac{-35}{100}$

(v) Recurring decimals, eg $0.1 = \frac{1}{9}, 0.13 = \frac{2}{15}\ etc$

(vi) Roots such as $\sqrt{4} = 2 = \frac{2}{1} and \sqrt{25} = 5 = \frac{5}{1}$

The square roots of these fractional numbers, referred as surds, results in irrational or non-rational numbers. Irrational/non-rational numbers are numbers when expressed as decimals neither repeat (recur) nor end (terminate). An irrational number cannot be written as a ratio of two integers, e.g $\pi = 3.141\ 592\ 654 \ldots\ and\ e = 2.718\ 282\ 828 \ldots\ (exponential)$

They are irritation because they do not have exact roots eg $\sqrt{2} = 1.414\ 213 \ldots, \sqrt{12} = 3.464\ 101 \ldots$

Note:

(i) All multiples of irrational numbers are irrational e.g $5\sqrt{3}, 2\sqrt{5}\ 4\sqrt{11}$ etc

(ii) All fractions of irrational numbers are irrational e.g$\sqrt{5}/_2, \frac{3\sqrt{2}}{5}, \frac{3\sqrt{11}}{3}\ etc$

Therefore, surd is the word that is used to refer to the square roots of numbers that are not perfect squares.

THE RULES GUIDING THE BASIC OPERATION WITH SURDS

1. $\sqrt{X} \times \sqrt{Y} = \sqrt{XY}$ e.g $\sqrt{9} \times \sqrt{4} = \sqrt{9 \times 4} = \sqrt{36} = 6$
   $\sqrt{9} \times \sqrt{4} = 3 \times 2 = 6$
2. $\sqrt{X} + \sqrt{Y} \neq \sqrt{XY}$ e.g $\sqrt{9} + \sqrt{4} \neq \sqrt{9+4} = \sqrt{13} =$
   $\sqrt{9} + \sqrt{4} = 3 + 2 = 5$
3. $\sqrt{X} \div \sqrt{Y} = \sqrt{X \div Y}$ e.g $\sqrt{16} \div \sqrt{4} = \sqrt{16 \div 4} = \sqrt{4} = 2$
   $\sqrt{16} \div \sqrt{4} = 4 \div 2 = 2$
4. $\sqrt{X} - \sqrt{Y} \neq \sqrt{X - Y}$ e.g $\sqrt{9} - \sqrt{4} \neq \sqrt{9-4} = \sqrt{5} =$
   $\sqrt{9} - \sqrt{4} = 3 - 2 = 1$
5. $\sqrt{X} + \sqrt{X} = 2\sqrt{X}$ e.g $\sqrt{2} + \sqrt{2} = 2\sqrt{2}$
6. $2\sqrt{X} + 3\sqrt{X} = 5\sqrt{X}$ e.g $2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$
7. $\sqrt{X} + \sqrt{Y} = \sqrt{X}$ e.g $\sqrt{2} + \sqrt{3} = \sqrt{2} + \sqrt{3}$
8. $\sqrt{X} \times \sqrt{X} = \sqrt{X^2} = X^{2\times\frac{1}{2}} = X$ e.g $\sqrt{5} \times \sqrt{5} = \sqrt{5^2} = 5^{2\times\frac{1}{2}} = 5$

Like surds*: Two or more surds are said to be like surds if the number under the square root sign are the same eg*$\sqrt{3}$, $7\sqrt{3}$, $\frac{\sqrt{3}}{5}$.

Examples

(1) We know that $\sqrt{36 \times 25} = \sqrt{900}$ ie 30. But $\sqrt{36} \times \sqrt{25} = 6 \times 5$ ie 30. This means $\sqrt{36 \times 25} = \sqrt{36} \times \sqrt{25}$. In general: $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

(2) We know that $\sqrt{\frac{36}{9}} = \sqrt{\frac{36}{9}} = \sqrt{4}$ i.e 2

But $\frac{\sqrt{36}}{\sqrt{9}} = \frac{6}{3}$ i.e 2.

Hence: $\sqrt{\frac{36}{9}} = \frac{\sqrt{36}}{\sqrt{9}}$

In general: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Class Activity

1. State which of the following pairs of expressions are equal
(a) $\sqrt{16} \times \sqrt{36} \times \sqrt{4}, \sqrt{16 \times 36 \times 4}$
(b) $\sqrt{17-14}, \sqrt{17} - \sqrt{14}$
(c) $\sqrt{64} + \sqrt{144}, \sqrt{64+144}$
(d) $\sqrt{\frac{18}{2}}, \frac{\sqrt{18}}{\sqrt{2}}$
2. If $a = 36, b = 9\ and\ c = -16$, work out the following pairs of expressions are equal.
(a) $\sqrt{a^2 b}, \sqrt{a^2} \times \sqrt{b}$
(b) $\sqrt{a+b}, \sqrt{a} + \sqrt{b}$
(c) $-c\sqrt{a}, \sqrt{ac^2}$
(d) $4\sqrt{c^2}, \sqrt{16c^2}$

ADDITION AND SUBTRACTION OF SURDS

We can only add or subtract surds which are alike or have the same form

Note: Reduce first to their basic forms if they are not

Examples:

1. Simplify the following:
   (a) $\sqrt{3} + \sqrt{3}$we have two $\sqrt{3}\ thus, 2\sqrt{3}\ Recall{:}\ x + x = 2x$
   $$\therefore \sqrt{3} + \sqrt{3} = 2\sqrt{3}$$
   (b) $\sqrt{4} + \sqrt{5} = 2 + \sqrt{5}$
   (c) $\sqrt{12} + \sqrt{48}\ ,\ \sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}\ ,\ \sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}$
   $$\therefore 2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$$
2. Simplify the following:
   (a) $4\sqrt{11} - 2\sqrt{11}\ , \quad = (4-2)\ 2\sqrt{11} = 2\sqrt{11}$
   (b) $4\sqrt{18} - \sqrt{200} + 3\sqrt{50}$
   $$= 4\sqrt{18} + 3\sqrt{50} - \sqrt{20} \quad = 4\sqrt{9 \times 2} + 3\sqrt{25 \times 2} - \sqrt{100 \times 2}$$
   $$= 4 \times 3\sqrt{2} + 3 \times 5\sqrt{2} - 10\sqrt{2} = 12\sqrt{2} + 15\sqrt{2} - 10\sqrt{2}$$
   $$= 27\sqrt{2} - 10\sqrt{2} \quad = 17\sqrt{2}$$
   (c) $3\sqrt{48} + \sqrt{192} + 3\sqrt{12} - \sqrt{147}\ = 3\sqrt{16 \times 3} + \sqrt{64 \times 3} + 3\sqrt{4 \times 3} - \sqrt{49 \times 3}$
   $$= 3 \times 4\sqrt{3} + 8\sqrt{3} + 3 \times 2\sqrt{3} - 7\sqrt{3}\ = 12\sqrt{3} + 8\sqrt{3} + 6\sqrt{3} - \sqrt{3}\ = 26\sqrt{3} - 7\sqrt{3}$$
   $$= 19\sqrt{3}$$

Class Activity

1. Simplify the following:
   (a) $\sqrt{5} + \sqrt{5} + \sqrt{5}$
   (b) $\sqrt{800} + \sqrt{200} - 2\sqrt{32}$
   (c) $4\sqrt{32} - \sqrt{192} + 3\sqrt{12} - \sqrt{147}$
   (d) $\sqrt{75} - 3\sqrt{48} + \sqrt{45} - 2\sqrt{12}$
   (e) $2\sqrt{12} - 5\sqrt{48} - \sqrt{75} - 3\sqrt{363}$
   (f) $\sqrt{28} - \sqrt{45} + \sqrt{175} - \sqrt{20} + \sqrt{245}$
   (g) $3\sqrt{2} + \sqrt{128} - \sqrt{27} - \sqrt{50} + \sqrt{75}$

MULTIPLICATION AND DIVISION OF SURDS

Note: To multiply surds:

(a) Simplify the surds, if possible.

(b) Group the numbers together, coefficient of surds together and then multiply.

(c) Simplify further if possible but divide surds:

(i) simplify the fraction if necessary.

(Ii) If the denominator has a surd, then rationalize it i.e to eliminate the surd in the denominator by multiplying both the numerator and the denominator of the fraction by the surd in the denominator. This will make the denominator a rational number.

(Iii)Simplify further if possible.

Examples:

1. Simplify the following:

(a) $\sqrt{24} \times \sqrt{72} \times 3\sqrt{5} = \sqrt{4 \times 6} \times \sqrt{36 \times 2} \times 3\sqrt{5} = 2\sqrt{6} \times \sqrt{2} \times 3\sqrt{5} = 2 \times 6 \times 3\sqrt{6} \times \sqrt{2} \times \sqrt{5} = 36\sqrt{60} = 36\sqrt{4 \times 15} = 36 \times 2\sqrt{15} = 72\sqrt{15}$

(b) $\sqrt{32} \times \sqrt{576} \times (\sqrt{4})^3 = \sqrt{16 \times 2} \times 24 \times 2^3 = 4\sqrt{2 \times 24} \times 8$

$$= 768\sqrt{2}$$

(c) $(96 \times 90)^{\frac{1}{2}}$

$$= \sqrt{96 \times 90} = \sqrt{48 \times 2 \times 90} = \sqrt{16 \times 6 \times 9 \times 10} = 4 \times 3\sqrt{6 \times 10} = 12\sqrt{60}$$

$$= 12\sqrt{4 \times 15} = 12 \times 2\sqrt{15} = 24\sqrt{15}$$

2. Simplify the following:

(a) $\frac{2}{\sqrt{11}} = \frac{2}{\sqrt{11}} \times \frac{11}{\sqrt{11}} = \frac{2\sqrt{11}}{11}$

(b) $\sqrt{\frac{25}{3}} = \sqrt{\frac{25}{1}} \times \sqrt{\frac{1}{3}} = \frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \times \frac{5\sqrt{3}}{3}$

(c) $\frac{\sqrt{7} \times 5\sqrt{44} \times \sqrt{12}}{\sqrt{20} \times \sqrt{77}}$

$$= \frac{\sqrt{7} \times 5\sqrt{4 \times 11} \times \sqrt{4 \times 3}}{\sqrt{4 \times 5} \times \sqrt{11 \times 7}}$$

$$= \frac{\sqrt{7} \times 5 \times 2\sqrt{11} \times 2\sqrt{3}}{2\sqrt{5} \times \sqrt{11} \times \sqrt{11} \times \sqrt{7}}$$

$$= \frac{5 \times 2\sqrt{3}}{\sqrt{5}}$$

$$= \frac{10\sqrt{3} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}}$$

$$= \frac{10 \times \sqrt{15}}{5}$$

$$= 2\sqrt{15}$$

(d) $(\sqrt{2} - 2\sqrt{3})(3\sqrt{2} + 4\sqrt{3})$

Recall: $a(x + y) = ax + ay$

Also $(a + b)(x + y) = ax + ay + bx + by$

Then: $(\sqrt{2} - 2\sqrt{3})(3\sqrt{2} + 4\sqrt{3}) - (2\sqrt{3} \times 4\sqrt{3})$

$$= 3\sqrt{4} + 4\sqrt{6} - 6\sqrt{6} - 8\sqrt{9}$$

$$= 3\sqrt{4} - 2\sqrt{6} - 8 \times 3$$

$$= 3 \times 2 - 2\sqrt{6} - 24$$

$$= 6 - 2\sqrt{6} - 24$$

$$= -18 - 2\sqrt{6}$$

(e) $(\sqrt{6.4} - \sqrt{2.5})^2$

$$= (\sqrt{6.4} - \sqrt{2.5})(\sqrt{6.4} - \sqrt{2.5})$$

$$= (\sqrt{6.4})^2 - \sqrt{6.4 \times 2.5} - \sqrt{2.5 \times 6.4} + (\sqrt{2.5})^2$$

$$= 6.4 - 2\sqrt{6.4} \times 2.5 + 2.5 = 6.4 - 2\sqrt{16} + 2.5$$

$$= 6.4 - 2 \times 4 + 2.5 = 6.4 - 8 + 2.5 = 8.9 - 8$$

$$= 0.9\ or\ (\sqrt{6.4} - \sqrt{2.5})^2 (perfect\ square)$$

$$= 6.4 - 2\sqrt{6.4} \times \sqrt{2.5} + (\sqrt{2.5})^2 = 6.4 - 2\sqrt{16} + 2.5\ (same\ as\ above)$$

$$= 6.4 - 2 \times 4 + 2.5 = 0.9$$

Class Activi

1. Simplify the following:
   (a) $\sqrt{2} \times \sqrt{3} \times \sqrt{5} \times \sqrt{15} \times \sqrt{20} \times \sqrt{60}$
   (b) $\sqrt{162} \times (\sqrt{2})^3$
   (c) $(54 \times 18)^{\frac{1}{2}}$
   (d) $\frac{2\sqrt{50} \times 3\sqrt{21}}{5\sqrt{2} \times 7\sqrt{42}}$
   (e) $\sqrt{48} - \frac{9}{\sqrt{3}} + \sqrt{75}$
   (f) $(3\sqrt{a} - 5\sqrt{a})(3\sqrt{a} + 5\sqrt{a})$
   (g) $(2\sqrt{3} + \sqrt{2})^2$
2. Evaluate without using tables:

$$(5\sqrt{2.5} - 10\sqrt{10})(\sqrt{0.4})[WAEC]$$

PRACTICE EXERCISE

1. Simplify the following surds expressions
   (a) $(4\sqrt{27} - 2\sqrt{8} + 3\sqrt{48} + 3\sqrt{94}$

   (b) $6\sqrt{7} - \sqrt{4} \times 7 - 5\sqrt{7} + \sqrt{245}$

   (c) $4\sqrt{3} - 2\sqrt{20} + \sqrt{108} + 3\sqrt{125}$

2. Expand and simplify the following:
   (a) $(5\sqrt{2} - 3\sqrt{5})(3\sqrt{2} + 3\sqrt{5})$
   (b) $(2\sqrt{5} + 3\sqrt{2})(2\sqrt{5} - 3\sqrt{2})$
3. (a) $3\sqrt{2} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{8}}$
   (b) $\frac{4\sqrt{5}\times 3\sqrt{2}}{\sqrt{32}\times\sqrt{100}}$

CONJUGATE OF BINOMIAL SURDS USING THE IDEA OF DIFFERENCE OF TWO SQUARES

A binomial surd is a surd expression that is made up of only two terms, example 3+√(2,) √5-2√(7,) 3√2+5,etc.

To rationalize a binomial surd, we use the numerator and the denominator to multiply both the numerator and the denominator.

√a+ √(b) and √a–√(b) are said to be conjugate surds because when they are multiplied together, the result gives a rational number. It operates in this way:

$(a+b)(a-b)= a^2-ab+ab+b^2$

$= a^2-b^2$ (difference of two squares)

$$\therefore = (\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2$$

$$= a - b$$

Examples

1. $\boldsymbol{Simplify\ the\ following}: (\boldsymbol{a})\quad (\mathbf{5}\sqrt{2} + 2\sqrt{8})(5\sqrt{2} - 2\sqrt{2})$

$= (5\sqrt{2})^2 - (2\sqrt{8})^2$

$= 5^2 \times \sqrt{4} - 2^2 \times \sqrt{64}$

$= 25 \times 2 - 4 \times 8$

$= 50 - 32$

$= 18$

$(\boldsymbol{b})\quad (3\sqrt{4} - 2)(3\sqrt{4} + 2)$

$= (3\sqrt{4})^2 - (2^2)$

$= 3^2 \times \sqrt{4 \times 4} - 2 \times 2$

$= 9 \times 4 - 4$

$= 36 - 4$

$= 32$

$2(a)\quad (4\sqrt{5} - \sqrt{9})(4\sqrt{5} - \sqrt{9})$

$sNote: (a-b)(a-b)$

$= a^2 - ab - ab + b^2$

$$= a^2 - 2ab + b^2$$

$$\rightarrow (4\sqrt{5})^2 - 2(4\sqrt{5})(\sqrt{9}) + (\sqrt{9})^2$$

$$= 4^2 \times (\sqrt{5})^2 - 2(4 \times \sqrt{45}) + (\sqrt{9})^2$$

$$= 16 \times 5 - 8 \times \sqrt{45} + 9$$

$$= 80 - 8 \times \sqrt{9} \times \sqrt{5} + 9$$

$$= 80 - 8 \times 3\sqrt{5} + 9$$

$$= 80 + 9 - 24\sqrt{5}$$

$$= 89 - 24\sqrt{5}$$

(b) $(\sqrt{0.81} - \sqrt{36})^2$

$$= (\sqrt{0.81} - \sqrt{36})(\sqrt{0.81} - \sqrt{36})$$

$$= (\sqrt{0.81})^2 - 2(\sqrt{0.81})(\sqrt{36}) + (\sqrt{36})^2$$

$$= 0.6561 - 2(0.9 \times 6) + 36$$

$$= 0.6561 - 2(5.4) + 36$$

$$= 36.6561 - 10.8$$

$$= 25.8561$$

(3) $(2\sqrt{54} + \sqrt{24})(\sqrt{6} + 3\sqrt{63})$

$$(2\sqrt{54})(\sqrt{6}) + (2\sqrt{54})(3\sqrt{63}) + (\sqrt{24})(\sqrt{6}) + (\sqrt{24})(3\sqrt{63})$$

$$= (2\sqrt{9 \times 6})(\sqrt{6}) + 2(\sqrt{9 \times 6})(3\sqrt{9 \times 7}) + (\sqrt{4 \times 6})(\sqrt{6}) + (\sqrt{4 \times 6})(3\sqrt{9 \times 7})$$

$$= 2 \times 3 \times 6 + 2 \times 3\sqrt{6} \times 3 \times 3\sqrt{7} + 2 \times 6 + 2 \times 3 \times 3 \times \sqrt{6} \times \sqrt{7}$$

$$= 36 + 12 + 2 \times 3 \times 3 \times 3 \times \sqrt{6} \times \sqrt{7} + 2 \times 3 \times 3 \times \sqrt{6} \times \sqrt{7}$$

$$= 48 + 54\sqrt{42} + 18\sqrt{42}$$

$$= 48 + 72\sqrt{42}$$

(4) *Simplification by rationalising the denominator*

$$\frac{2\sqrt{5} + \sqrt{3}}{3\sqrt{5} - \sqrt{3}}$$

$$= \frac{2\sqrt{5} + \sqrt{3}}{3\sqrt{5} - \sqrt{3}} = \frac{2\sqrt{5} + \sqrt{3}}{3\sqrt{5} - \sqrt{3}} \times \frac{3\sqrt{5} + \sqrt{3}}{3\sqrt{5} + \sqrt{3}}$$

$$= \frac{30 + 2\sqrt{15} + 3\sqrt{15} + 3}{(3\sqrt{5} - \sqrt{3})(3\sqrt{5} + \sqrt{3})}$$

$$= \frac{33 + 5\sqrt{15}}{(3\sqrt{5})^2 - (\sqrt{3})^2}$$

$$= \frac{33 + 5\sqrt{15}}{3^2 (\sqrt{5})^2 - (\sqrt{3})^2}$$

$$= \frac{33 + 5\sqrt{15}}{9 \times 5 - 3}$$

$$= \frac{33 + 5\sqrt{15}}{42}$$

Class Activity

1. Simplify the following:

(a)$\left(\sqrt{3}(\sqrt{27}-2\sqrt{3}+\sqrt{6}\right)$

(b) $\left(\sqrt{12}+\sqrt{20}\right)\left(\sqrt{12}+\sqrt{3}\right)$

(c) $\left(\sqrt{0.25}-\sqrt{100}\right)^2$

(d) $\left(3\sqrt{5}-2\right)\left(3\sqrt{5}+2\right)$

2. **Rationalise the denominators of the following:**

   (a) $\frac{3\sqrt{5}+2\sqrt{3}}{2\sqrt{5}-3\sqrt{3}}$

   (b) $\frac{10}{4\sqrt{18}-3\sqrt{48}}$

3. $\boldsymbol{Express\ the\ following\ in\ the\ form\ a+b\sqrt{5}}$:

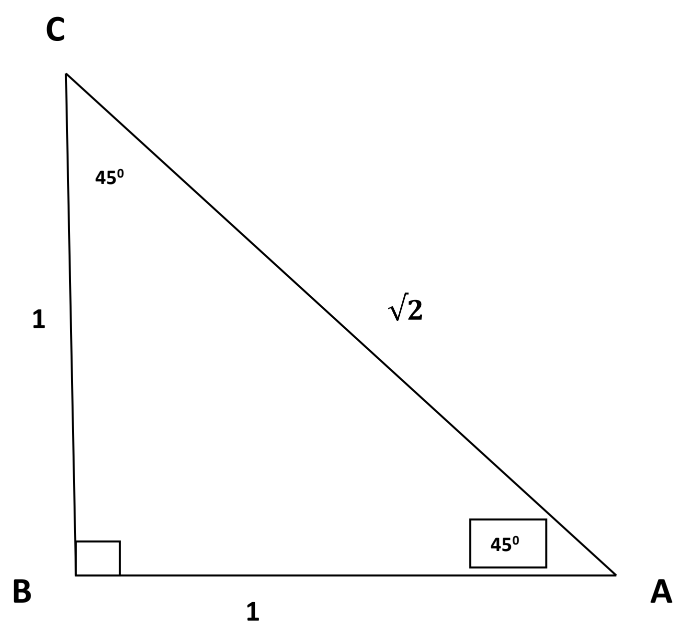
$$(\boldsymbol{a})\frac{\sqrt{5}}{\sqrt{7}+\sqrt{5}}$$

$$(\boldsymbol{b})\frac{\sqrt{3}-3\sqrt{2}}{2\sqrt{3}-2\sqrt{2}}$$

APPLICATION TO SOLVING TRIANGLES INVOLVING TRIGONOMETRIC RATIOS OF SPECIAL ANGLES $30°, 60, and\ 45°$.

In $\Delta ABC, there\ is\ a\ right\ angle\ at\ B\ and\ sides\ AB = BC = 1unit.$

Angle 45°



$$\left\{\begin{matrix} Note: AC^2 = AB^2 + BC^2 \\ = 1^2+1^2 \\ = 2 \\ \therefore AC = \sqrt{2}\, units. \end{matrix}\right\}$$

$$\therefore \text{Sin } 45° = \frac{opp}{hyp}$$

$$= \frac{1}{\sqrt{2}}$$

$$\text{Cos } 45° = \frac{adj}{hyp}$$

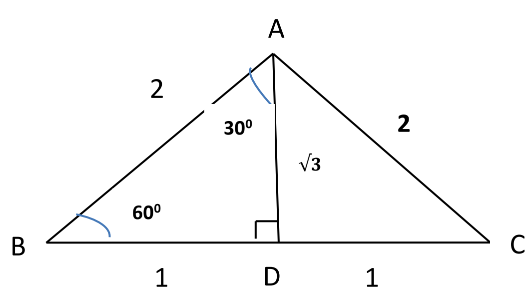
$$= \frac{1}{\sqrt{2}}$$

$$Tan\ 45° = \frac{opp}{adj}$$

$= \frac{1}{1}$ i.e. 1

Angles 30° *and* 60°

ΔABC *is an equilateral of side* 2*units*. *Ad is the altitude or height.*



$$In\ \Delta ABD, < BAD = 30°$$

$$\rightarrow AD^2 = AB^2 - BD^2 \{Pythagoras\ theorem\}$$

$$= 2^2 - 1^2$$

$$= 3$$

$\therefore AD = \sqrt{3}$units

$$\therefore Sin\ 60° = \frac{\sqrt{3}}{2}$$

$$Cos\ 60° = \frac{1}{2}$$

$$Tan\ 60° = \sqrt{3}$$

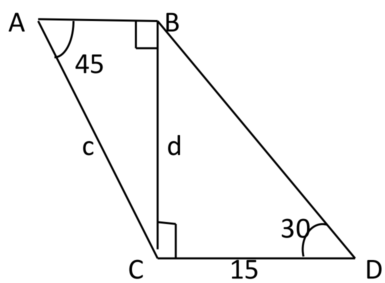
And $\sin 30° = \frac{1}{2}$

$$\cos 30° = \frac{\sqrt{3}}{2}$$
$$Tan\ 30° = \frac{1}{\sqrt{3}}$$

Examples

(1) Find the sides marked with letters. All answers must be left in surd form with rational denominators when necessary in cm.

(a)



Solution

$$In\ \Delta BCD,$$
$$Tan\ 30° = \frac{d}{15}$$
$$15tan30° = d$$
$$\frac{15}{1} \times \frac{1}{\sqrt{3}} = d$$
$$\frac{15 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

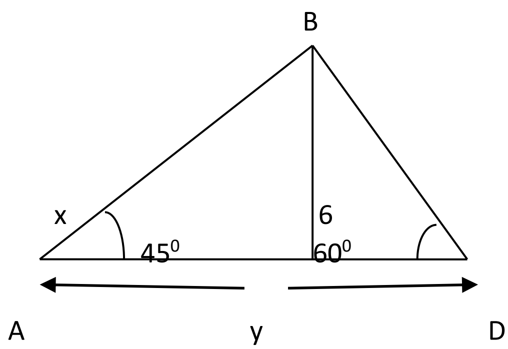
$\frac{15\sqrt{3}}{3}$ = d

$5\sqrt{3}$cm= d

$$In\ \Delta ABC$$
$$Sin\ 45° = \frac{d}{C}$$
$$Sin\ 45° = \frac{5\sqrt{3}}{C}$$
$$Csin45° = 5\sqrt{3}$$
$$C = \frac{5\sqrt{3}}{\frac{1}{\sqrt{2}}}$$
$$C = \frac{5\sqrt{3}}{1} \div \frac{1}{\sqrt{2}}$$
$$= \frac{5\sqrt{3} \times \sqrt{2}}{1}$$

$= 5\sqrt{6}$cm B

(b)



In $\Delta ABC, \sin 45° = \frac{6}{x}$

$$x sin 45° = 6$$

$$x = \frac{6}{\frac{1}{\sqrt{2}}}$$

$$= \frac{6}{1} \div \frac{1}{\sqrt{2}}$$

$$= \frac{6}{1} \times \frac{\sqrt{2}}{1}$$

$$= 6\sqrt{2} cm$$

In $\Delta ABC, \cos 45° = \frac{a}{x}$

$$x cos 45° = a$$

$$\frac{6\sqrt{2}}{1} \times \frac{1}{\sqrt{2}} = a$$

$$\frac{6\sqrt{2} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = a$$

$$\frac{6 \times 2}{2} = a$$

$$6 cm = a$$

$$In\ \Delta BCD, Tan\ 60° = \frac{6}{b}$$

$$b tan 60° = 6$$

$$b = \frac{6}{\tan 60°}$$

$$= \frac{6}{\sqrt{3}}$$

$$= \frac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

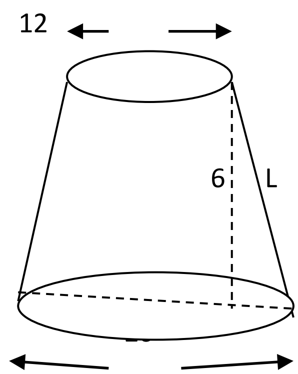
$$= \frac{6\sqrt{3}}{3}$$

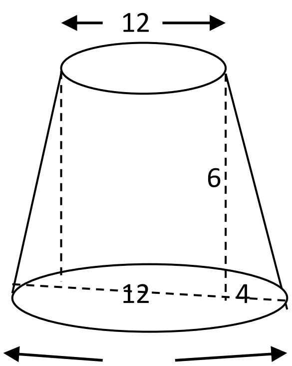
$$= 2\sqrt{3}cm$$

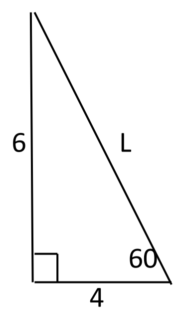
$$\therefore y = a + b$$

$$= 6 + 2\sqrt{3}cm$$

(C)







$In\ the\ \Delta,\qquad \sin 60° = \frac{6}{L}$

$$Lsin60° = 6$$

$$L = \frac{6}{sin60}$$

$$L = \frac{6}{\frac{\sqrt{3}}{2}}$$

$$= \frac{6}{1} \div \frac{\sqrt{3}}{2}$$

$$= \frac{6}{1} \times \frac{2}{\sqrt{3}}$$

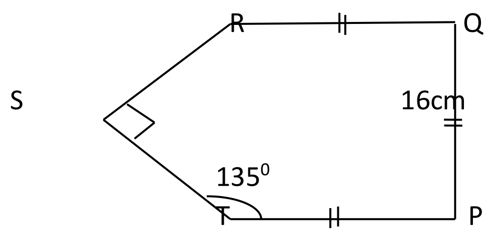
$$= \frac{12 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$= \frac{12\sqrt{3}}{3}$$
$$= 4\sqrt{3}cm$$

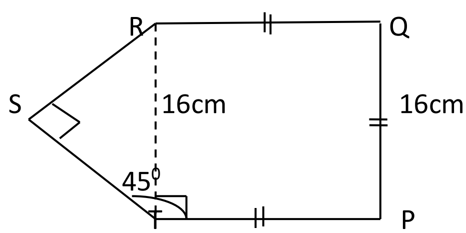
OR

$$Cos\ 30° \quad = \quad \frac{6}{L}$$
$$Lcos30° \ = \ 6$$
$$L \qquad = \ \frac{6}{cos30°} = \frac{6}{1} \times \frac{2}{\sqrt{3}}$$
$$= \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$= 4\sqrt{3}$$

$(d) PQRST\ is\ a\ pentagon. PQ = QR = PT = 16cm, and\ P\hat{T}S = 135°. Calculate\ RS\ and\ ST.$



Solution



$$In\ \Delta STR,$$
$$Sin\ 45° = \frac{RS}{16}$$
$$16sin45° = RS$$
$$\frac{16}{1} \times \frac{1}{\sqrt{2}} = RS$$
$$\frac{16 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} \ = RS$$
$$\frac{16\sqrt{2}}{2} = RS$$
$$8\sqrt{2}RS$$

Also, $\sin 45° = \frac{ST}{16}$

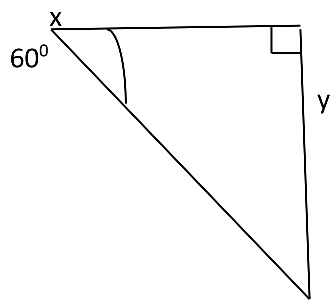
$$\frac{16 \times 1}{\sqrt{2}} ST$$

$$8\sqrt{2} = ST$$

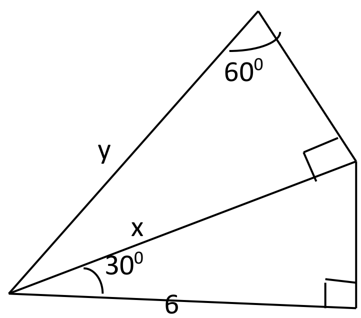
Class Activity

$\boldsymbol{F}ind\ the\ sides\ marked\ with\ letters.\ All\ lenghts\ are\ in\ cm.$

(a)



(b)



EVALUATION OF EXPRESSIONS INVOLVING SURDS

This topic exonerates the use of calculators or tables. When evaluating an expression with surds, it is useful to rationalise the denominator.

Examples

$(1) Given\ that\ \sqrt{2} = 1.414, \sqrt{3} = 1.732\ and\ \sqrt{5} = 2.236, without\ using\ tables\ os\ a\ calculator,$
$evaluate\ the\ following\ to\ 2\ d.p.$

$$(a) \frac{5}{\sqrt{3}}$$

__Solution__

$$\frac{5}{\sqrt{3}} = \frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{5\sqrt{3}}{3}$$

$$= \frac{5 \times 1.732}{3}$$

$$= \frac{8.660}{3}$$

$$= 2.886$$

$$= 2.89\ (2\ d.p)$$

$$(b)\frac{1}{\sqrt{45}}$$

Solution:

$$\frac{1}{\sqrt{45}} = \frac{1}{\sqrt{9 \times 5}}$$

$$= \frac{1}{3\sqrt{5}}$$

$$= \frac{1}{3 \times \sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{\sqrt{5}}{3 \times 5}$$

$$= \frac{2.236}{15}$$

$$= 0.149$$

$$= 0.15\ (2.\ dp)$$

(c) $\sqrt{\frac{4\times0.27\times3}{6\times2-4}}$

Solution

$=\sqrt{\frac{4\times0.9\times0.3\times3}{12-4}}$

$=\frac{2\times0.3\times0.3}{\sqrt{8}}$

$= \frac{2\times0.3\times0.3}{\sqrt{4}\times\sqrt{2}}$

$= \frac{1.8}{2\sqrt{2}}$

$$= \frac{1.8 \times \sqrt{2}}{2 \times \sqrt{2} \times \sqrt{2}}$$

$$= \frac{1.8 \times 1.414}{4}$$

$$= \frac{2.5452}{4}$$

$$= 0.6363$$

= 0.64

(d) $\left[\frac{\sqrt{50}}{\sqrt{162}-\sqrt{98}}\right]^2$

Solution

$\left[\frac{\sqrt{50}}{\sqrt{162}-\sqrt{98}}\right]^2 = \left(\frac{\sqrt{25}\times2}{\sqrt{81\times2}-\sqrt{2\times49}}\right)^2$

$$= \left(\frac{5\sqrt{2}}{9\sqrt{2}-7\sqrt{2}}\right)^2$$

$$= \left(\frac{5\sqrt{2}}{2\sqrt{2}}\right)^2$$

$$= \left(\frac{5\sqrt{2}\times\sqrt{2}}{2\times\sqrt{2}\times\sqrt{2}}\right)^2$$

$$= \left(\frac{5\times 2}{2\times 2}\right)^2$$

$= \frac{25}{4}$ i.e. $6\frac{1}{4}$ or 6.25

Examples

1. $Evaluate\ \sqrt{20}\times\left(\sqrt{5}\right)^3$ (WAEC)
2. $Without\ using\ a\ calculator\ or\ Mathematic\ tables, evaluate$:

   $(a)\frac{3}{\sqrt{3}}\left(\frac{2}{\sqrt{3}}-\frac{\sqrt{12}}{6}\right)$ (WAEC)

   (b) $\sqrt{1.225} = 1.107, \sqrt{12.25} = 3.5\ and\ \sqrt{100}$ = 10. Evaluate $\sqrt{1225}$ (WAEC)

   $$(c)\left(\frac{10\sqrt{32}}{\sqrt{18}-\sqrt{2}}\right)^2$$
3. $Given\ that\ \sqrt{2} = 1.414, \sqrt{3} = 1.732\ and\ \sqrt{5} = 2.236, without\ using\ tables\ or\ a\ calcu-lator, evaluate\ the\ following\ to\ 2\ d.p.$

   $$(a)\frac{2}{\sqrt{8}}$$

   $$(b)\sqrt{3}\left(\sqrt{9}+3\sqrt{27}\right)$$

   $$(c)\sqrt{\frac{2}{5}}\left(\sqrt{7.5}+\sqrt{30}\right)\sqrt{2}$$

PRACTICE EXERCISE

(1) $\boldsymbol{Ra}tionalize\ \frac{5\sqrt{7}-7\sqrt{5}}{\sqrt{7}-\sqrt{5}}$ (JAMB)

(2) $\boldsymbol{Simplify}$:

$\sqrt{50}-3\sqrt{2}\left(2\sqrt{2}-5\right)-5\sqrt{32}$ (WAEC)

(3) $Simplify\ without\ using\ table\ \frac{5-3\sqrt{2}}{6+5\sqrt{2}}$ (WAEC)

. $XYZ\ is\ an\ isosceles\ triangle\ with\ |XY| = |XZ| = 6cm\ and\ Y\hat{X}Z = 120°. Calculate\ the\ Length\ of\ \overline{YZ}.(WAEC).$

(4) From the top of a vertical mast 150m high,two huts on the same ground level are observed one due East and the other due West of the mast.Their angles of depression

are 60° and 45°,respectively.Find the distance between the huts.(JAMB).

(5) Theangle of elevation of a building from a measuring intrument placed on the ground is 30°.If the building is 40m high,how far is the instrument from the foot of the building? (JAMB)

ASSIGNMENT

1. Simplify $\frac{5}{\sqrt{3}} - \frac{3}{\sqrt{2}}$

   A. $\frac{1}{6}(5\sqrt{3} - 3\sqrt{2})$B. $\frac{1}{6}(15\sqrt{3} - 6\sqrt{2})$C. $\frac{1}{6}(3\sqrt{2} - \sqrt{3})$D. $\frac{1}{6}(10\sqrt{3} - 9\sqrt{2})$ (SSCE 2005)

2. Given that $\sqrt{128} - \sqrt{18} - \sqrt{k} = 7\sqrt{2}$, find k.
   A. 8 B. 16 C.32 D.48 (SSCE 2004)
3. K$\sqrt{28} + \sqrt{63} - \sqrt{7} = 0$, find K.
   A. -2 B. -1 C. 1 D. 2 (SSCE 1999)

4. Given that $\sqrt{5} = 2.236$, evaluate $\frac{2}{\sqrt{5}}$ to 2 decimal places
   A. 0.89 B. 1.89 C. 0.98 D. 1.98

5. Simplify $\frac{3}{4\sqrt{6}+5}$
6. Simplify $\frac{5}{\sqrt{7}+\sqrt{3}} + \frac{7}{\sqrt{7}-\sqrt{3}}$

WEEK 2
SUBJECT: MATHEMATICS
CLASS: SS 3
TOPIC: MATRICES AND DETERMINANTS
CONTENT:

- Definition, order and notation of matrix.
- Types of matrix.
- Addition and subtraction of matrix.
- Scalar multiplication of matrices

DEFINITION, ORDER AND NOTATION OF MATRIX

TOPIC: ROOTS OF QUADRATIC EQUATIONS 2

SUB-TOPICS:

(a) Quadratic functions (Simultaneous Equations: One linear, One quadratic)
(b) Solution of problems on roots of quadratic equation.
(c) Maximum and minimum values.

SUB-TOPIC 1

Quadratic functions (Simultaneous Equations: One linear, one quadratic)

We have discussed different ways but we need to mention that graphical solution is very important aspect of solving quadratic equations. This is because with graphical solution a lot of other problems can be solved.

The graph of the quadratic equation called parabola. Some call it cup or cap. The quadratic expression is equated to y and it is called a ***quadratic function***. The example below show the graphical solution of quadratic function.

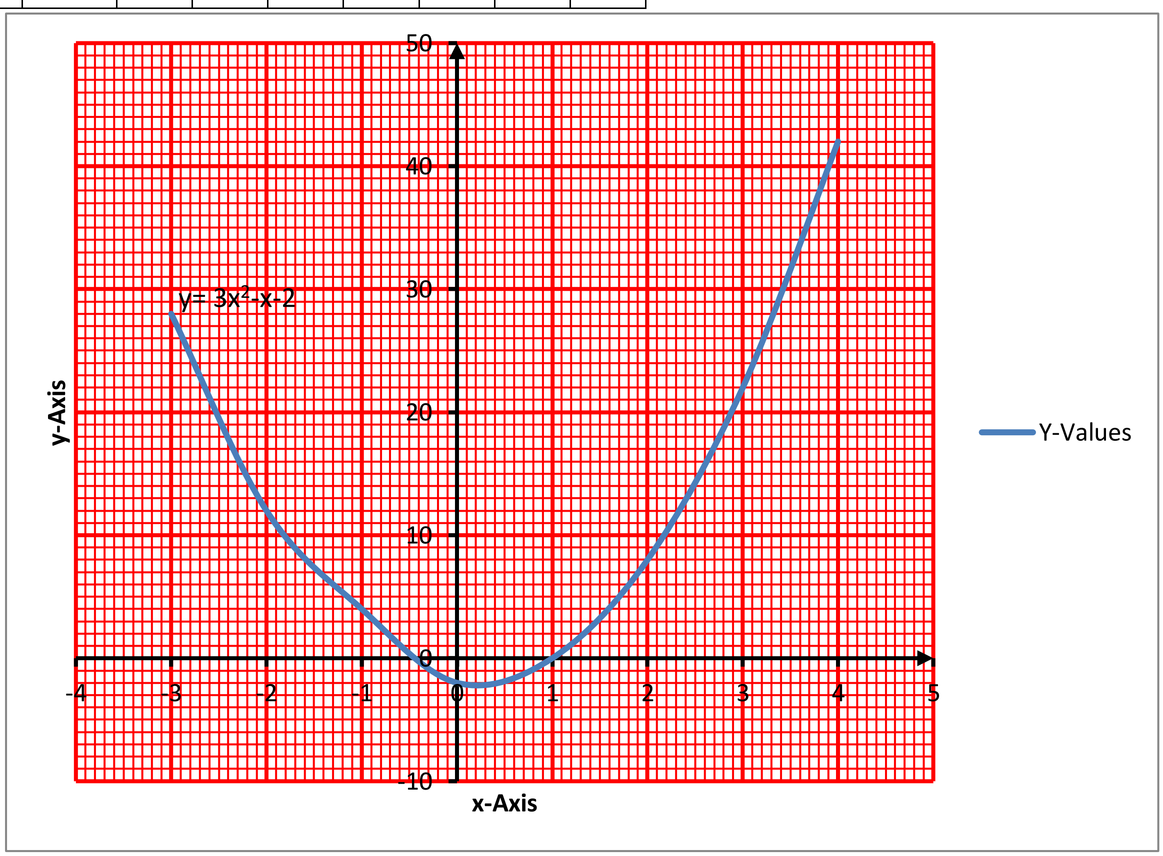
Example 1: Solve graphically, the equation $y = 3x^2 - x - 2$

Solution:

Draw the table of values for the equation $y = 3x^2 - x - 2$

x	-3	-2	-1	0	1	2	3	4
$3x^2$	27	12	3	0	3	12	27	48
$-x$	3	2	1	0	-1	-2	-3	-4
-2	-2	-2	-2	-2	-2	-2	-2	-2
y	28	12	4	-2	0	8	22	42

Choose a convient scale, on $x - axis$ and $y - axis$, on $x - axis$ let $2cm$ represents $1unit$ and on $y - axis$ $2cm$ represents $10units$.



From the graph we find the point here the curve intersects $x - axis$ at $x = -7$ and 1.

The graph is also useful to determine the minimum value of y. the minimum value of $y = -3$. we have minimum point when $a > 0$ and maximum point when $a < 0$.

Simultaneous Equations

When solving simultaneous equation (you are already used to solving it graphically). In situation where one equation is linear and the second is quadratic, it can be solved by substitution as well as solving graphically.

In graphical solution of one linear-one quadratic simultaneous equation, there are three possible relationships between the straight line (linear) and the parabola (quadratic). They are:

- Line intersecting with curve
- Line touching curve at a point (tangent)
- Line not intersecting the curve.

Example 2: Solve the simultaneous equations: $9x^2 - 4y^2 = 44 \quad and \quad 3x + 2y = 2$

Solution**:** By substitution:

$$9x^2 - 4y^2 = 44 \; \dots \dots \dots \dots \dots \dots \dots (i)$$

$$3x + 2y = 2 \dots \dots \dots \dots \dots \dots \dots (ii)$$

$$9x^2 - 4y^2 = 44 \Rightarrow (3x - 2y)(3x + 2y) = 44$$

Since $3x + 2y = 2$, then $(3x - 2y) \times 2 = 44 \dots \dots \dots \dots iii$

Hence, adding (ii) and (iii) together we get

$$6x - 24 \Rightarrow x = 4$$

From (ii) $3(4) + 2y = 2 \Rightarrow y = -5$.

Example 3: Given the simultaneous equations:

$y = x^2 + 5x + 6$ and $y = 2x + 3$

Show on the graph the points of interest. Hence write out the values of x and y.

Solution:

$$y = x^2 + 5x + 6$$

$$y = 2x + 3$$

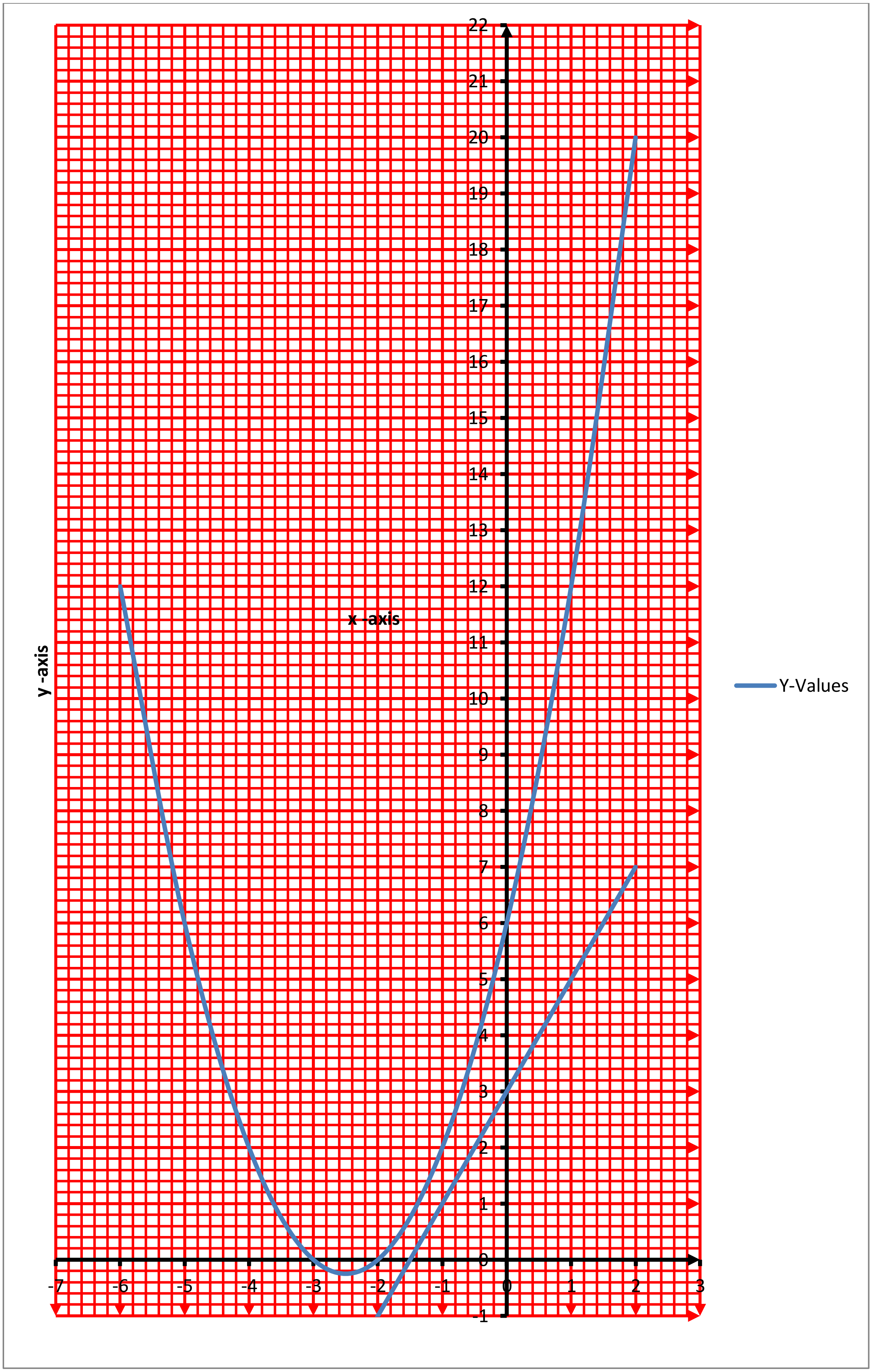
Table of values for $y = x^2 + 5x + 6$ and $y = 2x + 3$

$y = x^2 + 5x + 6$

x	-6	-5	-4	-3	-2	-1	0	1	2
x^2	36	25	16	9	4	1	0	1	4
$+5x$	-30	-25	-20	-15	-10	-5	0	5	10
$+6$	$+6$	$+6$	$+6$	$+6$	$+6$	$+6$	$+6$	$+6$	$+6$
y	12	6	2	0	0	2	6	12	20

$y = 2x + 3$

x	-4	-2	0	1	2
$2x$	-8	-4	0	2	4
$+3$	$+3$	$+3$	$+3$	$+3$	$+3$
y	-5	-1	3	5	7



x -axis
y -axis
Y-Values
22
21
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
-1
-7
-6
-5
-4
-3
-2
-1
0
1
2
3

From the above, there was no intersect of the curve and the straight line. The solutions to the two equations cannot be determined because there is no point of intersection.
The points of intersection give the solution.

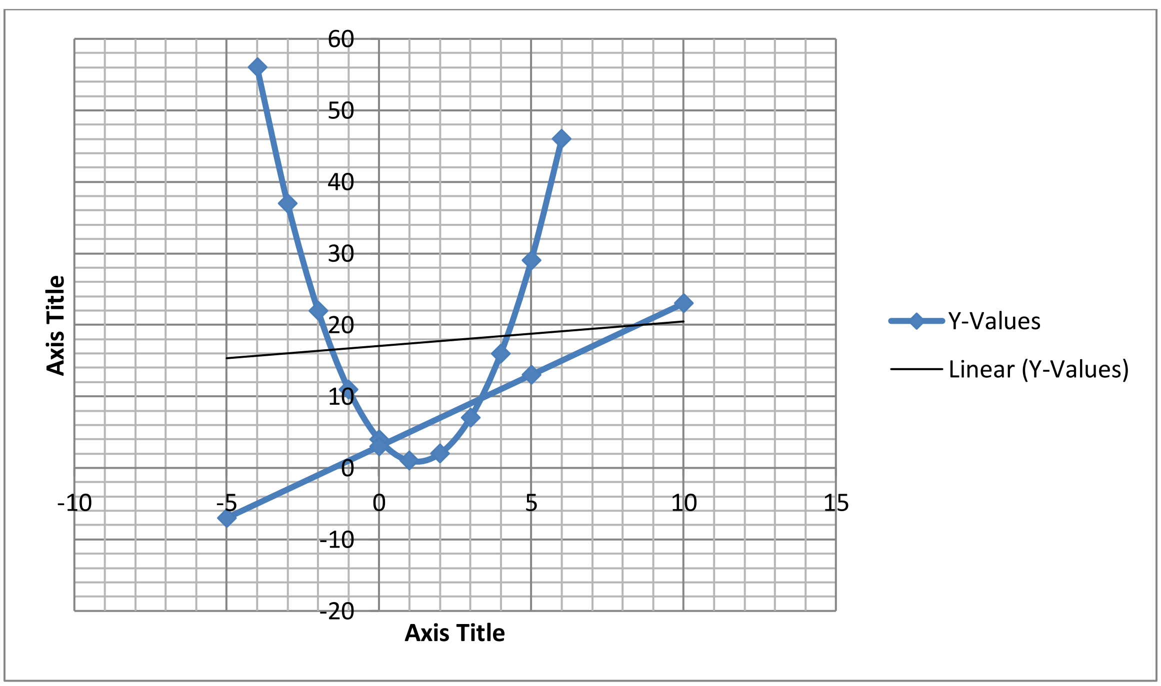
Example 3: On the same axes, plot the graph of $y = 2x^2 - 5 + 4$ and $y = 2x + 3$. Hence find the points of intersection of the two graphs.
Solution:

Prepare the table of values for the functions given above.

$y = 2x^2 - 5 + 4$

X	-4	-3	-2	-1	0	1	2	3	4	5	6
$2x^2$	32	18	8	2	0	2	8	18	32	50	72
-5x	20	15	10	5	0	-5	-10	-15	-20	25	-30
+4	+4	+4	+4	+4	+4	+4	+4	+4	+4	+4	+4
Y	56	37	22	11	4	1	2	7	16	29	46

Choose a convenient scale.



The points of intersections x = 0.2 and 3.3

The above example shows the case of the line intersecting with the curve.

Example 4: solve the simultaneous equation $y = x^2-2x + 2$ and $y = 4x -7$. Interpret your result geometrically.

Solution:

Eliminate y to obtain: $x^2-2x + 2 = 4x -7 \Rightarrow x^2 - 6x + 9 = 0$

By factorisation:

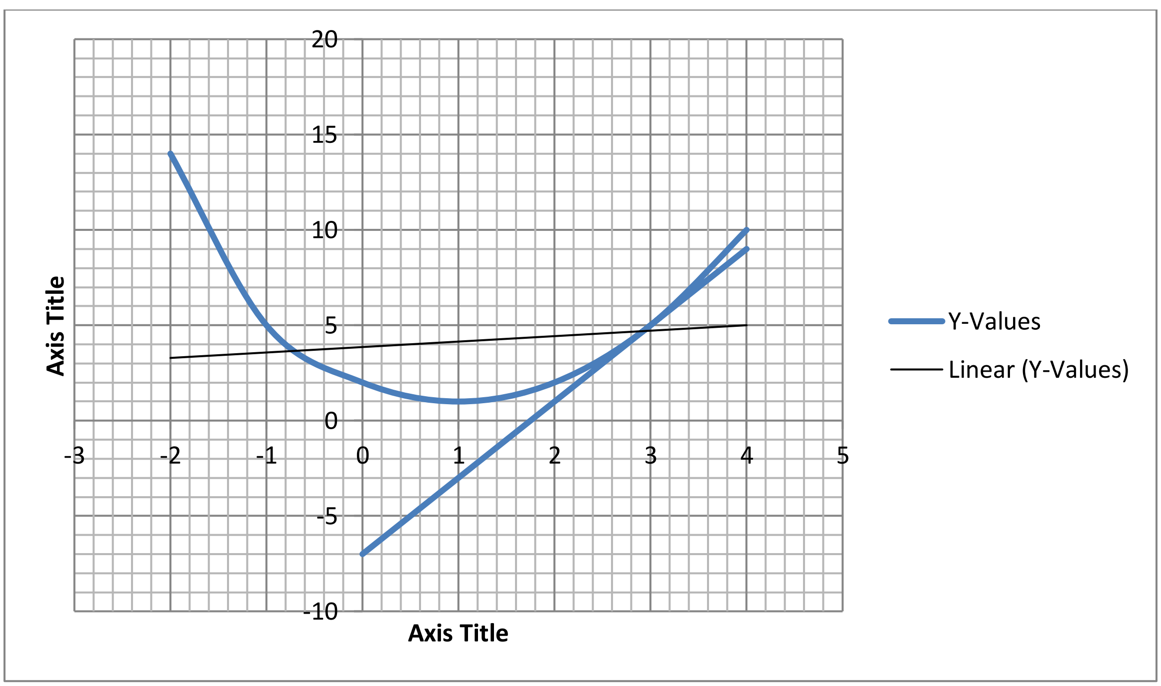
$(x - 3)(x - 3) = 0 \Rightarrow x = 3$(twice).

From $y = 4x - 7 = 4(3) -7 = 5$. The solution is $x = 3$ and $y = 5$.

Draw the graphs of the two equation to interpret it geometrically.

Table of values for $y = x^2 - 2x + 2$. $y = 4x - 7$

x	-2	-1	0	1	2	3	4			x	0	2	3	4
x^2	4	1	0	1	4	9	16			4x	0	8	12	16
-2x	8	2	0	-2	-4	-6	-8			-7	-7	-7	-7	-7
+2	+2	+2	+2	+2	+2	+2	+2			y	-7	1	5	9
y	14	5	2	1	2	5	10							



The line $y = 4x -7$ intersects the curve $y = x^2 + 2x + 2$ at only one point. Therefore, the solution to the equations is at the point $x = 3$ and $y = 7$.

Class activity

1. Solve the simultaneous equations y = 4x – 1 and y =$2x^2$ graphically and interpret your result geometrically.
2. Solve $y = 2x^2 - 9x - 1$ for -1≤ $x \leq 6$. Using a scale of 2cm to 1 unit on the x-axis and 2cm to represent 5 units on the y-axis.

SUB-TOPIC 2

Solutions of problems on roots of quadratic equation

Mathematics is important of life situation because of its application. You are used to problems leading to simple equations. We want to see the word problems leading to quadratic equations.

In order to solve such problems, you must take note of the following:

a. Express the ideas involved in mathematical symbols.
b. Write out the equation using the symbols.
c. Solve the equation.
d. Interprete your result.

Example 1: the product of two consecutive whole numbers is 506. Find the numbers.

Solution:

Let the numbers be x and (x + 1).

Then, x(x+1) = 506 ⇒ $x^2 + 1 = 506$ (this is now quadratic equation)

$X^2 + x - 506 = 0$

Solve by formula to find the values of x using the parameters below

a = 1 b = 1 c = -506

Example 2: There are two possible routes from Lagos to Ijebu Ode. One route is through Lagos/Ibadan express way which is 100km and the other is through Ikorodu-Epe covering a distance of 80km. A motorist going through express way can travel 10km per hour faster than the one going through Ikorodu and Epe and arrive Ijebu-Ode 5 minutes earlier as well. What is the time spent on the journey to Ijebu Ode by the motorist travelling through the express way?

Solution:

Let x be the speed of motorist going through Ikorodu/Epe and the speed of the one going through express way is x + 10.

Time taken by Ikorodu/Epe = 80/x.

Time taken by express way = 100/ (x + 10)

Hence, 80/x – 100/(x+10) = 1/12.

$$\frac{80(x+10)-100x}{x(x+10)} = \frac{1}{12} \Rightarrow \frac{80(x+10)-100x}{x(x+10)} = \frac{1}{12} \Rightarrow 12(80-20x) = x(x+10) \ldots\ldots\ldots\ldots\ldots i$$

Form a quadratic equation from (i) above and solve it using formula and conclude.

Class activity

1. The length of a rectangular field is 6m more than the width. If the area of the field is $72m^2$, find the dimensions of the field.
2. Two consecutive odd integers are such that the sum of their reciprocals is $\frac{8}{15}$. Find the odd integers.

SUB-TOPIC 3

Maximum and Minimum values

The graph of $y = ax^2 + bx + c$ as we have seen is a parabola. We have minimum point when $a > 0$ and maximum point when $a < 0$.
The maximum or minimum value (y) is

$$\frac{4ac - b^2}{4a}$$

The curve is symmetrical about the line $x = -\frac{b}{2a}$ which is called the ***axis of symmetry***.
If f(x) = 0, then,

i. the curve cuts the horizontal axis if $b^2 - 4ac > 0$
ii. the curve touches the horizontal axis if $b^2 - 4ac = 0$
iii. the curve does not cut the horizontal axis if $b^2 - 4ac < 0$

Example 1:

Find the minimum value of $y = 3x^2 + 5x - 2$ and the corresponding the value of x for which y is a minimum.

Solution:

$y = 3x^2 + 5x - 2$

$= 3\left[x^2 + \frac{5}{3}x\right] - 2 = 3[\left(x + \frac{5}{6}\right)^2 - \frac{25}{36}\right] - 2$

$3(x + \frac{5}{6})^2 - \frac{25}{12} - 2 = 3(x + \frac{5}{6})^2 - \frac{49}{12}$

When x = -5/6, the expression in the brackets will be zero, hence the minimum is -49/12.

The corresponding value of x for which y is minimum is -5/6.

Note that x = -5/6 is the axis of symmetry of the parabola. Alternative, let the minimum value of y be y_m then

$y_m = \frac{4ac-b^2}{4a}$ $for\ a = 3, b = 5, c = -2\ \therefore y_m = -\frac{49}{12}$

Also the equation of the line of symmetry is

X = -b/2a = -5/6.

General evaluation:

1. Solve the equations simultaneously and show the points of intersections
   $Y = 4 – 11x$ and $y = 2x^2-19$
2. Find the maximum value of $y = 5 + 4x – x^2$ and the coordinates at the point where the curve $y = 5 + 4x - x^2$, cuts the coordinates axes.
3. The formula $S = \frac{1}{2}n(n+1)$ gives the sum of n consecutive whole numbers. If $S = 325, find\ n$.
4. A father got his first son at 31 years. If the product of their ages is 816. Find the ages of the father and his son.