SCHEME THIRD TERM
SS2 MATHEMATICS

WEEK	TOPIC	CONTENT
1	**STATISTICS 1**	(a) Meaning and computations of mean, median, mode or ungrouped data. (b) Determination of the mean, median and the mode of grouped frequency data. (c) Comparison of mean, mode and median. (d) Rate and mixtures.
2	**STATISTICS 2**	(a) Definitions of: (i) Range, (ii) Variance, (iii) Standard deviation. (b) Calculation of range, variance and standard deviation. (c) Practical application in capital market reports; (i) Home (ii) Health studies (iii) Population studies.
3	**STATISTICS 3**	(a) Histograms of grouped data (Revision). (b) Need for grouping. (c) Calculation of; (i) class boundaries (ii) class interval (iii) class mark. (d) Frequency polygon. (e) Cumulative Frequency graph: (i) Calculation of cumulative frequencies. (ii) Drawing of cumulative frequency curve graph (Ogive). (f) Using graph of cumulative frequencies to estimate; (i) Median (ii) Quartiles (iii) Percentiles. (iv) Other relevant estimates. (g) Application of ogive to everyday life.
4	**PROBABILITY 1**	(a) Definitions and examples of: (i) Experimental outcomes, (ii) Random experiment. (iii) Sample space. (iv) Sample points. (v) Event space. (vi) Probability. (b) Practical example of each term. (c) Theoretical Probability. (d) Equiprobable sample space; Definition, Unbiasedness. (e) Simple probable on equiprobable sample space.
5	**PROBABILITY 2**	(a) Addition and multiplication rules of probability: (i) Mutually exclusive events and addition (“or”) rule. (ii) Complimentary events and probability rule. (iii) Independent events and multiplication (“and”) rules. (b) Solving simple problems on mutually exclusive, Independent and complimentary events. (c) Experiment with or without replacement. (d) Practical application of probability in; health, finance, population, etc.
6	**FUNCTIONS AND RELATIONS**	(a) Types of function (one-to-one, one-to-many, many-to-one, many-to-many). (b) Function as a mapping. (c) Determination of the rule of a given mapping/function.
7	**MID TERM BREAK**	
8	**VECTORS**	(a) Vectors as directed line segment. (b) Cartesian components of a vector. (c) Magnitude of a vector, Equal vectors, Addition and subtraction of vectors, zero vectors, parallel vectors, multiplication of a vector by a scalar.
9	**TRANSFORMATI ON GEOMETRY**	(a) Rotation of points and shapes on the Cartesian plane. (b) Translation of points and shapes on the Cartesian plane. (c) Reflection of points and shapes on the Cartesian plane. (d) Enlargement of points and shapes on the Cartesian plane.
10	**REVISION**	
11	**EXAMINATION**	

WEEK 1
SUBJECT: MATHEMATICS
CLASS: SS 2
TOPIC: STATISTICS 1
CONTENT:

(a) Meaning and computations of mean, median and mode of ungrouped data.
(b) Determination of the mean, median and the mode of grouped frequency data.
(c) Comparison of mean, mode and median.
(d) Rate and mixtures.

Meaning and computation of mean of ungrouped data

The mean, median and the mode are called measures of central tendency or measures of location. The mean is also known as the average, the median is the middle number while the mode is the most frequent element or data.

THE ARITHMETIC MEAN:

The arithmetic mean is the sum of the ungroup of items divided by the number of it. The mean of an ungrouped data can be calculated by using the formula;

$$\bar{x} = \frac{\sum x}{n}, \qquad \text{(when } n \text{ is small)}$$

(where the symbol $\sum x$ is called sigma meaning summation of all the given data)

Also, Mean, $\bar{x} = \frac{\sum fx}{\sum f}$ (when n is large)

$\sum fx =$ Sum of the product of scores and their corresponding frequencies

$\sum f =$ Sum of the frequencies

Example 1:

Find the arithmetic mean of the numbers 42, 50, 59, 38, 41, 86 and 56

Solution: Add all the numbers and divide by 7

$$\bar{x} = \frac{42+50+59+38+41+86+56}{7}$$

$$= \frac{378}{7}$$

$$\therefore \bar{x} = 54$$

Example 2:

The table below gives the frequency distribution of marks obtained by some students in a scholarship examination.

Scores(x)	15	25	35	45	55	65	75
Frequency	1	4	12	24	18	8	3

Calculate, correct to 3 significant figures the mean mark of the distribution (WAEC)

Solution:

Scores(x)	Frequency	fx
15	1	15
25	4	100
35	12	420
45	24	1080
55	18	990
65	8	520
75	3	225
	$\sum f = 70$	$\sum fx = 3350$

Since Mean; $\bar{x} = \frac{\sum fx}{\sum f}$

$$\bar{x} = \frac{3350}{70}$$

$$\bar{x} = 47.857143$$

$$\bar{x} = 47.9 \quad \text{(3s.f)}$$

Method 2: mean; $\bar{x} = \frac{\sum fx}{\sum f} = \frac{(1\times15)+(4\times25)+(12\times35)+(24\times45)+(18\times55)+(8\times65)+(3\times75)}{1+4+12+24+18+8+3}$

$$= \frac{15+100+420+1080+990+520+225}{1+4+12+24+18+8+3}$$

$$= \frac{3350}{70}$$
$$= 47.8571$$
$$= 47.9 \quad \text{(3s.f)}$$

Example 3:

The table below shows the scores of some students in a quiz

Scores	1	2	3	4	5	6
frequency	1	4	5	x	2	2

If the mean score is 3.5, calculate the value of x.

Solution:

x	f	fx
1	1	1
2	4	8
3	5	15
4	x	$4x$
5	2	10
6	2	12
	$\sum f = 14 + x$	$\sum fx = 46 + 4x$

Since, mean; $\bar{x} = \frac{\sum fx}{\sum f}$

But, $\bar{x} = 3.5$

$\Rightarrow \quad 3.5 = \frac{46+4x}{14+x}$

On cross multiplying

$$3.5(14 + x) = 46 + 4x$$
$$\frac{7}{2}(14 + x) = 46 + 4x$$
$$7(14 + x) = 2(46 + 4x)$$
$$98 + 7x = 72 + 8x$$
$$98 - 72 = 8x - 7x$$
$$6 = x$$
$$\therefore x = 6$$

Example 4:

The table below shows the mark distribution of an English language test in which the mean mark is 3. Find the value of$y$.

Mark (x)	1	2	3	4	5

Frequency(f)	y	3	y+3	3	4 –y

Solution:

Mean; $\bar{x} = \frac{\sum fx}{\sum f}$

x	f	fx
1	Y	y
2	3	6
3	y+3	3y + 9
4	3	12
5	4 –y	20 –5y
	$\sum f = y + 13$	$\sum fx = 47 - y$

But, mean; $\bar{x} = 3$

So we have that, $3 = \frac{47-y}{13+y}$

On cross multiplying

$3(13 + y) = 47 - y$

$39 + 3y = 47 - y$

$3y + y = 47 - 39$

$4y = 8 \quad \therefore y = 2$

Class Activity:

The table below shows the frequency distribution of marks obtained by a group of students in a test. If the mean is 5, calculate the value of x.

Marks	3	4	5	6	7	8
frequency	5	x –1	X	9	4	1

Meaning and computation of median of ungrouped data

The median is the value of the middle item when the items are arranged in order of magnitude either ascending or descending order.

Example 1;

Find the median of the following set of numbers; 16, 13, 10, 23, 36, 9, 8, 48, 24

Solution: Arrange in (either ascending or descending order)

8, 9, 10, 13, 16, 23, 24, 36, 48

The middle number is 16

$\therefore median = 16$

<u>Median from frequency distribution</u> (i.e when n is large)

Median = $\left(\frac{N+1}{2}\right) th$, when N is odd

Median = $\frac{\left(\frac{N}{2}\right)th+\left(\frac{N}{2}+1\right)th}{2}$, when N is even

Example 2:

The table below shows the distribution of marked scored by some students in a maths test

Marks %	22	24	36	42	45	48	56	60
Frequency	11	2	7	13	10	3	9	5

Solution:

To find the median, a cumulative frequency table is needed.

Marks %(x)	Frequency	Cumulative frequency
22	11	11
24	2	13
36	7	20
42	13	33
45	10	43
48	3	46
56	9	55
60	5	60

From the table, there are 60 members as indicated by the cumulative frequency.

Since 60 is even, Median $= \frac{\left(\frac{N}{2}\right)th+\left(\frac{N}{2}+1\right)th}{2}$

$= \frac{\left(\frac{60}{2}\right)th+\left(\frac{60}{2}+1\right)th}{2}$

$= \frac{30th+31st}{2}$

The 30th member is 42% and the 31st member is 42%

$$\therefore median = \frac{42\%+42\%}{2} = \frac{84}{2} = 42\%$$

Example 3:

Calculate the median age from the following data

Age(yrs)	10	12	13	14	16	17	18	19
No of students	7	15	11	7	12	9	4	6

Solution:

Ages (yrs)	No of students	Cumulative frequency
10	7	7
12	15	22
13	11	33
14	7	40
16	12	52

17	9	61
18	4	65
19	6	71

Since 71 is odd,

Median = $\left(\frac{N+1}{2}\right) th$ member

$= \left(\frac{71+1}{2}\right) th$

$= \frac{72}{2} th$

= 36th member

The 36th member falls within the cumulative frequency of up to 40 and this is under 14 years.

$\therefore median = 14 years$

Class Activity:

Calculate the median of the distribution below;

Marks (x)	10	20	30	40	50
Frequency (f)	13	18	34	60	10

Meaning and computation of mode of ungrouped data

The mode of a given data is the item which occurs most often in the distribution

Example 1;

The record of the marks scored by a number of students in an oral test in economics is as follows;

10, 10, 5, 9, 15, 10, 20, 10, 9, 5, 9, 10, 25, 9, 5, 25. Find the modal mark

Solution:

Marks	5	9	10	15	20	25
Frequency	3	4	5	1	1	2

From the table above, the highest frequency is 5 and this corresponds to a mark of 10

∴ the mode is 10

Example 2;

For a class of 30 students, the scores on a maths test out of 20 marks were as follows

8	10	14	4	6	12	10	10	16	18
10	8	4	6	14	18	16	14	14	14
6	8	10	10	4	6	12	14	14	4

Solution:

Marks	Frequency
4	4
6	4
8	3
10	6
12	2
14	7
16	2
18	2

The highest frequency is 7; $\therefore$ modal score = 14

Class Activity:

Find the mode of the following distributions

Age (years)	13	14	15	16	17	18
Frequency	3	10	15	21	5	5

1. Which of the following is not a measure of central tendency?
   (a) Mode
   (b) Range
   (c) Mean
   (d) Median
2. The table below shows the distribution of test scores in a class

Scores (x)	no of pupils
1	1
2	1
3	5
4	3
5	$k^2 + 1$
6	0
7	6
8	2
9	3

10	4

If the mean score of the test is 6, find the (a) values of k (b) median score

Mean Of Grouped Data

Mean for grouped data can be calculated in two ways;

(i) Mean for problems without assumed mean

$$\bar{x} = \frac{\sum fx}{\sum f}$$

where x is the class mark or class midpoint

(ii) Mean of problems with assumed mean

$\bar{x} = A + \frac{\sum fd}{\sum f}$, where A = assumed mean; d = deviation from mean $(x - A)$

Example;

The weights to the nearest kilogram of a group of 50 students in a college of technology are given below:

65 70 60 46 51 55 59 63 68 53 47 53 72 58 67 62 64 70 57 56 73 56 48 51 58

63 65 62 49 64 53 59 63 50 48 72 67 56 61 64 66 52 49 62 71 58 53 69 63 59

(a) Prepare a grouped frequency table with class intervals 45–49, 50–54, 55–59 etc

(b) Without the method of assumed, calculate the mean of the grouped data correct to one decimal place.

(c) Using an assumed mean of 62, calculate the mean of the grouped data, correct to one decimal place. (WAEC)

Solution:

(a)

Class interval	frequency
45 – 49	6
50 – 54	9
55 – 59	10
60 – 64	12
65 – 69	7
70 – 74	6

(b) Mean; $\bar{x} = \frac{\sum fx}{\sum f}$

Class interval	Class mark(x)	frequency (f)	fx
45 – 49	47	6	282
50 – 54	52	9	468
55 – 59	57	10	570
60 – 64	62	12	744
65 – 69	67	7	469
70 – 74	72	6	432
		$\sum f = 50$	$\sum fx = 2965$

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{2965}{50}$$
$$= 59.3 \ (1\ d.p)$$

(c) $mean, \bar{x} = A + \frac{\sum fd}{\sum f}$, where $d = x - A$

but A = 62,

Class interval	Class mark(x)	frequency (f)	$d = x - A$	fd
45 – 49	47	6	-15	-90
50 – 54	52	9	-10	-90
55 – 59	57	10	-5	-50
60 – 64	62	12	0	0
65 – 69	67	7	5	35
70 – 74	72	6	10	60
		$\sum f = 50$		$\sum fd = -135$

k

. $\bar{x} = A + \frac{\sum fd}{\sum f} = 62 + \left(\frac{-135}{50}\right)$

$= 62 - 2.7$

$= 59.3 \quad (1d.p)$

Class Activity:

The table below gives the masses in kg of 35 students in a particular school. (NECO)

45 43 54 52 57 59 65 50 61 50 48 53 61 66 47 52 48 40
44 60 68 51 47 51 41 50 62 70 58 42 51 49 55 71 60

(a) Using the above given data, construct a group frequency table with class interval 40 – 44, 45 – 49, 50 – 54 etc
(b) From the data above, calculate the mean of the distributions
(c) Using assumed mean of 52, calculate correct to two decimal places the mean of the distribution

The median of a grouped data

The median formula for grouped data is given as;

$$\text{Median} = L_1 + \left[\frac{\frac{n}{2} - C_{fb}}{f_m}\right] C$$

Where; L_1 = lower class boundary of the median class

n = total frequency

C_{fb} = cumulative frequency before the median class

f_m = frequency of the median class

C = size of the median class

Example 1;

The table below shows the marks obtained by forty pupils in a mathematics test

Marks	0 – 9	10 – 19	20 –29	30 – 39	40 – 49	50 – 59
No of pupils	4	5	6	12	8	5

Calculate the median of the distribution.

Solution:

Marks	Class boundaries	f	Cf
0 – 9	0 – 9.5	4	4
10 – 19	9.5 – 19.5	5	9
20 – 29	19.5 – 29.5	6	15
30 – 39	29.5 – 39.5	12	27
40 – 49	39.5 – 49.5	8	35
50 – 59	49.5 – 59.5	5	40

Median = $L_1 + \left[\frac{\frac{n}{2} - C_{fb}}{f_m}\right] C$

$$\frac{n}{2} = \left(\frac{40}{2}\right) th = 20^{th} \text{ member}$$

We find the class interval where the median lies, with the aid of the cumulative frequency 20 lies in the cf after 15. i.e class interval 30 – 39

$$L_1 = 29.5, \quad C_{fb} = 15, C = 10 \ (i.e\ 30 - 20), \quad f_m = 12$$

Median = $29.5 + \left[\frac{\frac{40}{2} - 15}{12}\right] 10$

$= 29.5 + \left[\frac{20-15}{12}\right] 10$

$= 29.5 + \left[\frac{5}{12}\right] 10$

= 29.5 + (0.147 x 10)

$= 29.5 + 4.17$

$= 33.67$

Therefore, median mark = 33.67

Class Activity:

1. The frequency distribution shows the marks of 100 students in a mathematics test.

Marks	No of students
1 – 10	2
11 – 20	4
21 – 30	9
31 – 40	13
41 – 50	18
51 – 60	32
61 – 70	13
71 – 80	5
81 – 90	3
91 – 100	1

Calculate the median mark. (WAEC)

2. The table below shows the weight distribution of 40 men in a games village.

Weight(kg)	110 – 118	119 – 127	128 – 136	137 – 145	146 – 154	155 – 163	164 – 172
frequency	**9**	**3**	**4**	**5**	**2**	**5**	**12**

Calculate the median of the distributions

The mode of grouped data

Mode formula for grouped data is given as;

$$\text{Mode} = L_1 + \left[\frac{\Delta_1}{\Delta_1 + \Delta_2}\right]C \quad OR \quad L_1 + \left[\frac{f_x}{f_x + f_y}\right]C$$

Where, L_1 = Lower class boundary of the modal class

Δ_1 or f_x = Difference between the modal frequency and the frequency of the next lower class i.e class before it

Δ_2 or f_y = Difference between the modal frequency and the frequency of the next highest class i.e class after it

C = Size of the modal class

Example 1:

The table below shows the weekly profit in naira from a mini – market

Weekly profit	1 – 10	11 – 20	21 – 30	31 – 40	41 – 50	51 – 60
frequency	6	6	12	11	10	5

What is the modal weekly profit?

Solution:

Weekly profit	Class boundaries	Frequency
1 – 10	0.5 – 10.5	6
11 – 20	10.5 – 20.5	6
21 – 30	20.5 – 30.5	12
31 – 40	30.5 – 40.5	11
41 – 50	40.5 – 50.5	10
51 – 60	50.5 – 60.5	5

The modal class is 21 – 30 (i.e class with the highest frequency)

Mode = $L_1 + \left[\frac{\Delta_1}{\Delta_1+\Delta_2}\right]C$, $L_1 = 20.5$, $\Delta_1 = 12 - 6 = 6$, $\Delta_2 = 12 - 11 = 1$, $C = 10$

Mode = $20.5 + \left[\frac{6}{6+1}\right]10$

$= 20.5 + \left[\frac{6}{7}\right]10$

$= 20.5 + 0.8571 \times 10$

$= 20.5 + 8.571$

= 29.07

∴ Modal profit is #29.07

Example 2:

The frequency distribution of the weights of 100 participants in a women conference held in Jupiter is shown below.

Weight(kg)	40 – 49	50 – 59	60 – 69	70 – 79	80 – 89	90 – 99	100 – 109
No of women	9	2	22	30	17	4	16

Calculate the modal weight of the women

Solution:

Weights (kg)	Class boundaries	No. of women (f)
40 – 49	39.5 – 49.5	9
50 – 59	49.5 – 59.5	2
60 – 69	59.5 – 69.5	22
70 – 79	69.5 – 79.5	30
80 – 89	79.5 – 89.5	17
90 – 99	89.5 – 99.5	4
100 – 109	99.5 – 109.5	16

Modal class = 70 – 79; $L_1 = 69.5$, $f_x = 30 - 22 = 8$, $f_y = 30 - 17 = 13$, $C = 79.5 - 69.5 = 10$

Mode = $L_1 + \left[\frac{f_x}{f_x+f_y}\right]C$

$= 69.5 + \left[\frac{8}{8+13}\right] \times 10$

$= 69.5 + \left[\frac{8}{21}\right] \times 10$

= 69.5 + 0.381 x 10

= 69.5 + 3.81

= 73.31

∴ Modal weight = 73.3kg (3s.f)

Class Activity:

The table below shows the age distributions of the members of a club.

Age (years)	10 – 14	15 – 19	20 – 24	25 – 29	30 – 34	35 – 39
frequency	7	18	25	17	9	4

Calculate the modal age. (WAEC)

PRACTICE EXERCISE:

1. If 8kg of coffee costing #2000 a kg is mixed with 12kg of another kind of coffee costing #2200 a kg, what is the cost of the mixture per kg?
2. Three kinds of tea at #1,160, #1,460 and #1,540 per kg are in the ratio 2:3:5. What is the mixture worth per kg.
3. Four ingredients costing #320 per kg, #240 per kg, #160 per kg and #80 per kg are mixed so that their masses are in ratio 4:1:2:3. Calculate the average cost per kg of the mixture.
4. A trader mixes three bags of sugar costing #900/bag with seven sacks of sugar which cost #700/bag. If she sells the mixture at #950/bag, calculate her percentage profit.
5. A trader bought three kinds of nuts at #100 per kg, #84 per kg and #60 per kg respectively. He mixed them in the ratio 3:5:4 respectively and sold the mixed nuts to make a profit of 25%. At what price per kg did sell them?

ASSIGNMENT:

1. The marks scored by 30 students in a particular subject are as follows;
   39 31 50 18 51 63 10 34 42 89 73 11 33 31 41
   25 76 13 26 23 29 30 51 91 37 64 19 86 9 20
   (a) Prepare a frequency table, using class intervals 1 – 20, 21 – 40 e.t.c
   (b) Calculate the mean mark
   (c) Calculate the modal score
2. The table below shows the monthly profit in #100,000 of naira of a super market

Monthly profit in #100,000	11 – 20	21 – 30	31 – 40	41 – 50	51 – 60	61 – 70

frequency	5	11	9	10	7	8

(a) What is the modal monthly?

(b) Estimate the mean and the median profit

WEEK 2

SUBJECT: MATHEMATICS

CLASS: SS 2

TOPIC: STATISTICS 2

CONTENT:

(a) Definitions of: (i) Range, (ii) Variance, (iii) Standard deviation.

(b) Calculation of range, variance and standard deviation.

(c) Practical application in capital market reports; (i) Home (ii) Health studies (iii) Population studies.

DEFINITION AND CALCULATION OF RANGE

Measures of Dispersion

The measure of dispersion (also called measure of variation) is concerned with the degree of spread of the numerical value of a distribution.

Range: This is the difference between the maximum and minimum values in the data.

Examples 1:

Find the range of the data 6, 6, 7, 9, 11, 13, 16, 21 and 32

Solution: The maximum item is 32

The minimum item is 6

∴ Range = 32 – 6 = 26

Example 2:

Find the range of the distributions below 65,62,62,61,61,60,60,59,58,52

Solution: Range = 65 – 52 = 13

Deviation from the mean:

If the mean of a distribution is subtracted from any value in the distribution, the result is called the DEVIATION of the value from the mean.

Consider the table below (set of examination marks)

65	62	62	61	61
60	60	59	58	52

The mean = $\frac{65 + 62 + 62 + 61 + 61 + 60 + 60 + 59 + 58 + 52}{10}$

$= \frac{600}{10}$

$= 60$

Deviation from the mean = $X - \bar{X} = 65 - 60 = +5$

= 62 – 60 = +2

= 62 – 60 = +2

= 61 – 60 = +1

= 61 – 60 = +1

= 60 – 60 = 0 e.t.c

The deviations of the scores from the mean are +5, +2, +2, +1, +1, 0, 0, -1, -2, -8

The sum of these deviations = 0

Class Activity:

(1) Calculate the range of the following distributions

(a) 72, 78, 72, 90, 72, 83, 79.

(b) 3.9, 4.0, 4.2, 3.9, 3.8, 4.0

(2) Calculate the mean deviation of (1a) and (1b) above

DEFINITION AND CALCULATION OF VARIANCE

The variance is the arithmetic mean of the squares of the deviation of the observations from the true mean. It is also called the **mean squared deviation**.

The formula for variance is (a) $\frac{\sum(x-\bar{x})^2}{n}$ for an ordinary distribution (ungrouped)

(b) $\frac{\sum f(x-\bar{x})^2}{\sum f}$, for a frequency distribution table (grouped)

Example 1:

Calculate the variance of the following distributions of the ages of 50 pupils in a secondary school

Age (years)	10	12	13	14	15	16
Number of pupils	18	4	6	12	6	4

Age (x)	Freq (f)	fx	$\lvert x-\bar{x}\rvert$	$\lvert x-\bar{x}\rvert^2$	$f\lvert x-\bar{x}\rvert^2$

10	18	180	2.6	6.76	121.68
12	4	48	0.6	0.36	1.44
13	6	78	0.4	0.16	0.96
14	12	168	1.4	1.96	23.52
15	6	90	2.4	5.76	34.56
16	4	64	3.4	11.56	46.24
	50	628			228.4

Mean $\bar{X} = \frac{\sum fx}{\sum f}$

$= \frac{628}{50}$

$= 12.56$

= 12.6

Variance = $\frac{\sum f(x-\bar{x})^2}{\sum f}$

$= \frac{228.4}{50}$

= 4.568

= 4.6 approximately

Example 2:

Calculate the variance of the distribution below.

90, 80, 72, 68, 64, 56, 52, 48, 36, 34

Solution:

Mean $\bar{x}$ $or\ m = \frac{\sum x}{n}$

= 60

x	$x - m = d$	d^2
90	+30	900
80	+20	400
72	+12	144
68	+8	64
64	+4	16
56	-6	16
52	-8	64
48	-12	144
36	-24	576
34	-26	676
		Total = 3000

Variance = $\frac{\sum d^2}{n}$

$= \frac{3000}{10}$

$= 300$

Class Activity:

Calculate the mean and variance of the ages of 12 students aged 16, 17, 18, 16.5, 17, 18, 19, 17, 17, 18, 17.5 and 16

Definition and Calculation of standard deviation

Standard deviation (S.D) is the square root of variance.

The formula for S.D are: (a) $\sqrt{\frac{\sum(x-\bar{x})^2}{n}}$ and (b) $\sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}}$

Example 1:

Find the variance and standard deviation of the set of numbers 2,5,6,3 and 4

Solution: Variance = $\frac{\sum(x-\bar{x})^2}{n}$

But mean = 4

x	$x-\bar{x}$	$(x-\bar{x})^2$
2	-2	4
5	1	1
6	2	4
3	-1	1
4	0	0
		$\sum(x-\bar{x})^2 = 10$

Variance = $\frac{10}{5}$ = 2

Standard deviation, S.D = $\sqrt{\frac{\sum(x-\bar{x})^2}{n}}$

$= \sqrt{2}$

$= 1.414$

Example 2:

Calculate the standard deviation of the distribution

Age (years)	10	12	13	14	15	16
Frequency	18	4	6	12	6	4

Solution:

Reference to example 2 n page 3 and 4

Standard Deviation = $\sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}}$

$$= \sqrt{\frac{228.4}{50}}$$

$$= \sqrt{4.568}$$

$$= 2.14$$

Class Activity:

Compute (i) **the variance** (ii) **the standard deviation** of the data.

1. In a college, the number of absentees recorded over a period of 30days was a shown in the frequency distribution table.

Number of absentees	0 – 4	5 – 9	10 – 14	15 – 19	20 – 24
Number of days	1	5	10	9	5

2. The table shows the distribution of ages of workers in a company

Age (in yrs)	17 – 21	22 – 26	27 – 31	32 – 36	37 – 41	42 – 46	47 – 51	52 – 56
Frequency	12	24	30	37	45	25	10	7

PRACTICAL APPLICATION IN CAPITAL MARKET REPORT

EXAMPLE :

Two groups of eight students in a class were given a test in English. Group A had the following marks; 60, 70, 50, 48, 68, 72, 80 and 56

Group B had the following marks: 50, 90, 40, 58, 90, 82, 60 and 44.

(a) Calculate the mean, range, variance and standard deviation of each group.

(b) Which group had less variation in its marks?

Solution:

(a) Group A

x	$x - \bar{x}$	$\lvert x - \bar{x} \rvert$	$\lvert x - \bar{x} \rvert^2$
60	-3	3	9
70	7	7	49
50	-13	13	169
48	-15	15	225
68	+5	5	25
72	+9	9	81
80	+17	17	289
56	-7	7	49
			896

Mean $\bar{X} = \frac{60+70+50+48+68+72+80+56}{8}$

$= \frac{504}{8}$

$= 63$

Range = 70 50 = 20

Variance (v) $= \frac{\sum(x-\bar{x})^2}{n}$

$= \frac{896}{8}$

$= 112$

S.D = $\sqrt{\frac{\sum(x-\bar{x})^2}{n}}$

$= \sqrt{\frac{896}{8}}$

$= \sqrt{112}$

$= 10.5830$

$= 10.58$ (2 d.p)

GROUP B:

x	$\lvert x-\bar{x}\rvert$	$\lvert x-\bar{x}\rvert^2$
50	14.25	203.0625
90	25.75	663.0625
40	24.25	588.0625
58	6.25	39.0625
90	25.75	663.0625
82	17.75	315.0625
60	4.25	18.0625
44	20.25	410.0625
		2899.5

Mean = 64.25

Mean = 64.25

Variance = 362.43

S.D = 19.04 (2 d.p)

(b) Group A

Class Activity:

1. The rainfall in millimetres from June to November in two towns is given below

	June	July	Aug	Sept	Oct	Nov
Town A	1.8	2.7	1.4	2.4	2.8	1.5
Town B	3.4	3.6	2.2	2.5	2.8	1.2

(a) Compare the means and standard deviations of rainfall in towns A and B

(b) In which town is rainfall less widely spread during the period?

2. Compute the (i) Variance
   (ii) Standard deviations
   (iii) Range of the following distributions

Score	95	85	80	75	70	65	55	40
frequency	1	1	1	4	1	3	1	3

WEEK 3

SUBJECT: MATHEMATICS

CLASS: SS 2

TOPIC: STATISTICS 3

CONTENT:

Histograms of grouped data (Revision)**:** (a) Need for grouping (b) Calculation of; (i) class boundaries (ii) class interval (iii) class mark. **(**b**)** Frequency polygon **(**c**)** Cumulative Frequency graph**:** (a) Calculation of cumulative frequencies. (b) Drawing of cumulative frequency curve graph (Ogive). (c) Using graph of cumulative frequencies to estimate; (i) Median (ii) Quartiles (iii) Percentiles. (iv) Other relevant estimates. (d) Application of ogive to everyday life.

Let the record below be the mass of some people (in kg)

66	48	71	61	39	68	33	60	52	44
33	49	81	58	59	71	42	88	68	91
80	66	70	26	96	63	76	46	51	61
54	32	50	59	41	55	38	56	86	62
50	69	23	84	77	33	71	42	69	93

Should bar chart be drawn for the different masses above, there would be too many bars, so the data may be grouped into class intervals and then a frequency distribution table prepared. Appropriate class intervals are : 21 – 30, 31 – 40, 41 – 50, …
Each data belongs to one of the class intervals. Each data is first represented by a stroke in the tally column. Every fifth stroke is used to cross the first four counted. The number of tally in each class interval gives the frequency

Class interval	Tally	Frequency
21 – 30	//	2
31 – 40	~~////~~ /	6
41 – 50	~~////~~ ////	9
51 – 60	~~////~~ ////	9
61 – 70	~~////~~ ~~////~~ /	11
71 – 80	~~////~~ /	6
81 – 90	////	4
91 – 100	///	3

The modal class is the one with the highest frequency.

Class

Activity:

1. Prepare a frequency table, using class intervals 1 - 20, 21 – 40, … for the scores by 30 students.

26	23	29	30	91	51
37	64	86	9	20	19
39	31	50	18	51	63
33	13	31	25	41	76
10	34	42	89	73	11

2. The marks scored by fifty students in an examination paper are given below:

43	27	31	43	22	31	47	34	18	15
30	45	48	55	39	25	31	12	18	21
26	19	38	10	44	43	51	33	59	54
41	35	37	41	46	33	51	37	48	58
17	19	23	26	29	38	57	36	35	44

Prepare a frequency table, using class intervals 10 – 19, 20 – 29, 30 – 39, e.t.c
What is the modal class?

Calculation of (i) class boundaries
(ii) class interval
(iii) class mark

Grouped data can be represented using a kind of rectangles called histogram. The width of these rectangles is determined by the class interval while the height is proportional to the frequency in that interval. To close up the gaps between the class intervals, the class interval at both ends to have a common boundary in-between two intervals. From the last frequency table above we get this table.

Class	Frequency	Class

intervals		boundaries
21 – 30	2	20.5 – 30.5
31 – 40	6	30.5 – 40.5
41 – 50	9	40.5 – 50.5
51 – 60	9	50.5 – 60.5
61 – 70	11	60.5 – 70.5
71 – 80	6	70.5 – 80.5
81 -90	4	80.5 – 90.5
91 – 100	3	90.5 – 100.5

To get a common boundary between two class interval, the upper class limit of a class is added to the lower class limit of the next class and divide the sum by 2.

e.g $\frac{20+21}{2} = \frac{41}{2} = 20.5$

$\frac{30+31}{2} = \frac{61}{2} = 30.5$ e.t.c

The upper class boundary of a class is the lower class boundary of the next class. This gives a continuous horizontal axis.

Another thing to consider is the class mark or class centre. This may be used in finding the mean. For any class interval, the class center is the average of the upper and lower limits of that particular class interval.

Class center of interval 21 – 30 is $\frac{21+30}{2} = \frac{51}{2} = 25.5$

Class mark for class interval 31 – 40 is $\frac{31+40}{2} = \frac{71}{2} = 35.5$

The class mid-values (class centre) are used in plotting frequency polygon.

CUMULATIVE FREQUENCY GRAPH

The Cumulative frequency of a given class or group is the sum of the frequency of all the classes below and including the class itself.

Cumulative frequency curve or Ogive is a statistical graph gotten by plotting the upper class boundaries against cumulative frequencies. It is used to determine among the others: Median, Percentiles (100 divisions), Deciles (10 divisions), Quartiles (4 divisions)

The cumulative frequencies are placed along the y – axis, while the scores or class boundaries are placed along the x-axis

Calculation of cumulative frequencies and Drawing of cumulative frequency curve graph (Ogive)

Example 1;

The table below shows the frequency distributions of the lengths (in cm) of fifty planks cut by a machine in the wood – processing factory of kara sawmill (Nigeria)

Class interval	21 – 30	31 – 40	41 – 50	51 – 60	61 – 70	71 – 80	81 – 90	91 – 100
frequency	2	6	9	9	11	6	4	3

(a) Prepare a cumulative frequency table for the distribution

(b) Draw the cumulative frequency curve (Ogive) for the distribution

Scale: 2cm to represent 10 units on the frequency axis

2cm to represent 10 units on the length axis

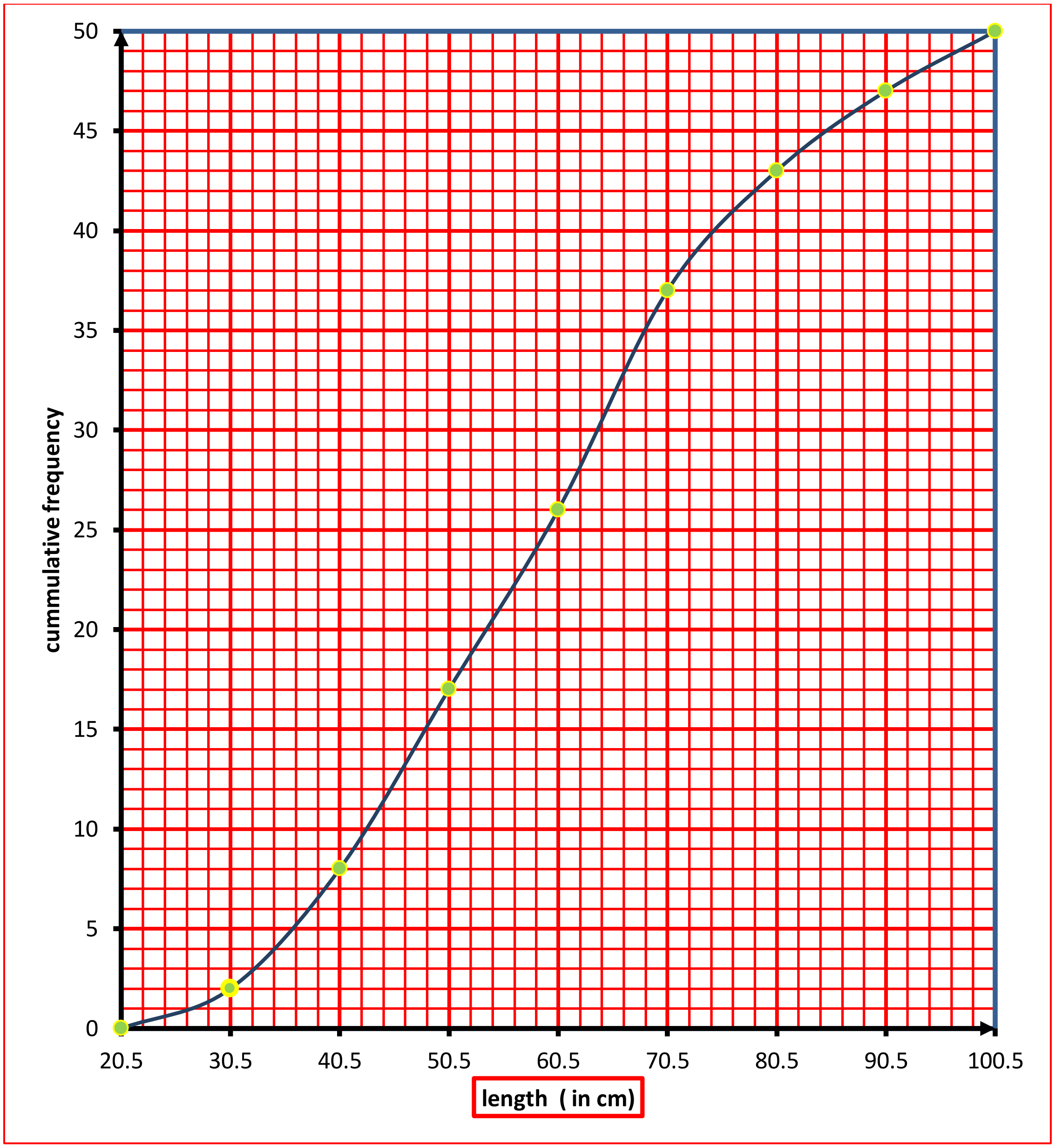
Solution:

The cumulative frequency table is given below as;

Class interval	Class boundaries	Frequency	Cumulative frequency
21 – 30	20.5 – 30.5	2	2
31 – 40	30.5 – 40.5	6	6 + 2 = 8
41 – 50	40.5 – 50.5	9	9 + 8 = 17
51 – 60	50.5 – 60.5	9	9 + 17 = 26
61 – 70	60.5 – 70.5	11	11 + 26 = 37
71 – 80	70.5 – 80.5	6	6 + 37 = 43
81 – 90	80.5 – 90.5	4	4 + 43 = 47
91 – 100	90.5 – 100.5	3	3 + 47 = 50

To plot the graph, it is advisable to use a suitable scale. The graph should be drawn big, because the bigger the graph the more accurate the answers that would be obtained from the graph.

Cumulative frequency curve



Using graph of cumulative frequencies to estimate median, quartiles, percentiles etc

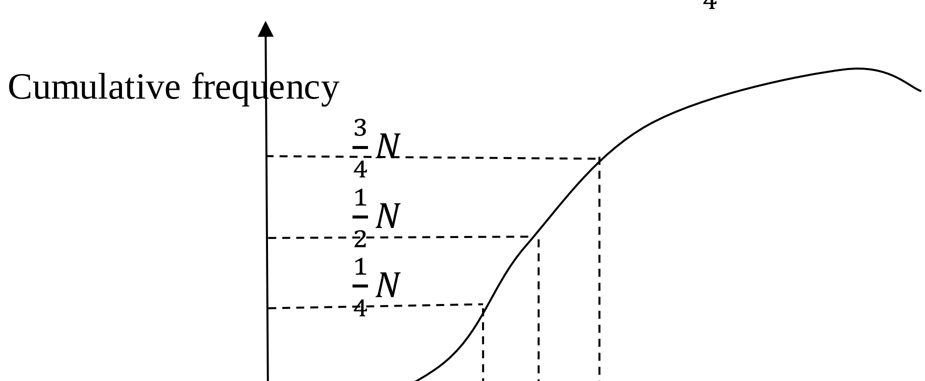
To estimate median and quartiles from the Ogive or cumulative frequency curve, we take the following steps;

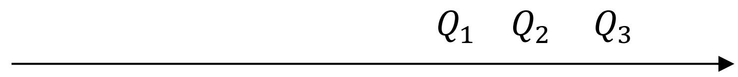
STEP 1: Compute to find their position on the cumulative frequency (CF) axis using the following formulae,

(a) For lower quartile or first quartile (Q_1) we use $\frac{1}{4}N$

(b) For median quartile or second quartile (Q_2), we use $\frac{1}{2}N$

(c) For upper quartile or third quartile (Q_3), we use $\frac{3}{4}N$ (Total frequency or last CF)





Upper class boundaries

STEP 2: Locate the point on the cumulative frequency axis and draw a horizontal line from this point to intersect the Ogive.

STEP 3: At the point it intersect the Ogive, draw a line parallel to the cumulative frequency axis to intersect the horizontal axis.

STEP 4: Read the value of the desired quartile at the point of intersection of the vertical line and the horizontal axis.

Inter-quartile range = $Q_3 - Q_1$

Semi inter-quartile range $\frac{Q_3 - Q_1}{2}$

Percentile

This is the division of the cumulative frequency into 100 points. For instance;

$$75\% = \frac{75}{100} \times N$$

$$20\% = \frac{20}{100} \times N$$

Then, we trace the required values to the graph (curve) then to the class boundaries to get the required answer.

Example 1:

The frequency distribution of the weight of 100 participants in a high jump competition is as shown below:

(a) Construct the cumulative frequency table

Weight (kg)	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69	70 –79
No of participants	10	18	22	25	16	9

(b) Draw the cumulative frequency curve

(c) From the curve, estimate:

(i) The median

(ii) The lower quartile

(iii) The upper quartile

(iv) The inter-quartile range

(v) The semi inter-quartile range

(vi) 65 percentile

(vii) 4th decile

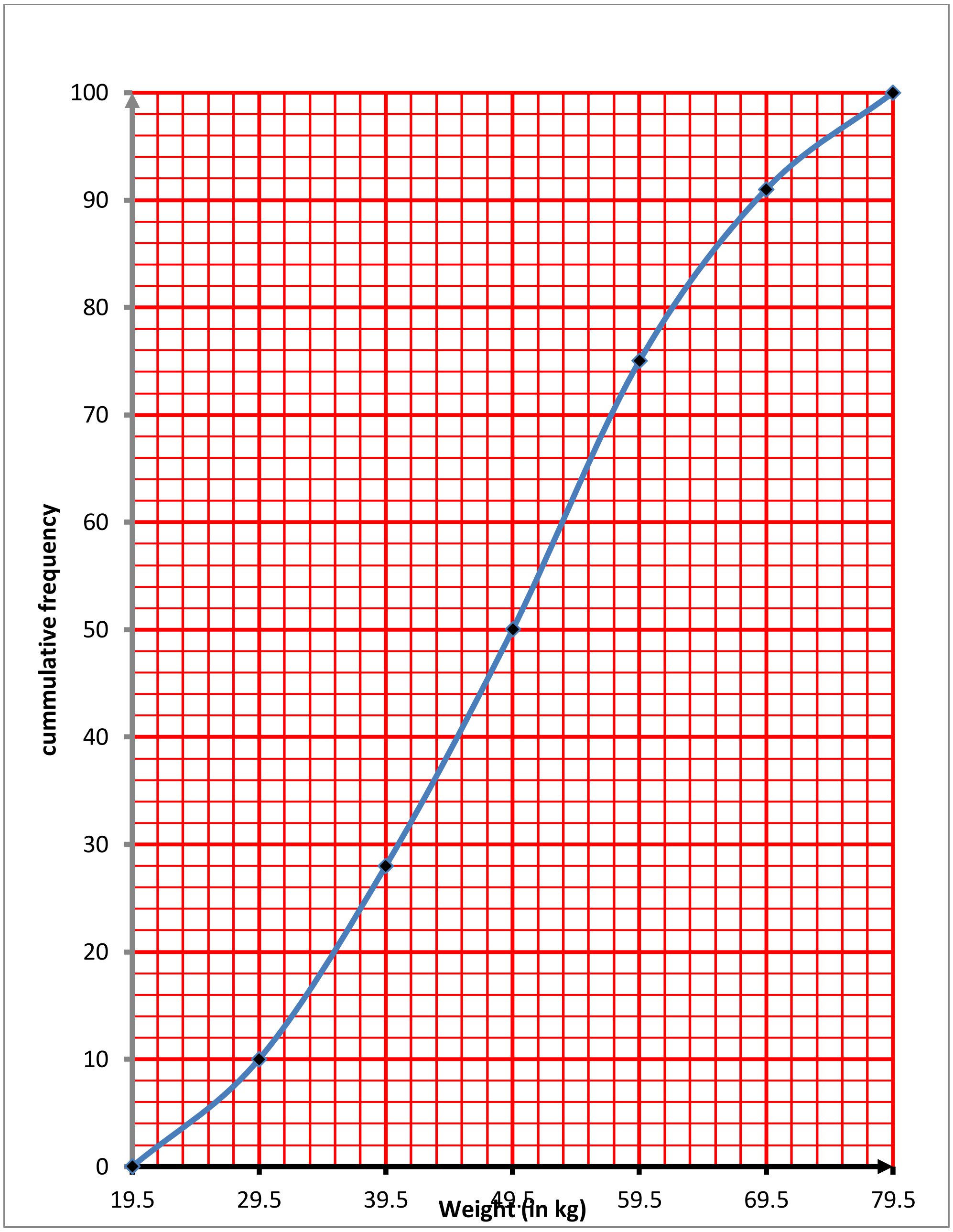
(viii) The probability that a participant chosen at random weighs at least 60kg

Solution:

Class interval	Class boundary	Frequency	Cumulative Frequency
20 – 29	19.5 – 29.5	10	10

30 – 39	29.5 – 39.5	18	28
40 – 49	39.5 – 49.5	22	50
50 – 59	49.5 – 59.5	25	75
60 – 69	59.5 – 69.5	16	91
70 – 79	69.5 – 79.5	9	100

(b)



(c i.) From the curve, median is half way up the distribution. This is obtained by using $\frac{1}{2}N$, where N is the total frequency. Median = $\frac{N}{2} = \frac{100}{2}$

$$Q_2 = 50th\ position$$

Median is at point Q_2 on the graph, i.e median = 49.5kg

ii. Lower quartile is one-quarter of the way up the distribution; lower quartile = $\frac{1}{4}N = \frac{100}{4}$ = 25

∴ Q_1 = 25th position

Lower quartile is at point Q_1 on the graph. i.e lower quartile = 37.5kg

iii. Upper quartile is three-quarters way up the distribution;

$$\text{Upper quartile} = \frac{3}{4}N$$
$$= \frac{3}{4} \times 100$$
$$= \frac{300}{4}$$
$$Q_3 = 75^{th} \text{ position}$$

Upper quartile is at the point Q_3 on the graph. i.e Upper quartile = 59.5kg

iv. Inter-quartile range (IQR) = Upper quartile – Lower quartile

$$= Q_3 - Q_1$$
$$= 59.5\text{kg} - 37.5\text{kg}$$
$$= 22\text{kg}$$

v. Semi inter-quartile range (SIQR) = $\frac{\text{Inter-quartile range (IQR)}}{2}$

$$= \frac{Q_3 - Q_1}{2}$$

∴ SIQR = 11kg

vi. 65 percentile = $\frac{65}{100} \times N$

$$= \frac{65}{100} \times 100$$

∴ P_{65} = 65th position

65 percentile is at point p on the graph = 54.5kg

vii. 4th deciles = $\frac{4}{10} \times N$

$$= \frac{4}{10} \times 100$$

∴ D_4 = 40th position

4th deciles is at point d on the graph i.e 44.5kg

viii. Probability of at least 60kg = $\frac{25}{100} = \frac{1}{4}$

Application of Ogive to everyday life

Example 1;

The table below shows the frequency distribution of the marks of 800 candidates in an examination

Marks	Frequency
0 – 9	10
10 – 19	40
20 – 29	80

30 – 39	140
40 – 49	170
50 – 59	130
60 – 69	100
70 – 79	70
80 – 89	40
90 – 99	20

(ai.) Construct a cumulative frequency table

ii. Draw the Ogive

iii. Use your Ogive to determine the 50th percentile

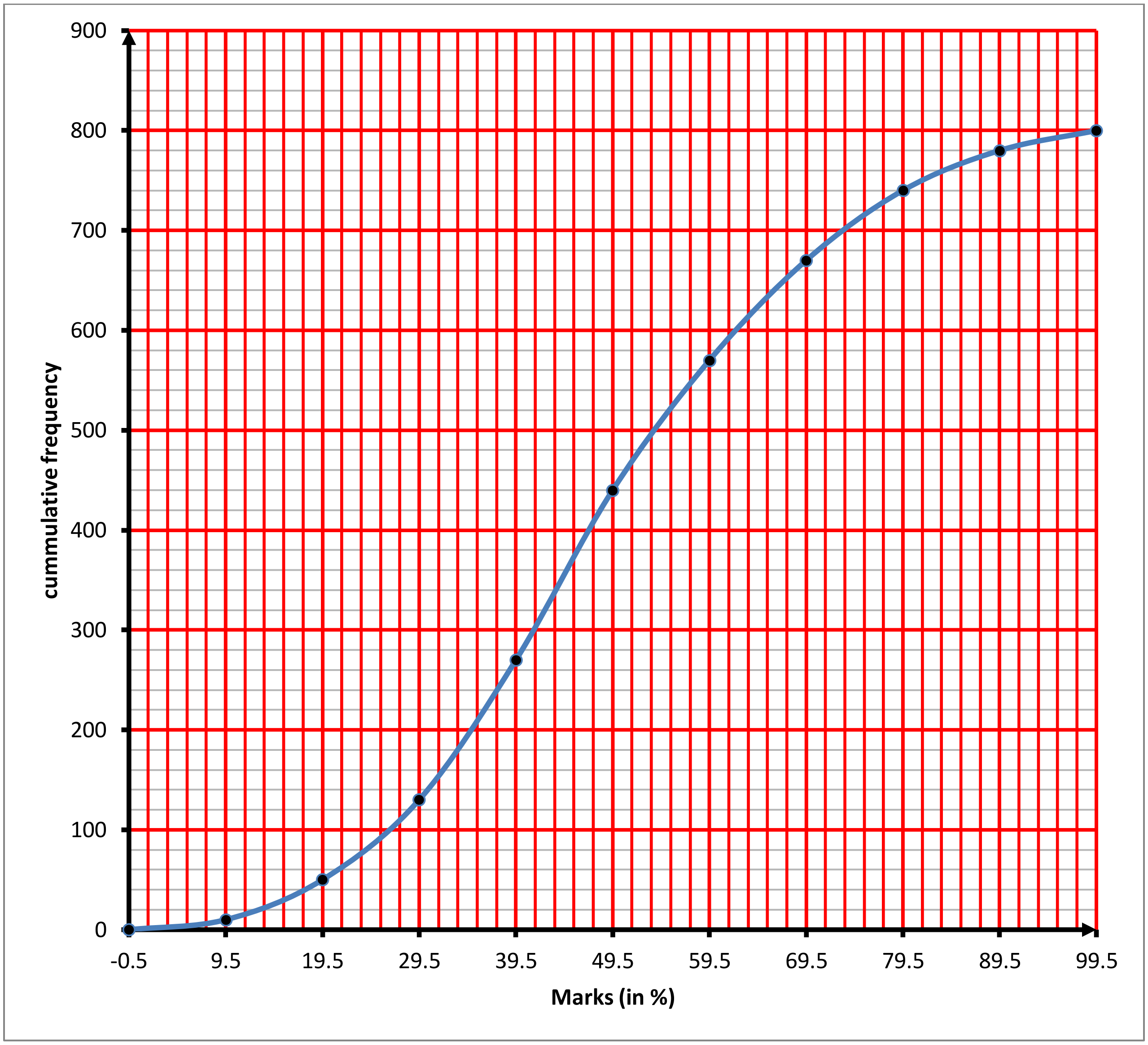
(b.) The candidates that scored less than 25% are to be withdrawn from the institution, while those that scored more than 75% are to be awarded scholarship. Estimate the number of candidates that will be retained, but will not enjoy the award

(c.) If 300 candidates are to be admitted out of the 800 candidates for a particular course in the institution, what will be the cut of mark for the admission?

(d.) if a candidate is picked from the population, what is the probability that the candidate scored above 40%?

Solution: (ai.)

Marks (%)	Class Boundary	Frequency	Cumulative frequency
0 – 9	– 0.5 – 9.5	10	10
10 – 19	9.5 – 19.5	40	50
20 – 29	19.5 – 29.5	80	130
30 – 39	29.5 – 39.5	140	270
40 – 49	39.5 – 49.5	170	440
50 – 59	49.5 – 59.5	130	570
60 – 69	59.5 – 69.5	100	670
70 – 79	69.5 – 79.5	70	740
80 – 89	79.5 – 89.5	40	780
90 – 99	89.5 – 99.5	20	800



iii. 50^{th} percentile = $\frac{50}{100} \times 800$

= 400 position

50^{th} percentile is at the point Q_2 on the graph = 47.5%

(b.) To get the number of candidate that scored less than 25%, we would read from the mark axis at the point of 25% to the frequency axis for the number of candidates.

From the graph, this is at the point number 80. Therefore 80 candidates are to be withdrawn from the institution.

Those that scored more than 75% would also be read from the mark axis to the frequency axis. From the graph, this is 720;

Number of candidates = 800 – 720

= 80 candidates

∴ 80 candidates are to be awarded scholarship, the number of candidates that will be retained without award = 800 – (80 + 80)

= 800 – 160 = 640 candidates

(c.) If 300 candidates are to be registered for the course, then the 300 candidates would be obtained from the top of the frequency axis. This is read from the point C on the graph

i.e 800 – 300 = 500 position

∴ The cut-off mark from the graph is 55.5%

(d.) Reading from the mark axis at 40.5%, we get the value 290 from the graph

∴ Those that scored 40% and below = 290 candidates

Those that scored above 40% = 800 – 290 = 510 candidates

Therefore, probability that the candidate scored above 40% = $\frac{510}{800} = \frac{51}{80}$

ASSIGNMENT:

1. In the test conducted in a particular school, the students are graded according to the marks scored as given in the table below; this is the scores of 2000 candidates

Marks (%)	11 – 20	21 – 30	31 – 40	41 – 50	51 – 60	61 – 70	71 – 80	81 – 90
Pupil's no	68	184	294	402	480	310	164	98

(a) Prepare a cumulative frequency table and draw the cumulative frequency curve for the distribution.

(b) Use your curve to estimate the; (i) cut off mark if 300 candidates are to be offered admission (ii) probability that a candidate picked at random scored at least 45%

2. The table below shows the marks scored by a group of students in a test

Marks	1 – 10	11 – 20	21 – 30	31 – 40	41 – 50	51 – 60	61 – 70	71 – 80	81 – 90	91 - 100
Frequency	4	6	9	12	20	15	7	5	0	2

(a) Construct the cumulative frequency table

(b) Draw the ogive

(c) From your ogive, find the: (i) Median (ii) Lower quartile

(d) A student was picked at random from the group, what is the probability that the students (using o-give) (i) Obtain a distinction grade of 75% and above (ii) failed the test if the pass mark is 40%

WEEK 4

SUBJECT: MATHEMATICS

CLASS: SS 2

TOPIC: PROBABILITY

CONTENT:

(a) Definitions and examples of: (i) Experimental outcomes, (ii) Random experiment. (iii) Sample space. (iv) Sample points. (v) Event space. (vi) Probability.
(b) Practical example of each term.
(c) Theoretical Probability.
(d) Equiprobable sample space; Definition, Unbiasedness.
(e) Simple probable on equiprobable sample space.

SAMPLE SPACE: Any result of an experiment in probability is usually called an outcome. If we cannot predict before hand, the outcome of an experiment, the experiment is called a random experiment.

The set of all possible outcomes of any random experiment will be called a sample space and it will denoted by S. The number of outcomes in S or the number of elements in the sample space will be denoted n(S).

EVENT SPACE: A subset of the sample space which may be a collection of outcomes of a random experiment is called an event space. We shall denote an event space by E, and the number of outcomes or elements in E by N(E).

The probability of an event E denoted Pr(E) is defined as Pr(E) = $\frac{n(E)}{n(S)}$

Since the empty set θ is a subset of the sample space, n(θ) = 0

$$\Pr(\theta) = \frac{n(\theta)}{n(S)} = 0 \quad \text{or Pro. (S)} = \frac{n(S)}{n(S)} = 1$$

Example 1:

In a single throw of a fair coin, find the probability that:

i) a head appears
ii) a tail appears

solution

Let S be the sample space, then

S = $\{H, T\}$

n(S) = 2

Let E_1 be the event that a head appears,

$E_1 = \{H\}$

$n(E_1) = 1$

Prob(E_1) = $\frac{n(E1)}{n(S)} = \frac{1}{2}$

Let E_2 be the event that a tail appears, then E_2

$E_2 = \{T\}$

$n(E_2) = 1$

Prob.(E_1) = $\frac{1}{2}$

Example 2: In a single throw of two fair coins, find the probability that:

a) two heads appears
b) two tails appears
c) one head and one tail appears

Solution:

Let S be the sample space then,
S = [HH, TT, HT, TH]

n(S) = 4

a) Let E_1 be the event that two heads appears, then
$E_1 = \{HH\}$, $n(E_1) = 1$
$\text{Prob}(E_1) = \frac{n(E1)}{n(S)} = \frac{1}{4}$
b) Let E_2 be the event that two tails appears, then
$E_2 = \{TT\}$ $n(E_2) = 1$

$\text{Prob}(E_2) = \frac{n(E2)}{n(S)} = \frac{1}{4}$
c) Let E_3 be the event that one head and one tail appear, then

$E_3 = \{TH, HT\}$, $n(E_3) = 2$

$\text{Prob}(E_2) = \frac{n(E3)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

PROPERTIES OF PROBABILITY

The following are some fundamental properties of probability for finite sample space.
1). For every event E, $0 \leq P(E) \leq 1$. That is all probabilities lie between 0 and 1.
2). P(S) = 1, Where S is the sample space. That is the probability of a sure event i

Example 1:

A bag containing 3 blue balls, 2 black balls and 5 red balls. A ball was selected, what is the probability that it is (a) Red b) Blue c) not black

Solution: sample space = total number of balls
= 3 + 2 + 5
= 10
n(S) = 10

a) Let R be the event of red balls
n(R) = 5
P(R) = n(R) / n(S) = 5/10 = ½
b) Let B be the event of blue balls
n(B) = 3
P(B) = n(B)/n(S) = 3/10
c) Let E be the event of black balls → not black balls will be Ē

P(Ē) = 1 – P(E)
n(E) = 2
P(E) = n(E)/n(S) = 2/10 = ½

$P(\acute{E}) = 1 - \frac{1}{5} = \frac{4}{5}$

Example 2: The probability that John and Dara pass a mathematics examination is 0.4 and 0.8 respectively. What is the probability that

i) Both pass ii) none pass

iii) Only one pass iv) at least one pass.

Solution

Let E_1 be the event that john passed

Let E_2 be the event that Dara passed

Since E_1 and E_2 are mutually exclusive

$P(E_1) = 0.4$ $\quad$ $P(E_2) = 0.8$

$P(\acute{E}) = 1- 0.4$ $\quad$ $P(\acute{E}) = 1- 0.8$

$P(\acute{E}) = 0.6$ $\quad$ $P(\acute{E}) = 0.2$

1) Prob. (both passed) $= P(E_1) \times P(E_2)$

$= P\ E_1\ n\ E_2$

$= 0.4 \times 0.8$

$= 0.32$

2) Prob. (none passed) $\rightarrow$ both failed

$P(\acute{E}_1)\ n\ P(\acute{E}_2) = P(\acute{E}_1) \times P(\acute{E}_2)$

$= 0.6 \times 0.2$

$= 0.12$

3) Prob. That only one passed

Only one passed could mean $P\ (E_1\ n\ \acute{E}_2)$ or $P\ (\acute{E}_1\ n\ \acute{E}_2)$

$P\ (E_1\ n\ \acute{E}_2) = 0.4 \times 0.2 = 0.08$

$P\ (\acute{E}_1\ n\ E_2) = 0.6 \times 0.8 = 0.48$

$P\ (E_1\ n\ \acute{E}_2)\ U\ P\ (\acute{E}_1\ n\ E_2) = 0.08 + 0.48$

$= 0.56$

4) Prob. That at least one passed

$P\ (E_1\ n\ \acute{E}_2)$ **U** $P\ (\acute{E}_1\ n\ E_2)\ n\ E_2)$ **U** $P\ (E_1\ n\ E_2)$

$= 0.08 + 0.48 + 0.32$

$= 0.88$

Example 3:

A bag contains 3 black, 5 white and 7 yellow balls. If a ball is picked, what is the probability that it is either black or yellow?

Solution: sample space S = total number of balls

$= 3 + 5 + 7 = 15$

Let B represent black balls, n (B) = 3

Prob. (B) $= \frac{n(B)}{n(S)} = \frac{3}{15} = \frac{1}{5}$

Let Y represent yellow balls, n (Y) = 7

Prob. (Y) $= \frac{n(Y)}{n(S)} = \frac{7}{15}$

Prob. (B or Y) = P (B) + P (Y) = $\frac{1}{5} + \frac{7}{15}$

$= \frac{10}{15}$

$= \frac{2}{3}$

Example 4: in example 3 above, if the two balls are picked at random one after the other without replacement, find the probability that they are both white.

Solution:

Let the two events, picking the first white ball be E_1 and the second white ball be E_2

$P(E_1) = \frac{5}{15} = \frac{1}{3}$

$P(E_2/E_1) = \frac{5-1}{15-1} = \frac{4}{14} = \frac{2}{7}$

$P(E_1 \cap E_2) = P(E_1) \times P(E_2/E_1)$ and E and E are dependent.

$= \frac{1}{3} \times \frac{2}{7} = \frac{2}{21}$

Class Activity:

1. A bag contains 15 clips that differ in colours, 5 are White, 4 are Pink, and 6 are Blue. If a clip is selected from the bag at random, what is the probability that it is

(a) white
(b) blue
(c) white or pink
(d) Not white?

2. A Crate contains 24 bottles of soft drinks, 7 are Fanta, 8 are Coke, 6 are Sprite and 3 are Soda. If one bottle of soft drink is taken from the crate, what is the probability of picking a

(i) Fanta
(ii) Coke
(iii) Sprite or a Soda
(iv) Neither Coke nor soda?

3. The data below shows the number of workers employed in the various sections of a construction company in Lagos

Carpenters	24	Labourers	27
Plumbers	12	Plasterers	15
Painters	9	Messengers	3
Bricklayers	18		

(i) If one of the workers is absent on a certain day, what is the probability that he is a bricklayer?

(ii) If a worker is retrenched, what is the probability that he is a plumber or plasterer? ***(WAEC)***

(4) If two fair coins are thrown once, what is the probability of having

(i) A head and a tail
(ii) At least one head
(iii) Two tails

(5) If three fair coins are thrown once, what is the probability of having?

(i) At least two heads
(ii) A head and two tails
(iii) The three showing the same face.

(6) A number is picked at random from the set 25 to 40 inclusive. What is the Probability that it is a

(i) Prime number
(ii) Number divisible by 3.
(iii) Perfect square.

Further Examples

Example 3:

If a letter is picked from the alphabet, what is the probability that

(i) It is a vowel.
(ii) It is NOT a letter of the word "BEAUTIFUL".
(iii) It is a letter of the word "SMALL".

Solution:

$\xi = \{a, b, c, d, ---, x, y, z\}$, $n(\xi) = 26$

(i) Let Q be the Set of vowels in the alphabet.

$Q = \{a, e, i, o, u,\}$

$n(Q) = 5$

$\therefore$ Prob. $(Q) = \frac{n(Q)}{n(\xi)}$

$= {}^{5}/_{26}$

(ii) The word BEAUTIFUL is made up of 8 different letters not 9, because we have 2 of the letter U.

Prob. of the letter of the word BEAUTIFUL $= {}^{8}/_{26}$

$= {}^{4}/_{13}$

$\therefore$ Prob. that it is NOT in the word BEAUTIFUL $= 1 - {}^{4}/_{13}$

$= {}^{9}/_{13}$

(iii) The word "SMALL" is made up of 4 different letters, not 5 since L is written twice.

$\therefore$ Prob. of the letter of the word SMALL $= {}^{4}/_{26}$

$= {}^{2}/_{13}$

Class Activity:

(1) The table below shows the total number of goals scored by 4 players in a league match played in the year 2000.

Names of Players	**Ade**	**John**	**Musa**	**Chidi**
Number of Goals	**6**	**11**	**5**	**8**

If a football march is to be played by the team, what is the probability that

(i) Musa would score a goal,

(ii) John would Not score any goal,

(iii) Ade or Chidi would score a goal?

(2) A pack of 52 playing cards is shuffled and a card is draw at random. Calculate the probability that it is either a five or a red nine.

[Hint: there are 4 fives and 2 red nines in a pack of 52 cards]. ***SSCE JUNE, 1995 NO.4a (WAEC)***

Theoretical probability

Experimental probability is based on numerical data of past experiences to predict the future. But prediction cannot be taken to be perfectly accurate, therefore, the determining probability has been further clarified with the introduction of the **theoretical concept**.

Theoretical probability in its new cases, bases its result and occurrence on exact values that are dependent on the physical nature of the situation under consideration. For instance, if the probability of an event happening is p, the p lies between 0 and 1, i.e $0 \leq p \leq 1$, but the probability that an event is not happening is 1 – p, then it follows that the sum of an event happening and event not happening is always equal to one, (1). That is, p + (1 – p) = p + 1 – p = p – p + 1 = 1.

Probability can also be denoted in set language: if probability of an event happening is Pr(R), then Pr(R) = $\frac{n(R)}{n(U)}$, where R is the required outcome and U is the number of possible outcomes or universal set.

Equiprobable sample space

Equiprobable events are those events whose chances of occurring are the same, e.g if a coin is tossed once, the chance for each of a head and a tail is the same which is ½, likewise, when a fair die is thrown once, each of the numbers 1,2,3,4,5 and 6 on the die has equal chances, or has equiprobable at one out of 6, i.e 1/6 to show up.

Experimental probability bases its result on the actual experiment carried out, and the outcome will therefore, be based on the number of attempts made. Experimental probability uses past numerical records of occurrences in order to arrive at the future occurrences of an event.

OUTCOME TABLES

For some probability problems, all possible outcomes can be obtained by the use of outcome tables, which gives a picture of what the possible outcomes of an experiment should be.

Example :

If two dice are thrown simultaneously, find probability of obtaining

(i) a total of 10
(ii) at least a total of 9
(iii) at least one three.

Solution

The outcome table is given below as follows

1st die

2nd die	**1**	**2**	**3**	**4**	**5**	**6**
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4, 3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

From the table above, there are 36 possible outcomes

(i) Number of required outcome = 3 i.e. {(4,6), (5,5), (6,4) }

Pr {a total of 10 } = $\frac{3}{36}$

= $^{1}/_{12}$

(ii) Number of required outcome = 10

i.e. { (3,6). (4,5) (5,4), (6,3), (4,6), (5,5), (6,4), (5,6), (6,5), (6,6) }

Pr {at least a total of 9 } = $\frac{10}{36}$

= $^{5}/_{18}$

(iii) Number of required outcome = 11

i.e. { (1,3), (2,3), (3,3), 4,3), (5,3), (6,3), 3,1), (3,2), (3,4), (3,5), (3,6) }

∴ Pr {at least one three} = $\frac{11}{36}$

Example :

The Probability that two hunters P and Q hit their target are $^2/_3$ and ¾ respectively. The two hunters aim at a target together.

(a) What is the probability that they both miss the target?

(b) If the target is hit, what is the probability that (i) Only hunter P hits it.

(ii) Only one of them hits it.

(iii) Both hunters hit the target?

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Solution

Let P be the event that P hits target

$P^/$ be the event that P misses target

Q be the event that Q hits target

$Q^/$ be the event that Q misses target.

The outcome of P and Q are independent

$Pr(P) = {}^2/_3$

$\therefore\ Pr(P^/) = 1 - {}^2/_3$

$= {}^1/_3$

$Pr(Q) = ¾$

$\therefore\ Pr(Q^/) = 1 - ¾$

$= ¼$

(a) Pr{that both miss}

$= Pr(P^/) * Pr(Q^/)$

$= {}^1/_3 \text{ x } ¼$

$= {}^1/_{12}$

(b) (i) If only hunter P hits target, it means that hunter Q misses target

∴ Pr{ only hunter P hits target }

$= Pr(P) * Pr(Q^/)$

$= {}^2/_3 \text{ x } ¼$ $= {}^2/_{12}$

$= {}^1/_6$

(i) Since only one of them hits it, the one is not specified. Hence it is either (P hits and Q misses) or (P misses and Q hits)

∴ Pr{only one hits it} = $Pr(P) * Pr(Q^/) + Pr(P^/) * Pr(Q)$

$= \left(\frac{2}{3} \text{ x } \frac{1}{4}\right) + \left(\frac{1}{3} \text{ x } \frac{3}{4}\right)$

$$= \frac{2}{12} + \frac{3}{12}$$

$$= \frac{5}{12}$$

(ii) Pr{both hunter hit target}

$= Pr(P) * Pr(Q)$

$= {}^{2}/_{3} \times ¾$

$= {}^{6}/_{12}$

$= ½$

ASSIGNMENT:

(1) In a contest, Ama, Kwaku and Musa are asked to solve a problem. The probabilities that they solve the problem correctly are respectively ${}^{1}/_{5}$, ${}^{2}/_{3}$, and ${}^{2}/_{5}$.

Calculate the probabilities that:

(i) None of them solves the problem correctly,

(ii) At least one of them solves the problem correctly,

(iii) Only one of them solves the problem correctly.

(WAEC)

(2) The probability that a seed from a certain packet of sunflower seeds will germinate when planted is ${}^{2}/_{5}$. If two seeds are selected at random from this packet. Find the probability that:

(i) The two seeds germinate

(ii) Neither of the two seeds germinates

(iii)Exactly one of the two seeds germinate ***(WAEC)***

(3) Two dice are thrown together. What is the probability of getting

(i) a total score of at least 6,

(ii) a double (i.e. the same number on each die).

(iii) A total score greater than 7

(iv) A double or a total score greater than 7?

(WAEC)

(4) (a) A pair of fair dice each numbered 1 to 6 is tossed. Find the Probability of getting a sum of at least 9

(5) A number selected at random from each of the sets {2, 3, 4,} and {1, 3, 5}. What is the probability that the sum of the two numbers will be less than 7 but greater than 3? ***(WAEC)***

6. If two dies are thrown and the product of the outcome is recorded, what is the probability that it is a

(i) perfect square

(ii) number divisible by 4

number divisible by 9?

WEEK 5

SUBJECT: MATHEMATICS

CLASS: SS 2

TOPIC: PROBABILITY

CONTENT:

(a) Addition and multiplication rules of probability: (i) Mutually exclusive events and addition ("or") rule. (ii) Complimentary events and probability rule. (iii) Independent events and multiplication ("and") rules.
(b) Solving simple problems on mutually exclusive, Independent and complimentary events.
(c) Experiment with or without replacement.
(d) Practical application of probability in; health, finance, population, etc.

ADDITION LAW OF PROBABILITY SHOWN:

Probability of Event A "OR" Event B i.e. Pr (A∪B) *(for intersecting Sets).*

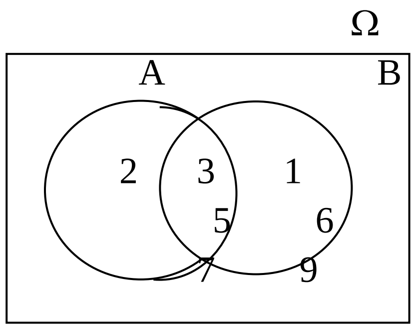
Given any sample space Ω = {1, 2, 3, …, 8, 9, 10} and Event A = {2, 3, 5, 7 } and Event B = {1, 3, 5, 6, 7, 9 }

If a number is picked from the sample space Ω, the probability of picking a number that forms the Set A or B denoted by A ∪ B is explained as follows

From above, Note that

A∪B = {1, 2, 3, 5, 6, 7, 9} **n (A∪B) = 7**

$\therefore$ Prob. (A∪B) = $\frac{7}{10}$ -------------------(1)



Note that A and B are intersecting, hence suppose

$Pr(A \cup B) = Pr(A) + Pr(B)$ *But* $Pr(A) = \frac{4}{10}$

$= \frac{n(A)}{n(\Omega)} + \frac{n(B)}{n(\Omega)}$ $Pr(B) = \frac{6}{10}$

$= \frac{4}{10} + \frac{6}{10}$

$= \frac{10}{10} = 1$ But $Pr(A \cup B) = \frac{7}{10}$

from equation **(1)**

$\therefore \quad Pr(A \cup B) \neq Pr(A) + Pr(B)$

This is because the Set {3, 5, 7} was counted twice. i.e. in A and in B.

More appropriately therefore

$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$

$= \frac{4}{10} + \frac{6}{10} - \frac{3}{10}$

$= \frac{4 + 6 - 3}{10}$

$= \frac{7}{10}$ as in Equation **(1)** above

Hence $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$

--- Thus generally if A and B are intersecting Sets, the probability of A or B is

$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$

— Similarly, it can be shown that if A, B and C are Three intersecting Sets, the probability of A or B or C is:

$Pr(A \cup B \cup C) = Pr(A) + Pr(B) + Pr(C) - Pr(A \cap B) - Pr(A \cap C) - Pr(B \cap C) + Pr(A \cap B \cap C)$

ADDITION LAW OF PROBABILITY SUMMARIZED

From section 5.2A and section 5.2B, the addition law of probability can be summarized as follows:

(1) If A and B are intersecting sets, the probability of A or B denoted by $Pr(A \cup B)$ is given as: $Pr(A\cup B) = Pr(A) + Pr(B) - Pr(A\cap B)$ ---(1)

(2) If A, B and C are intersecting sets, the probability of A or B or C denoted by $Pr(A \cup B \cup C)$ is given as: $Pr(A\cup B\cup C) = Pr(A) + Pr(B) + Pr(C) - Pr(A\cap B) - Pr(A\cap C) - Pr(B\cap C) + Pr(A\cap B\cap C)$ ----------------- (2)

5.2C APPLICATION OF THE ADDITION LAWS OF PROBABILITY.

The addition laws of probability stated above are used to solve problems that contains the word "OR" or "EITHER/OR".

Example 4:

If a number is chosen at random from the integers 10 to 30 inclusive, find the probability that the number is

(i) a multiple of 3 or 5.

(ii) a number divisible by 2 or 3, or 5.

Solution:

$\xi = \{10, 11, 12, 13, \ldots, 28, 29, 30\}$, $n(\xi) = 21$

Let A be the Set of multiples of 3

Let B be the Set of multiples of 5

$A = \{12, 15, 18, 21, 24, 27, 30\}$ $Pr(A) = {}^{7}/_{21}$

$B = \{10, 15, 20, 25, 30\}$ $Pr(B) = {}^{5}/_{21}$

$A\cap B = \{15, 30\}$ $Pr(A\cap B) = {}^{2}/_{21}$

$\therefore Pr((A\cup B) = Pr(A) + Pr(B) - Pr(A\cap B)$

$$Pr(A\cup B\} = \frac{7}{21} + \frac{5}{21} - \frac{2}{21}$$

$$= \frac{7 + 5 - 2}{21}$$

$$= \frac{10}{21}$$

(ii) Let A be the Set of numbers divisible by 3

B be the Set of numbers divisible by 5

C be the Set of numbers divisible by2

$\xi = \{ 10, 11, 12, 13, \ldots, 28, 29, 30\}$ $n(\xi) = 21$

$A = \{12, 15, 18, 21, 24, 27, 30\}$, $Pr(A) = {}^{7}/_{21}$ $B = \{10, 15, 20, 25, 30\}$, $Pr(B) = {}^{5}/_{21}$

$C = \{10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30\}$, $Pr(C) = {}^{11}/_{21}$

$A\cap B = \{15, 30\}$, $Pr(A\cap B) = {}^{2}/_{21}$

$A \cap C = \{12, 18, 24, 30\}$, $Pr(A \cap C) = {}^{4}/_{21}$

$B \cap C = \{10, 20, 30\}$, $Pr(B \cap C) = {}^{3}/_{21}$

$A \cap B \cap C = \{30\}$, $Pr(A \cap B \cap C) = {}^{1}/_{21}$

$Pr(A \cup B \cup C) = Pr(A) + Pr(B) + Pr(C) – Pr(A \cap B) – Pr(A \cap C) – Pr(B \cap C)$
$+ Pr(A \cap B \cap C)$

$= {}^{7}/_{21} + {}^{5}/_{21} + {}^{11}/_{21} – {}^{2}/_{21} – {}^{4}/_{21} – {}^{3}/_{21} + {}^{1}/_{21}$

$= \dfrac{7 + 5 + 11 – 2 – 4 – 3 + 1}{21}$

$= \dfrac{15}{21}$

$= \dfrac{5}{7}$

Example 5:

Find the probability that a number chosen at random from the integer between 10 and 20 inclusive is either a prime number or a multiple of 3.

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Solution:

$\xi = \{10\ 11, 12, \ldots 18, 19, 20\}$ $n(\xi) = 11$

Let A be the Set of prime numbers

B be the Set of multiples of 3

$A = \{11, 13, 17, 19\}$, $Pr(A) = {}^{4}/_{11}$

$B = \{12, 15, 18\}$, $Pr(B) = {}^{3}/_{11}$

Since $n(A \cap B) = 0$

$A \cap B = \{\ \}$ or φ $Pr(A \cap B) = 0$

$\therefore Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$

$= \dfrac{4}{11} + \dfrac{3}{11} - 0$

$= \dfrac{7}{11}$

Example 6:

Out of the 27 students in a class, 17 offer Chemistry, 12 offer Physics and 2 offer non of the subjects. If a student is picked from the class, what is the probability that the student offers

(i) The two Subjects.
(ii) Chemistry or Physics
(iii) Physics only.?

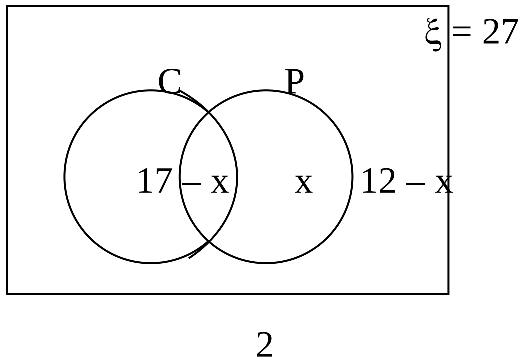
Solution:

$n(\xi) = 27$

$n(C) = 17$

$n(P) = 12$

Let $n(C \cap P) = x$



To get x i.e. Those that offer the two subjects.

$$17 - x + x + 12 - x + 2 = 27$$

$$31 - x = 27$$

$$31 - 27 = x$$

$$\therefore \quad x = 4$$

$\therefore$ 4 Students offer the two subjects i.e. $n(C \cap P) = 4$

(i) $\text{Prob.}(C \cap P) = \frac{4}{27}$

(ii) Prob. of Physic or Chemistry,

$Pr(C \cup P)$ is given as

$$Pr(C \cup P) = Pr(C) + Pr(P) - Pr(C \cap P)$$

$$= \frac{17}{27} + \frac{12}{27} - \frac{4}{27}$$

$$= \frac{17 + 12 - 4}{27}$$

$$= \frac{25}{27}$$

(iii)Physics only $= 12 - x$

$$= 12 - 4$$

$$= 8$$

$\therefore \text{Prob. } (C' \cap P) = \frac{8}{27}$ (Prob. of physics only)

Example 7:

In a community of 50 people, 26 speak Hausa and 29 speak Yoruba. If 13 speak none of these two languages and 18 people speak both languages. If a person is to be chosen from the community for an award, what is the probability that the person speaks

(i) Hausa or Yoruba;

(ii) Only one language;

(iii) Yoruba only?

Solution

Let H be Hausa and Y be Yoruba

(i) $n(\xi) = 50$

$n(H) = 26$

$n(Y) = 29$

$n(H \cap Y) = 18$ $n(H \cup Y)' = 13$

The probability that the person speak Hausa or Yoruba denoted by $Pr(H \cup Y)$ is

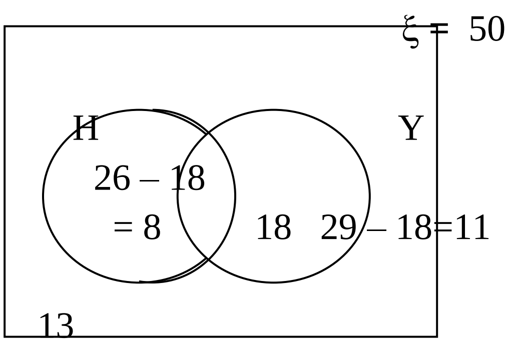
$Pr(H \cup Y) = Pr(H) + Pr(Y) - Pr(H \cap Y)$

$$= \frac{26}{50} + \frac{29}{50} - \frac{18}{50}$$

$$= \frac{26 + 29 - 18}{50}$$

$$= \frac{37}{50}$$

(ii) Those that speak only one language are shown in the Venn diagram below



Hausa only = 8 i.e. $n(H \cap Y') = 8$

Yoruba only = 11 i.e. $n(H' \cap Y) = 11$

Those that speak only one language = 8 + 11

= 19

$\therefore$ Prob. of one language only $= Pr(H \cap Y') + Pr(H' \cap Y)$

$$= \frac{8}{50} + \frac{11}{50}$$

$$= \frac{19}{50}$$

(iii) Yoruba only = 11

i.e. $n(H' \cap Y) = 11$

$$Pr(H' \cap Y) = \frac{11}{50}$$

Class Activity:

(1) A number is chosen at random from the integers 1 to 10. Find the probability that the number is

(i) Prime

(ii) a multiple of 2

(iii) Either a prime or a multiple of 2

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(2) If a number is chosen at random from the matrix

$$A = \begin{pmatrix} 3 & 7 & 9 \\ 4 & 2 & 30 \\ 8 & 15 & 1 \end{pmatrix}$$

what is the probability that it is

(i) a prime number

(ii) a perfect Cube.

(iii) divisible by 2 or 3,

(iv) a perfect Square or divisible by 3?

(3)A number is chosen at random from the integers 5 to 25 inclusive, find the probability that the number is a multiple of 5 or 3. ***SSCE NOV. 1990 NO. 9a (WAEC)***

(4) If a number is chosen at random from the integers 1 to 20 inclusive, find the probability that the number is

(i) a prime number,

(ii) divisible by 2 or 3 ,

(iii) divisible by 2 or 3 or 5 .

(5) In a class of 25 students, 7 can play scrabble game, 9 can play draft, 2 can play both games. If 11 can play none of the two games, find the probability that a student chosen from the class can play

(i) Scrabble or Draft

(ii) Scrabble only

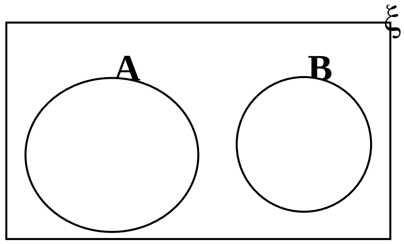
(iii) None of the two games.

Probability of Event A "0R" Event B i.e. Pr(A∪B) ***(For Mutually exclusive events)***

Two events A and B are said to be mutually exclusive if the occurrence of A excludes B. e.g. Head and Tail of coin are mutually exclusive because when a coin is tossed the occurrence of a Head automatically excludes the Tail.

Recall that $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$ ---(1) above

For mutually exclusive events $A \cap B = \varnothing$ i.e. $n(A \cap B) = 0$



$\therefore \quad Pr(A \cup B) = Pr(A) + Pr(B)$

Similarly, **this law can be extended to three or more events, hence**

Recall also that: $Pr(A \cup B \cup C) = Pr(A) + Pr(B) + Pr(C) – Pr(A \cap B) - Pr(A \cap C) –$
$Pr(B \cap C) + Pr(A \cap B \cap C)$ ----------------- (2)above

If A, B and C are mutually exclusive,
The Probability of A or B or C is
$Pr(A \cup B \cup C) = Pr(A) + Pr(B) + Pr(C)$
This is known as the addition law of Probability for mutually exclusive event.

Class Activity:

(1) Find the Probability that a number chosen at random from the integers between 10 and 20 inclusive is either a prime or a multiple of 3. ***SSCE NOV. 1996 NO. 2b.***

(2) A fair die is tossed once. What is the probability of scoring (a) 3 or 6 (b) 4 or 5 (c) neither 6 nor 1

(3) A bag contains 3 black, 4 yellow and 7 red balls. A ball is picked at random from the bag. What is the probability that it is

(a) Black or yellow
(b) Black or red
(c) Neither black nor red

Independent and Complementary events:

<u>Probability of Event A "AND" B i. e. Prob. (A ∩ B)</u>

the coin $Pr(A) = {}^3/_6$
$= ½$
$Pr(B) = ½$

Probability of getting both **independent events:**

Two events are said to be independent if the outcome of one has no effect on the other. e.g. the tossing of a coin and throwing of a die simultaneously. The outcome of the coin does not affect the outcome of the die.

In the case of independent events, the separate probabilities are multiplied to give the combined probability.

PRODUCT LAW

If events A and B are independent, the probability of A and B happening denoted by $Pr(A \cap B)$, is the product of their individual probabilities. i.e.

$PrA \cap B) = Pr(A) \times Pr(B)$

In general, If A, B, C, D, … are independent, the probability of A and B and C and D and … happening is the product of their individual probabilities. I.e.

$Pr(A \cap B \cap C \cap \ldots) = Pr(A) \times Pr(B) \times Pr(C) \times Pr(D) \times \ldots$

NB:

The Product law is used to solve problems with the word "AND" or "BOTH/AND"

Example 9:

If a coin is tossed and a die is thrown, what is the probability of getting a head and a Prime number?

Solution

Since the task of getting a head and a Prime number involves two events which have no effect on each other, the individual probabilities are found and multiplied

A die = {1, 2, 3, 4, 5, 6}

A coin = {H, T}

Let A be the events of getting a Prime number from the die.

∴ B be the events of getting a head from

$Pr(A \cap B) = Pr(A) \times Pr(B)$

$= \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$

∴ $Pr(A \cap B) = \frac{1}{4}$

Example :

A bag contains 7 identical balls, which differ in colour. 4 are white and 3 are Blue. If two balls are drawn from the bag one after the other without replacement, what is the probability that

(i) both are white;

(ii) both are blue?

Solution

1st choice

(iii) There is a total of 7 balls in the bag.

4 are white

∴ Pr {1st is white} = $^4/_7$

For 2^{nd} choice

There are 6 balls left in the bag

3 white are left since one was picked in the first choice

∴ Pr{2^{nd} is white} = $^3/_6$

∴ = ½

∴ Pr{both are white} = $^4/_7$ x ½

= $^2/_7$

(iv) For 1^{st} choice

There are 7 balls in the bag 3 are blue

Pr{1^{st} is blue} = $^3/_7$

For 2nd choice**:**

There are 6 balls left in the bag

2 blue are left in the bag after the first choice

Pr{ 2^{nd} is Blue} = $^2/_6$

= $^1/_3$

∴ Pr{both are Blue} = $\frac{3}{7} \times \frac{1}{3}$

= $^1/_7$

Example :

The Probability that two hunters P and Q hit their target are $^2/_3$ and ¾ respectively. The two hunters aim at a target together.

(c) What is the probability that they both miss the target?

(d) If the target is hit, what is the probability that (i) Only hunter P hits it.

(ii) Only one of them hits it.

(iii)Both hunters hit the target?

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Solution

Let P be the event that P hits target

$P^/$ be the event that P misses target

Q be the event that Q hits target

$Q^/$ be the event that Q misses target.

The outcome of P and Q are independent

Pr(P) = $^2/_3$

∴ Pr($P^/$) = 1 – $^2/_3$

= $^1/_3$

$Pr(Q) = ¾$

$\therefore Pr(Q') = 1 - ¾$

$= ¼$

(b) Pr{that both miss}

$= Pr(P') * Pr(Q')$

$= {}^{1}/_{3} \times ¼$

$= {}^{1}/_{12}$

(b) (i) If only hunter P hits target, it means that hunter Q misses target

$\therefore$ Pr{ only hunter P hits target }

$= Pr(P) * Pr(Q')$

$= {}^{2}/_{3} \times ¼$

$= {}^{2}/_{12}$

$= {}^{1}/_{6}$

(v) Since only one of them hits it, the one is not specified. Hence it is either (P hits and Q misses) or (P misses and Q hits)

$\therefore$ Pr{only one hits it} $= Pr(P) * Pr(Q') + Pr(P') * Pr(Q)$

$$= \left[\frac{2}{3} \times \frac{1}{4}\right] + \left[\frac{1}{3} \times \frac{3}{4}\right]$$

$$= \frac{2}{12} + \frac{3}{12}$$

$$= \frac{5}{12}$$

(vi) Pr{both hunter hit target}

$= Pr(P) * Pr(Q)$

$= {}^{2}/_{3} \times ¾$

$= {}^{6}/_{12}$

$= ½$

Example 12:

The probabilities that three boys pass an examination are ${}^{2}/_{3}$, ${}^{5}/_{8}$ and ¾ respectively. Find the probability that:

(i) *All the three boys passed;*
(ii) *None of the boys passed;*
(iii) *Only two of the boys passed.*

(WAEC)

Solution

Let A be the event that the first boy passed

$A^{/}$ be the event that the first boy did not pass
B be the event that the 2^{nd} boy passed
$B^{/}$ be the event that the 2^{nd} boy did not pass
C be the event that the 3^{rd} boy passed.
$C^{/}$ be the event that the 3^{rd} boy did not pass

$$Pr(A) = {}^{2}/_{3}$$

$$\therefore Pr(A^{/}) = 1 - {}^{2}/_{3}$$

$$= {}^{1}/_{3}$$

$$Pr(B) = {}^{5}/_{8}$$

$$Pr(B^{/}) = 1 - {}^{5}/_{8}$$

$$= {}^{3}/_{8}$$

$$Pr(c) = ¾$$

$$Pr(C^{/}) = ¼$$

(i) Pr{all three passed}

$$= Pr(A) * Pr(B) * Pr(C)$$

$$= {}^{2}/_{3} \text{ x } {}^{5}/_{8} \text{ x } ¾$$

$$= {}^{5}/_{16}$$

(ii)Pr{none of the boys passed}

$$= Pr(A^{/}) * Pr(B^{/}) * Pr(C^{/})$$

$$= {}^{1}/_{3} \text{ x } {}^{3}/_{8} \text{ x } ¼$$

$$= {}^{1}/_{32}$$

(iii)Probability that only two passed. The two are not Specified, hence

$Pr\{only\ two\ passed\} = Pr(A)*Pr(B)* Pr(C^{/}) + Pr(A)*Pr(B^{/})*Pr(C) + Pr(A^{/})*Pr(B)*Pr(C)$

$$= ({}^{2}/_{3} \text{ x } {}^{5}/_{8} \text{ x } ¼) + ({}^{2}/_{3} \text{ x } {}^{3}/_{8} \text{ x } ¾) + ({}^{1}/_{3} \text{ x } {}^{5}/_{8} \text{ x } ¾)$$

$$= \frac{10}{96} + \frac{18}{96} + \frac{15}{96}$$

$$= \frac{43}{96}$$

OUTCOME TABLES

For some probability problems, all possible outcomes can be obtained by the use of outcome tables, which gives a picture of what the possible outcomes of an experiment should be.

Example 13:

If two dice are thrown simultaneously, find probability of obtaining

(iv) a total of 10
(v) at least a total of 9
(vi) at least one three.

Solution

The outcome table is given below as follows

1st die

2nd die	**1**	**2**	**3**	**4**	**5**	**6**
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

From the table above, there are 36 possible outcomes

(ii) Number of required outcome = 3 i.e. {(4,6), (5,5), (6,4) }

Pr {a total of 10 } = $\frac{3}{36}$

= $^1/_{12}$

(ii) Number of required outcome = 10

i.e. { (3,6). (4,5) (5,4), (6,3), (4,6), (5,5), (6,4), (5,6), (6,5), (6,6) }

Pr {at least a total of 9 } = $\frac{10}{36}$

= $^5/_{18}$

(iii) Number of required outcome = 11

i.e. { (1,3), (2,3), (3,3), 4,3), (5,3), (6,3), 3,1), (3,2), (3,4), (3,5), (3,6) }

∴ Pr {at least one three} = $\frac{11}{36}$

PRACTICE EXERCISE:

(1)In a contest, Ama, kwaku and Musa are asked to solve a problem. The probabilities that they solve the problem correctly are respectively $^1/_5$, $^2/_3$, and $^2/_5$.

Calculate the probabilities that:

(iv) None of them solves the problem correctly,

(v) At least one of them solves the problem correctly,

(vi) Only one of them solves the problem correctly.

(WAEC)

(2)The probability that a seed from a certain packet of sunflower seeds will germinate when planted is $^2/_5$. If two seeds are selected at random from this packet. Find the probability that:

(i) The two seeds germinate

(ii) Neither of the two seeds germinates

(iii)Exactly one of the two seeds germinate ***(WAEC)***

(3)Two dice are thrown together. What is the probability of getting

(i) a total score of at least 6,

(ii) a double (i.e. the same number on each die).

(iii) A total score greater than 7

(iv) A double or a total score greater than 7?

(WAEC)

(4)(a) A pair of fair dice each numbered 1 to 6 is tossed. Find the Probability of getting a sum of at least 9

(5)A number selected at random from each of the sets {2, 3, 4,} and {1, 3, 5}. What is the probability that the sum of the two numbers will be less than 7 but greater than 3?

(WAEC)

CONDITIONAL PROBABILITY

Conditional probability helps to link the probability of two or more events. This probability can be calculated when the conditions surrounding the outcome of each event are stated e.g

- The probability of A given that B has occurred
- The Probabilities of events carried out with or without replacement etc

We shall be considering two methods of generating the outcomes of such events. They are

A) The tree diagram approach

B) m^n possible outcome method.

METHOD 1

THE TREE DIAGRAM

The tree diagram helps us to generate all the possible outcome of an experiment and the corresponding probabilities of picking one item from a group of items with or without replacement.

In determining probabilities of joint events for events occurring in a natural sequence, such as in ***example*** below, it is sometimes convenient to represent the probabilities in a tree diagram, as illustrated in the solution below, where each branch of the tree represents a possible outcome at that particular point and the number on each branch represents the probability of that particular event. The probability of being at the end of a particular branch is simply the product of the probabilities on the path, which was traveled to get there. However, for more complicated events tree diagrams becomes impractical

Example

Suppose a bag contains 7 balls out of which 4 are white and 3 are Blue. Represent in a tree diagram the possible outcomes and the corresponding probabilities of picking two balls from the bag one after the other

(I) WITH REPLACEMENT

(II) WITHOUT REPLACEMENT

Solution

(I) WITH REPLACEMENT

∴ Let W represent white
B represent Blue..

White Balls = 4

Blue Balls = $\underline{3}$

7 Ball

1st Choice	2nd Choice	possible Outcomes	corresponding probabilities
W ($\frac{4}{7}$)	W ($\frac{4}{7}$)	WW	$Pr(WW) = \frac{4}{7} \times \frac{4}{7} = \frac{16}{49}$
	B ($\frac{3}{7}$)	WB	$Pr(WB) = \frac{4}{7} \times \frac{3}{7} = \frac{12}{49}$
B ($\frac{3}{7}$)	W ($\frac{4}{7}$)	BW	$Pr(BW) = \frac{3}{7} \times \frac{4}{7} = \frac{12}{49}$
	B ($\frac{3}{7}$)	**BB**	$\mathbf{Pr(BB) = \frac{3}{7} \times \frac{3}{7} = \frac{9}{49}}$

NOTE THAT

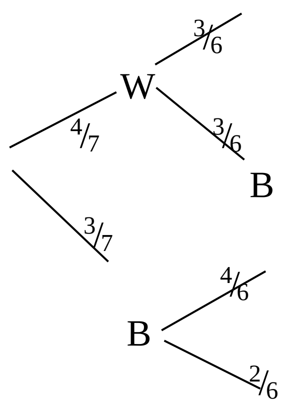
For convenience,

Pr (WW) ⇒ Pr (W ∩ W) , **Pr (WB) ⇒ Pr (W∩ B) etc**

(ii) **WITHOUT REPLACEMENT**

If the balls are drawn from the bag WITHOUT REPLACEMENTS, *the tree diagram would be as shown below.*

1st choice 2nd choice possible corresponding outcomes probabilities



WW $Pr(WW) = \frac{4}{7} \times \frac{3}{6} = \frac{2}{7}$

B WB $Pr(BW) = \frac{4}{7} \times \frac{3}{6} = \frac{2}{7}$

W BW $Pr(BW) = \frac{3}{7} \times \frac{4}{6} = \frac{2}{7}$

B BB $Pr(BB) = \frac{3}{7} \times \frac{2}{6} = \frac{1}{7}$

NB

The possible outcomes and the corresponding probabilities are recorded in front of the tree diagram.

Example 15:

Suppose the bag contains 12 balls out of which 5 are white, 4 are Blue and 3 are red. Represent on tree diagram the possible outcomes and the corresponding probabilities of picking two balls from the bag one after the other.

(i) With replacement

(ii) Without replacement.

Solution

(i) **WITH REPLACEMENT**

Let W be white , B be Blue, R be Red.

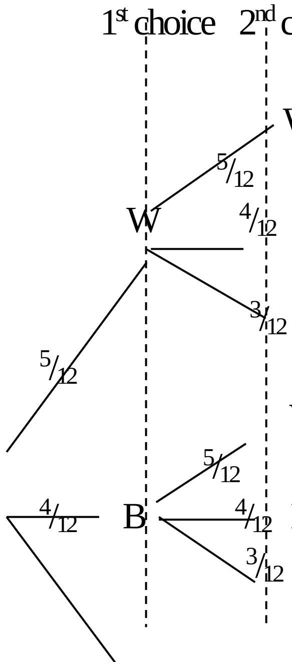
White = 5

Blue = 4

Red = 3 …..

12 Balls.

1st choice 2nd choice possible Outcome corresponding Probabilities



W WW $Pr(WW) = \frac{5}{12} \times \frac{5}{12} = \frac{25}{144}$

B WB $Pr(WB) = \frac{5}{12} \times \frac{4}{12} = \frac{20}{144}$

R WR $Pr(WR) = \frac{5}{12} \times \frac{3}{12} = \frac{15}{144}$

W BW $Pr(BW) = \frac{4}{12} \times \frac{5}{12} = \frac{20}{144}$

B BB $Pr(BB) = \frac{4}{12} \times \frac{4}{12} = \frac{16}{144}$

R BR $Pr(BR) = \frac{4}{12} \times \frac{3}{12} = \frac{12}{144}$

$\frac{3}{12}$ W RW $Pr(RW) = \frac{3}{12} \times \frac{5}{12} = \frac{15}{144}$

$\frac{5}{12}$

R $\frac{4}{12}$ B RB $PR(RB) = \frac{3}{12} \times \frac{4}{12} = \frac{12}{144}$

$\frac{3}{12}$ R RR $Pr(R,R) = \frac{3}{12} \times \frac{3}{12} = \frac{9}{144}$

(ii) **<u>WITHOUT REPLACEMENTS</u>**

1st choice	2nd choice	possible Outcome	corresponding Probabilities
W ($\frac{5}{12}$)	$\frac{4}{11}$ W	WW	$Pr(WW) = \frac{5}{12} \times \frac{4}{11} = \frac{20}{132}$
	$\frac{4}{11}$ B	WB	$Pr(WB) = \frac{5}{12} \times \frac{4}{11} = \frac{20}{132}$
	$\frac{3}{11}$ R	WR	$Pr(WR) = \frac{5}{12} \times \frac{3}{11} = \frac{15}{132}$
B ($\frac{4}{12}$)	$\frac{5}{11}$ W	BW	$Pr(BW) = \frac{4}{12} \times \frac{5}{11} = \frac{20}{132}$
	$\frac{3}{11}$ B	BB	$Pr(BB) = \frac{4}{12} \times \frac{3}{11} = \frac{12}{132}$
	$\frac{3}{11}$ R	BR	$Pr(BR) = \frac{4}{12} \times \frac{3}{11} = \frac{12}{132}$
R ($\frac{3}{12}$)	$\frac{5}{11}$ W	RW	$Pr(RW) = \frac{3}{12} \times \frac{5}{11} = \frac{15}{132}$
	$\frac{4}{11}$ B	RB	$PR(RB) = \frac{3}{12} \times \frac{4}{11} = \frac{12}{132}$
	$\frac{2}{11}$ R	RR	$Pr(R,R) = \frac{3}{12} \times \frac{2}{11} = \frac{6}{132}$

Comments

This tree diagram approach may not be convenient in an examination situation especially when you have 3 different items, to make a choice of 3 or more items. This invariably will lead to a very large tree diagram with 27 possible outcomes. For this reason, I wish to introduce a new approach to obtaining the possible outcomes of such experiments and the corresponding probabilities.

Class Activity:

(1). A Car dealer Sells four brands of Cars, Toyota, Mazda, Nissan and Lexus. If he has 7 Toyota Cars, 4 Mazda, 6 Nissan and 3 Lexus .Two were purchased one after the other, what is the probability that the Cars purchased were

(i) all Toyota Cars,

(ii) all of the same Brand,

(iii) at most one is Lexus ?

(2) A Crate contains 7 bottles of Coke, 4 bottles of Fanta and 3 bottles of Sprite. If three bottles are drawn from the Crate without replacement, what is the Probability that

(i) all are Fanta

(ii) at least 2 are Coke

(iii) all of different brand ?

(3). A bag Contains 5 grapes, 3 Oranges and 2 Mangoes. If three fruits are drawn from the bag one after the other and replaced what is the Probability that

(i) all are grapes

(ii) one of each fruit is drawn,

(iii) at least two Mangoes?

ALTERNATIVE METHOD

METHOD 2

m^n POSSIBLE OUTCOME METHOD

The m^n can be used to obtain the possible outcomes, where m and n are integers and m, n ≥ 2.

m represents the different brands of items to be chosen from.

n is the number of times you are making a choice with or without replacement.

Using the example above i.e.

"A bag contains 7 balls out of which 4 are white and 3 are blue. Show the possible outcome of drawing two balls from the bag.

(I) WITH REPLACEMENT

There are two different colours in the bag ∴ m = 2

Two balls are going to be chosen from the bag ∴ n = 2

∴ Possible outcome is $m^n = 2^2$

$= 4$

NOTE THE DISTRIBUTION OF THE 4 POSSIBLE OUTCOMES

1st choice can be shared down the column allocating $^4/_2$ =2 to each colour as shown in the 1st choice column below, until the 4 outcomes are covered.

2nd choice can be shared down the column allocating $^2/_2$ = 1 to each colour as shown in the 2nd choice column below, until the 4 outcomes are covered

SHORT CUT: { $^4/_2$=2, $^2/_2$=1 }

where ↷ means 'move answer over to compute for the next choice

WITH REPLACEMENT

1st choice	2nd choice	Corresponding Probabilities
W	W	Pr(WW) = 4 x 4 = 16

7 7 49

W	B	$Pr(WB) = \frac{4}{7} \times \frac{3}{7} = \frac{12}{49}$
B	W	$Pr(BW) = \frac{3}{7} \times \frac{4}{7} = \frac{12}{49}$
B	B	$Pr(B,B) = \frac{3}{7} \times \frac{3}{7} = \frac{9}{49}$

Comments

(i) *The denominator at each stage remains constant because probability is with replacement.*

(ii) The outcomes of the table above correspond with the outcomes displayed by the tree diagram

WITHOUT REPLACEMENT:

1^{st} choice	2^{nd} choice	Corresponding Probabilities
W	W	$Pr(WW) = \frac{4}{7} \times \frac{3}{6} = \frac{2}{7}$
W	B	$Pr(WB) = \frac{4}{7} \times \frac{3}{6} = \frac{2}{7}$
B	W	$Pr(BW) = \frac{3}{7} \times \frac{4}{6} = \frac{2}{7}$
B	B	$Pr(B,B) = \frac{3}{7} \times \frac{3}{6} = \frac{1}{7}$

Comment:

The total at the denominator for the second choice reduces because it is without replacement.

Suppose the bag contains two different colours of balls, white and Blue, to make a choice of three balls with or without replacement then we would have

$m^n = 2^3$ possible outcomes {*There are 2 colours in the bag to select 3 times*}.

$2^3 = 8$

NOTE THE DISTRIBUTION OF THE 8 POSSIBLE OUTCOMES

1^{st} choice can be shared down the column, allocating $^8/_2 = 4$ to each colour as shown in the 1^{st} choice column below, until the 8 outcomes are covered.

2^{nd} choice can be shared down the column, allocating $^4/_2 = 2$ to each colour as shown in the 2^{nd} choice column below until the 8 outcomes are covered

3^{rd} choice can be shared down the column allocating $^2/_2 = 1$, to each colour as shown in the 3^{rd} choice column, below until the 8 outcomes are covered.

SHORT CUT: $^8/_2 = 4$ **;** $^4/_2 = 2$ **;** $^2/_2 = 1$

where ↷ means 'move answer over to compute for the next choice

1^{st} choice	2^{nd} choice	3^{rd} choice	corresponding Probabilities
W	W	W	Pr (WWW) =
W	W	B	Pr (WWB) =
W	B	W	Pr (WBW) =
W	B	B	Pr (WBB) =
B	W	W	Pr (BWW) =
B	W	B	Pr (BWB) =
B	B	W	Pr (BBW) =
B	B	B	Pr (BBB) =

Considering the possible outcome table above, it would be observed that the same outcomes would be displayed if a tree diagram is used.

Suppose the bag contains three colours, white, Blue and Red to make a choice of 2 balls from the bag with or without replacement, then the possible outcome table would be prepared as follows.

$m^n = 3^2$ possible outcomes

$3^2 = 9$

1^{st} Choice can be shared down the column, allocating $^9/_3 = 3$ to each colour as shown in the first Choice column below, until the 9 outcomes are covered.

2^{nd} Choice can be shared down the column allocating $^3/_3 = 1$ to each colour as shown in the 2^{nd} choice column below until the 9 outcomes are covered.

SHORT CUT: $^9/_3 = 3$ **;** $^3/_3 = 1$

where ↷ means 'move answer over to compute for the next choice

1st choice	2nd choice	corresponding Probabilities
W	W	Pr (WW) =
W	B	Pr (WB) =
W	R	Pr (WR) =
B	W	Pr (BW) =
B	B	Pr (BB) =
B	R	Pr (BR) =
R	W	Pr (RW) =
R	B	Pr (RB) =
R	R	Pr (RR) =

EXAMPLE OF PROBABILITY WITH REPLACEMENT

Example :

A bag contains 12 balls, out of which 5 are white, 4 are Blue and 3 are Red. If two balls are drawn from the bag one after the other with replacement, what is the probability that

(i) *both are white;*
(ii) *all are the same colour;*
(iii) *one is Blue;*
(iv) *at least one is Blue?*

Solution

Let W be white, B be Blue, R be Red.

White = 5

Blue = 4

Red = 3

Total = 12 Balls

Using m^n possible outcome method.

$m^n = 3^2$ possible outcomes.

$3^2 = 9$ i.e. *There are three Colours in the bag* $\therefore$ $m = 3$

We are Choosing two balls from the bag

$\therefore$ $n = 2$

1st choice: $^9/_3 = 3$, to be shared 3 to a colour down the 1st choice column.

2nd choice: $^3/_3 = 1$, to be shared 1 to a colour down the 2nd choice column.

SHORT CUT: $\left\{ ^9/_3 = 3 \;;\; ^3/_3 = 1 \right\}$

where ↷ means 'move answer over to compute for the next choice

POSSIBLE OUTCOME TABLE

1st choice	2nd choice	Corresponding Probabilities
W	W	$Pr(WW) = \frac{5}{12} \times \frac{5}{12} = \frac{25}{144}$
W	B	$Pr(WB) = \frac{5}{12} \times \frac{4}{12} = \frac{20}{144}$
W	**R**	**Pr (W,R) =**
B	W	$Pr(B,W) = \frac{4}{12} \times \frac{5}{12} = \frac{20}{144}$
B	B	$Pr(B,B) = \frac{4}{12} \times \frac{4}{12} = \frac{16}{144}$
B	R	$Pr(B,R) = \frac{4}{12} \times \frac{3}{12} = \frac{12}{144}$
R	W	Pr (R,W) =
R	B	$Pr(R,B) = \frac{3}{12} \times \frac{4}{12} = \frac{12}{144}$
R	**R**	$\mathbf{Pr(R,R) = \frac{3}{12} \times \frac{3}{12} = \frac{9}{144}}$

<u>Comments</u>

You may choose to use the tree diagram instead of this outcome table. The same solutions would be obtained.

Since the balls are drawn from the bag with replacements, the denominator of all the corresponding probabilities remains constant.

(i) Pr{both are white} = Pr(WW)

$$= \frac{5}{12} \times \frac{5}{12}$$

$$= \frac{25}{144}$$

(ii) Since the two balls could be any of the three colours i.e. white and white or Blue and Blue or Red and Red.

∴ Pr { all are same colour }= Pr(WW) + Pr(BB) + Pr(RR)

$$= \left(\frac{5}{12} \times \frac{5}{12}\right) + \left(\frac{4}{12} \times \frac{4}{12}\right) + \frac{3}{12} \times \frac{3}{12}$$

$= \frac{25}{144} + \frac{16}{144} + \frac{9}{144}$

$= \frac{25 + 16 + 9}{144}$

$= \frac{50}{144}$

$= \frac{25}{72}$

(iii) To get the probability that one is blue, since the outcome could be white and Blue or Blue and white or Blue and Red or Red and Blue.

$\therefore$ Pr{one Blue}= Pr(WB) + Pr(BW) + Pr(BR) + Pr(R,B)

$= \left(\frac{5}{12} \times \frac{4}{12}\right) + \left(\frac{4}{12} \times \frac{5}{12}\right) + \left(\frac{4}{12} \times \frac{3}{12}\right) + \frac{3}{12} \times \frac{4}{12}$

$= \frac{20}{144} + \frac{20}{144} + \frac{12}{144} + \frac{12}{144}$

$= \frac{64}{144}$

$= \frac{4}{9}$

(iv) Since the word "at least" is used, we can have more than one blue

$\therefore$ Pr{ at least one Blue}

= Pr(WB) + Pr(BW) + Pr(B,R) + Pr(R,B) + Pr(BB)

$= \left(\frac{5}{12} \times \frac{4}{12}\right) + \left(\frac{4}{12} \times \frac{5}{12}\right) + \left(\frac{4}{12} \times \frac{3}{12}\right) + \left(\frac{3}{12} \times \frac{4}{12}\right) + \frac{4}{12} \times \frac{4}{12}$

$= \frac{20}{144} + \frac{20}{144} + \frac{12}{144} + \frac{12}{144} + \frac{16}{144}$

$= \frac{80}{144}$

$= \frac{5}{9}$

WITHOUT REPLACEMENT:

Example :

A bag contains 12 balls, out of which 5 are white, 4 are blue and 3 are red. If two balls are picked from the bag one after the other WITHOUT REPLACEMENT, What is the probability that

(i) both are white

(ii) all are the same colour

(iii) One is blue

(iv) *At least one is blue*

Solution

White (W) = 5

Blue (B) = 4

Red (R) = 3 .

Total = 12

POSSIBLE OUTCOME TABLE

1st choice	2nd choice	**Corresponding Probabilities**
W	W	Pr(WW) = $\frac{5}{12} \times \frac{4}{11}$
W	B	Pr(WB) = $\frac{5}{12} \times \frac{4}{11}$
W	**R**	**Pr(W,R) =**
B	W	Pr(B,W) = $\frac{4}{12} \times \frac{5}{11}$
B	B	Pr(B,B) = $\frac{4}{12} \times \frac{3}{11}$
B	R	Pr (B,R) = $\frac{4}{12} \times \frac{3}{11}$
R	W	Pr (R,W) =
R	B	Pr (R,B) = $\frac{3}{12} \times \frac{4}{11}$
R	**R**	**Pr (R,R) =** $\frac{3}{12} \times \frac{2}{11}$

N.B

Since the balls are drawn from the bag without replacements, the denominator of the corresponding probabilities is reduced from 12 to 11

(i) Pr{both white} = Pr(WW)

$$= \frac{5}{12} \times \frac{4}{11}$$

$$= \frac{5}{33}$$

(ii) Pr{all are same colours}

= Pr(WW) + Pr(BB) + Pr(RR)

$$= \left(\frac{5}{12} \times \frac{4}{11}\right) + \left(\frac{4}{12} \times \frac{3}{11}\right) + \left(\frac{3}{12} \times \frac{2}{11}\right)$$

$$= \frac{20}{132} + \frac{12}{132} + \frac{6}{132}$$

$$= \frac{38}{132}$$

$$= \frac{19}{66}$$

(iii) The outcome could be any of the following, white and Blue or Blue and white or Blue and Red or Red and Blue, Hence

Pr{one Blue}= Pr(WB) + Pr(BW) + Pr(BR) + Pr(RB)

$$= \left(\frac{5}{12} \times \frac{4}{11}\right) + \left(\frac{4}{12} \times \frac{5}{11}\right) + \left(\frac{4}{12} \times \frac{3}{11}\right) + \frac{3}{12} \times \frac{4}{11}$$

$$= \frac{20}{132} + \frac{20}{132} + \frac{12}{132} + \frac{12}{132}$$

$$= \frac{64}{132}$$

$$= \frac{16}{33}$$

(iv) Probability of at least one blue implies that we can have more than one blue, hence Pr {at least one blue}

= Pr(WB) + Pr(BW) + Pr(BR) + Pr(RB) + Pr(B,B)

$= ({}^5/_{12} \times {}^4/_{11}) + ({}^4/_{12} \times {}^5/_{11}) + ({}^4/_{12} \times {}^3/_{11})$
$+ ({}^3/_{12} \times {}^4/_{11}) + ({}^4/_{12} \times {}^3/_{11})$

$$= \frac{20}{132} + \frac{20}{132} + \frac{12}{132} + \frac{12}{132} + \frac{12}{132}$$

$$= \frac{76}{132}$$

$$= \frac{19}{33}$$

ASSIGNMENT:

(1) If an unbiased Coin is tossed three times,

(a) List all the possible outcomes.

(b) What is the Probability of obtaining two heads and one tail.

(2) Suppose a bag contains 7 fruits out of which 4 are Oranges and 3 are Mangoes. If three fruits are Selected from the bag one after the other WITHOUT REPLACEMENT, what is the Probability that

(i) all are Oranges

(ii) two are Mangoes
(iii) at least two are Mangoes?

(3)A Sachet contains 15 Chocolates that differs in Colour. 5 are Blue, 6 are white and 4 are green. If two Chocolates were picked from the bag one after the other and were eaten, what is the Probability that

(i) both are white,
(ii) they are of the same Colour
(iii) at least one is green?

(4) A box Contains 10 marbles, 7 of which are black and 3 are red. Two marbles are drawn one after the other without replacement.
(i) List all the possible outcomes.

Find the Probability of getting

(ii) a red, then a black marble

two black marbles. ***SSCE JUNE, 1992. NO. 5b (WAEC)***

WEEK 6

SUBJECT: MATHEMATICS

CLASS: SS 2

TOPIC: FUNCTIONS AND RELATIONS

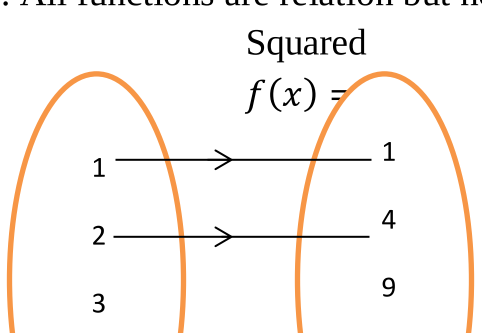
CONTENT:

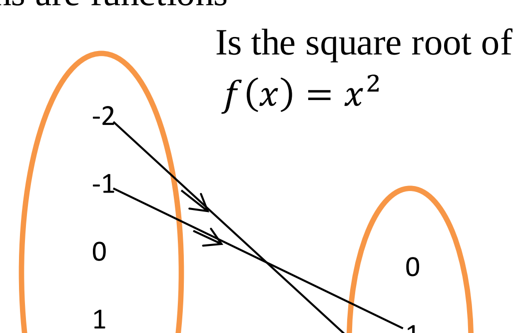
(a) Types of function (one-to-one, one-to-many, many-to-one, many-to-many)
(b) Function as a mapping
(c) Determination of the rule of a given mapping/function.

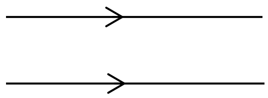
A mapping is simply an association or a relation between two sets
A function is a relation in which each element of the domain has one and only one image in the co – domain. One –to – one and many – to – one relation are therefore functions.
Note: 1. if there exist at least an element in the domain that does not have an image in the co-domain, then it is not a mapping.

2. If an element in the domain has 2 or more images in co-domain, then it is not a mapping.

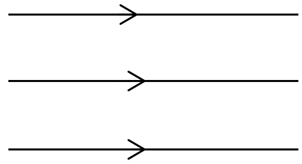
Thus, for a relation to be a mapping; it must be that:
*Every element of the domain has an image in the co-domain
*The image of every element of the domain is unique
Note: All functions are relation but not all relations are functions



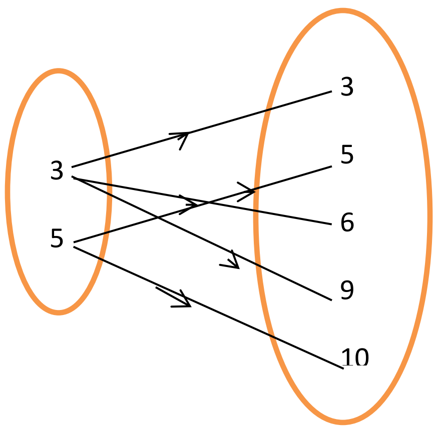




One – to – one



Many – to – one



One – to – many

Many – to – many relation is not a function since some elements of the domain have more than one image.

Examples:

1. Let the function $f: w \rightarrow R$ be defined by $f: x^2 - x - 2$, where $W = \{-1,0,2,5,11\}$ and R the set of real numbers. Find the range of f.

   Solution:

   $f: x^2 - x - 2$

   Taking the value of the domain $W = \{-1,0,2,5,11\}$ one by one

   $f(-1) = (-1)^2 - (-1) - 2 = 0$

   $f(0) = (0)^2 - (0) - 2 = -2$

   $f(2) = (2)^2 - (2) - 2 = 0$

   $f(5) = (5)^2 - (5) - 2 = 18$

   $f(11) = (11)^2 - (11) - 2 = 108$

   $Thus\ , the\ required\ set = \{0, -2, 0, 18, 108\}$

2. Find the domain of $f: x \rightarrow x^2 + 6$, if the range of f is $\{6,7,10,15\}$

Here we are to reverse the process,

When range = 6:

$x^2 + 6 = 6, find\ x$

$x = 0$

When range = 7:

$x^2 + 6 = 7, find\ x$

$x = -1, 1$

When range = 10:

$x^2 + 6 = 10, find\ x$

$x = -2, 2$

When range = 15:

$x^2 + 6 = 15, find\ x$

$x = -3, 3$

$Thus\ the\ domain\ here = \{-3, -2, -1, 0, 1, 2, 3, \}$

Class Activity:

1. If the domain of $f: x \to x^2 - 2$ is $\{-2, -1, 0, 1\}$, find its range.
2. Find the domain of $f: x \to x^2 + 1$ if the range is $\{1, 2, 5, 10\}$

PRACTICE EXERCISE:

1. Find the domain of $f(x) = \frac{x}{3 - x}$, where$x \in R$, the set of real numbers.
2. Given that P = (x : x is a factor of 6) is the domain of g(x) = x^2 + 3x – 5, find the range of g(x)
3. Which of the following functions is/are one – to – one?
   $f: x \to x^2$, $g: x \to 2x + 1$, $h: x \to \sqrt{x}$
4. Two functions f and g are defined by f : x → $3x - 1$ and g : x → $2x^3$, evaluate fg(-2)
5. The functions f and g are defined on the set R of real numbers by
   f : x → $2x^2 - 3$ and g : x→ $4 - x$, Find fog.

ASSIGNMENT:

1. If $f(x) = \frac{x-3}{2x-1}$, $x \neq \frac{1}{2}$ $and\ g(x) = \frac{x-1}{x+1}$, $x \neq -1$, find gof
2. Given that f(x) = 2x – 1 and g(x) = x^2 + 1
   (i) Find f(1 + x);
   (ii) Find the range of values of x for which f(x) < -3
   (iii) Simplify f(x) – g(x)
3. The function f and g are defined as f : x → $x - 2$ and g : x→ $2x^2 - 1$,
   Solve (i) f(x) = g$\left(\frac{1}{2}\right)$ (ii) f(x) + g(x) = 0
4. Given that f : x→ $3x - 1$ $and\ g : x \to x^2 + 1$
   (i) Evaluate g(-2) – f(-1)
   (ii) Simplify f(x) + g(x)
   (iii) Solve f(x) = g(x)
5. Write short notes on the following
   (i) Identity mapping
   (ii) Constant mapping
   (iii) Surjective mapping
   (iv) Composition of mapping
   (v) Ordered pair mapping

WEEK 7
MID-TERM BREAK

WEEK 8

SUBJECT: MATHEMATICS

CLASS: SS 2

TOPIC: VECTORS

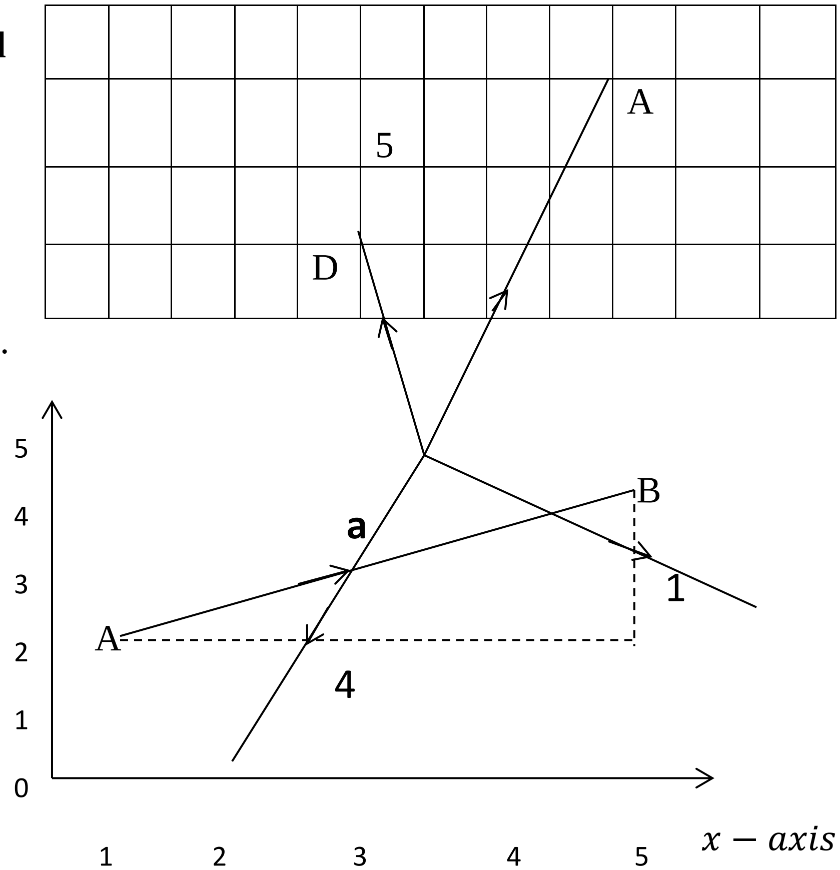
CONTENT:

(a) Vectors as directed line segment.

(b) Cartesian components of a vector.
(c) Magnitude of a vector, Equal vectors, Addition and subtraction of vectors, zero vectors, parallel vectors, multiplication of a vector by a scalar.

Vectors as directed line segment

A vector is any quantity which has direction as well as magnitude or size. Displacement, velocity, force, acceleration are all examples of vectors.

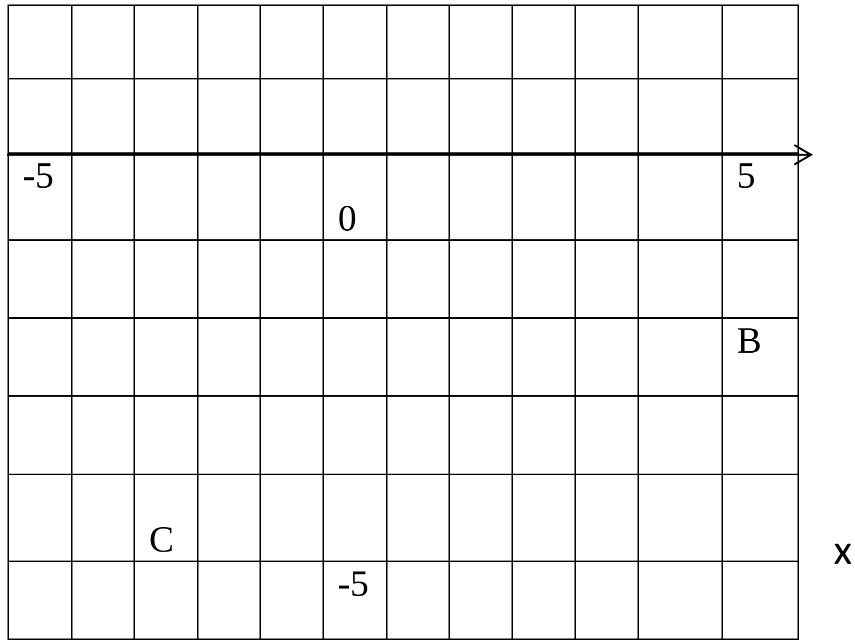


Since the points are on a Cartesian plane, AB can also be written as a column matrix, or column vector:

AB = a = $\begin{pmatrix}4\\2\end{pmatrix}$, Direction is important. BA is in the opposite direction to AB, although they are both parallel and have the same size: $BA = -AB = -\begin{pmatrix}4\\2\end{pmatrix} = \begin{pmatrix}-4\\-2\end{pmatrix}$

A displacement vector is a movement in a certain direction without turning.
The vector '**a**' is called the position vector of AB

Hence if a point has coordinates (x , y), its position vector is $\begin{pmatrix}x\\y\end{pmatrix}$. The figure shows the position vectors $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}, \overrightarrow{OD}$



In the figure above, the position vectors are as follows:

$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ $\overrightarrow{OB} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ $\overrightarrow{OC} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$ $\overrightarrow{OD} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

Class Activity:

Draw line segments to represent the following vectors.

1. $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}, \overrightarrow{CD} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}, \overrightarrow{EF} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}, \overrightarrow{GH} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$
2. $\overrightarrow{KL} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \overrightarrow{MN} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}, \overrightarrow{PQ} = \begin{pmatrix} -7 \\ -8 \end{pmatrix}$

The component of a vector in the Cartesian plane is denoted by $i\ and\ j$, given the component $ai + bj$, the 'a' is the i-component of the x-axis while the 'b' is the j-component of the y-axis.

Magnitude of a vector;

If $= \begin{pmatrix} x \\ y \end{pmatrix}$, then $|a| = \sqrt{x^2 + y^2}$, where $|a|$ is the magnitude of a. Notice that the magnitude of a vector is always given as a positive number of units.

Class Activity:

Find the magnitudes or modulus of the following vectors;

(a) $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

(b) $\begin{pmatrix} -5 \\ -12 \end{pmatrix}$

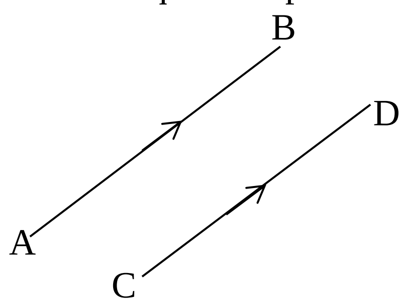
(c) $\begin{pmatrix} -15 \\ 8 \end{pmatrix}$

(d) $If\ \overrightarrow{AB} = \begin{pmatrix} 11 \\ 5 \end{pmatrix} + \begin{pmatrix} -2 \\ 7 \end{pmatrix}, find\ |\overrightarrow{AB}|$

(e) $p = \begin{pmatrix} 7 \\ -3 \end{pmatrix}\ and\ q = \begin{pmatrix} -6 \\ 2 \end{pmatrix}, find\ |p - q|$

Equal vectors and parallel vectors;

Two or more vectors are equal and parallel if they have the same magnitude and direction.



In the figure above, $\overrightarrow{AB} = \overrightarrow{CD}. This\ implies\ that\ /\ \overrightarrow{AB}/=/\overrightarrow{CD}/$ $and\ \overrightarrow{AB}\ and\ \overrightarrow{CD}\ have\ the\ same\ direction,$ i.e they are parallel.

Addition and subtraction of vectors;

Vectors are said to be added or subtracted component wise. A vector can be added or subtracted from another if they have equal number of components.

Examples:

1. Given the vectors $u = \begin{pmatrix} 4 \\ -5 \end{pmatrix}\ and\ v = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$, find (i) u + v (ii) u – v (iii) v – u

   Solution:

   (i) u + v = $\begin{pmatrix} 4 \\ -5 \end{pmatrix} + \begin{pmatrix} -2 \\ -3 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$

   (ii) u – v = $\begin{pmatrix} 4 \\ -5 \end{pmatrix} - \begin{pmatrix} -2 \\ -3 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$

   (iii) v – u = $\begin{pmatrix} -2 \\ -3 \end{pmatrix} - \begin{pmatrix} 4 \\ -5 \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$

2. Given that A = (3,4) and B = (7,-24), find (i) the addition of A and B (ii) subtract B from A (iii) subtract A from B.

   Class Activity:

   1. If $x = \begin{pmatrix} 6 \\ 3 \end{pmatrix}, y = \begin{pmatrix} -1 \\ 5 \end{pmatrix}, z = \begin{pmatrix} -4 \\ 2 \end{pmatrix}, w = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$, find;

      (a) x – y

      (b) x – y + z

      (c) z – x – w

      (d) w – y + x

      (e) (x + y) – (z – w)

   2. Draw OP = $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$ and OQ = $\begin{pmatrix} 6 \\ 7 \end{pmatrix}$

      (a) Use your drawing to find PQ

      (b) Use any method to find OQ – OP

Multiplication of a vector by a scalar.

If any vector is multiplied by a scalar, say 3, the result is a vector 3 times as big as the initial vector. Also, if multiplied by a scalar, say $\frac{1}{2}$, the result is a vector half its initial size.

Note: A scalar is simply a numerical multiplier.

Examples:

Given the following vectors; AB = $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and CD= $\begin{pmatrix} 6 \\ 9 \end{pmatrix}$, find (i) 2AB (ii) $-$3BA (iii) $-\frac{1}{3}CD$

Solution:

(i) 2AB = $2\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \times 2 \\ 2 \times -1 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$

(ii) Note that BA = $-$AB,

$-3\text{BA} = -3\begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \times -2 \\ -3 \times 1 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$

(iii) $-\frac{1}{3}\begin{pmatrix} 6 \\ 9 \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} \times 6 \\ -\frac{1}{3} \times 9 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$. (note that the entries of a vector can also be in fractional form or decimal)

Class Activity:

1. Given that $a = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and $b = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$, express each of the following as column vectors,
   (i) 2a + 3b
   (ii) – 2b – 5a
   (iii) $-\frac{1}{2}b + 4a$
   (iv) $-\frac{1}{3}(a + 3b)$
2. What is the resultant of the vectors $\overrightarrow{RS}, -\overrightarrow{QP}$ and $\overrightarrow{QR}$?

PRACTICE EXERCISE:

1. If $BC = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, CD = \begin{pmatrix} -1 \\ -2 \end{pmatrix}, DA = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$, $show\ that\ ABCD\ is\ a\ parallelogram$
2. What is the sum of $\overrightarrow{PQ}, -\overrightarrow{PS}, -\overrightarrow{RQ}$ and $\overrightarrow{SR}$?
3. If PQ = u and PR = v, find PM where M is the mid-point of QR.
4. A vector b is such that $\begin{pmatrix} 7 \\ 2 \end{pmatrix} + b = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$, $find\ b$
5. The coordinates of the vertices of a parallelogram QRST are Q(1,6), R(2,2), S(5,4) and T(x,y).
   (a) Find the vectors QR and TS and hence determine the values of x and y.
   (b) Calculate the magnitudes of RS and QT.

WEEK 9

SUBJECT: MATHEMATICS

CLASS: SS 2

TOPIC: TRANSFORMATIONS

CONTENT:
(a) Translation of points and shapes on the Cartesian plane.
(b) Reflection of points and shapes on the Cartesian plane.
(c) Rotation of points and shapes on the Cartesian plane.
(d) Enlargement of points and shapes on the Cartesian plane.

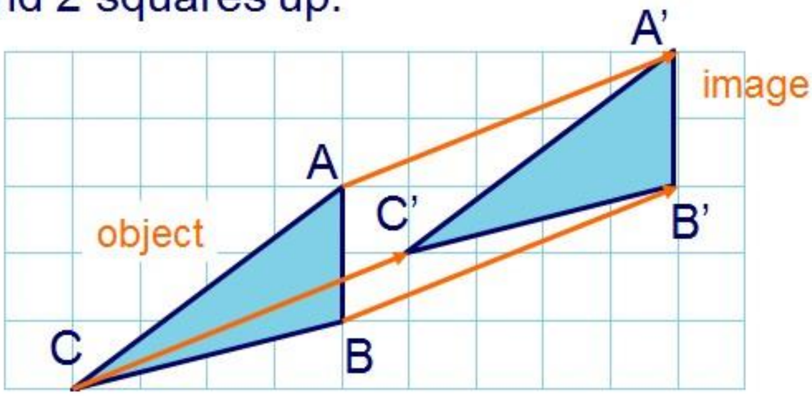
When the position or dimensions (or both) of a shape changes, we say it is **transformed**. The image is the figure which results after transformation of the shape. If the image has the same dimension as the original shape, the transformation is called a congruency. (Two shapes are congruent if their corresponding dimensions are congruent). A transformation is a mapping between two shapes.

Translation of points and shapes on the Cartesian plane.
A Translation is a movement in a straight line. Under a translation every point in a line or plane shape moves the same distance in the same direction by a fixed translation or displacement vector. Note: $\binom{x}{y} = (x, y)$

In general, if the position vector of a point $\binom{x}{y}$ is given by the translation $\binom{a}{b}$, the position vector of its image is $\binom{x+a}{y+b}$. We write $\binom{x}{y} \to \binom{x+a}{y+b}$ and say $\binom{x}{y}$ maps to $\binom{x+a}{y+b}$

When an object is moved in a straight line in a given direction we say that it has been **translated**.

As an example, we can translate triangle ABC 5 squares to the right and 2 squares up:



Every point in the shape moves the same distance in the same direction.

Examples:

1. A translation maps (5, -4) on to (3, -6).

   (a) What is the displacement vector?

   (b) What is the image of (-2, 7) under this translation?

   Solution:

   (a) Let the displacement vector be $\begin{pmatrix} a \\ b \end{pmatrix}$,

   Point + displacement = image

   $$\begin{pmatrix} 5 \\ -4 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$$

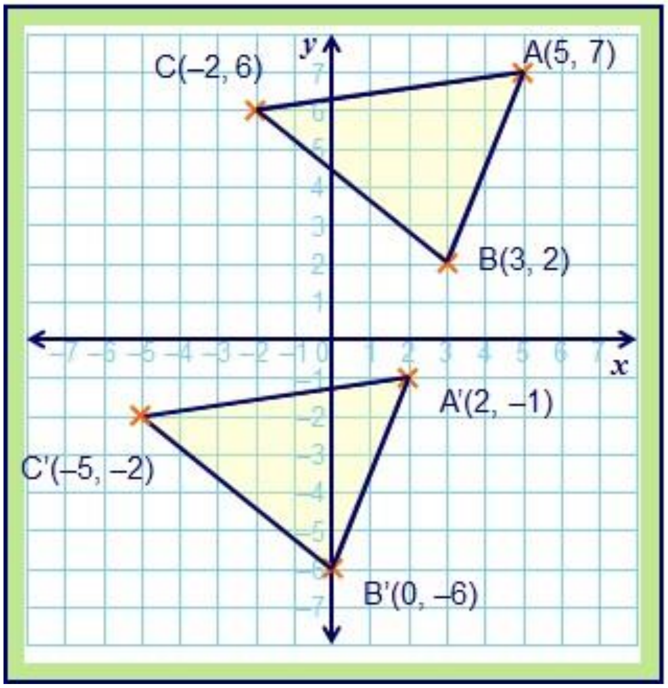
   $$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \end{pmatrix} - \begin{pmatrix} 5 \\ -4 \end{pmatrix}$$

   $$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

   (b) $\begin{pmatrix} -2 \\ 7 \end{pmatrix} + \begin{pmatrix} -2 \\ -2 \end{pmatrix} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$,

   Hence, Image under this translation is $\begin{pmatrix} -4 \\ 5 \end{pmatrix}$

2.

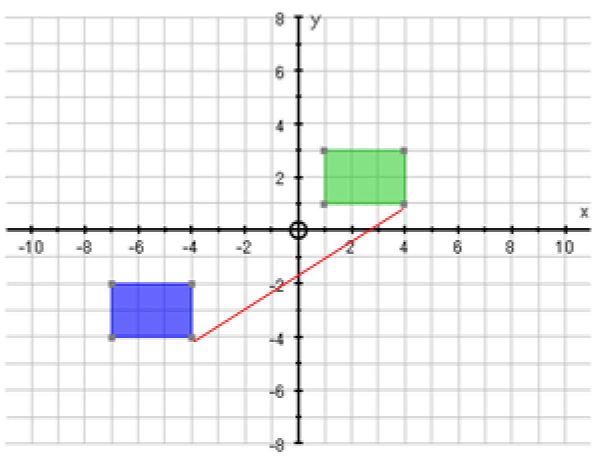


The vertices of a triangle lie on the points A(5, 7), B(3, 2) and C(–2, 6).

Translate the shape 3 squares left and 8 squares down. Label each point in the image.

What do you notice about each point and its image?

3.



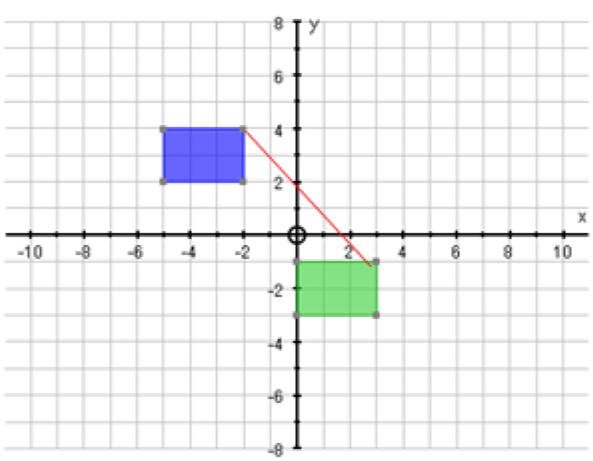
If we translate the blue object by the vector:

$$\begin{pmatrix} 8 \\ 5 \end{pmatrix}$$

8 to the right
5 up

We end up with the green object

Notice: If you pick any co-ordinate on the blue shape and translate it by the same vector, you end up with the matching corner on the green shape



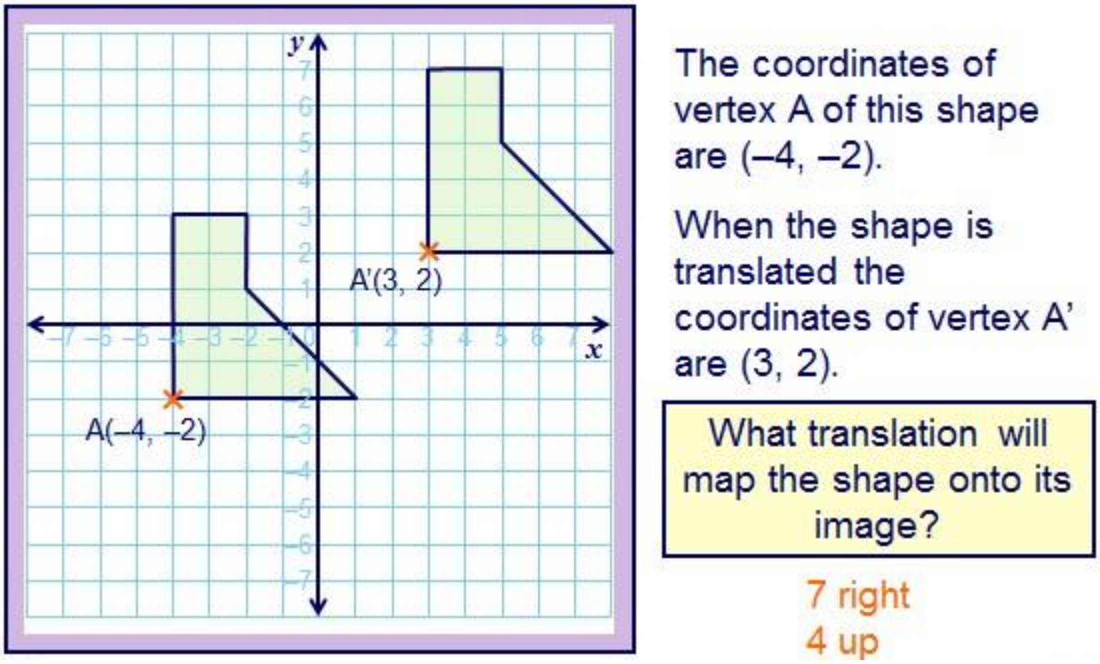
If we translate the blue object by the vector:

$$\begin{pmatrix} 5 \\ -5 \end{pmatrix}$$

5 to the right
5 down

We end up with the green object

4.

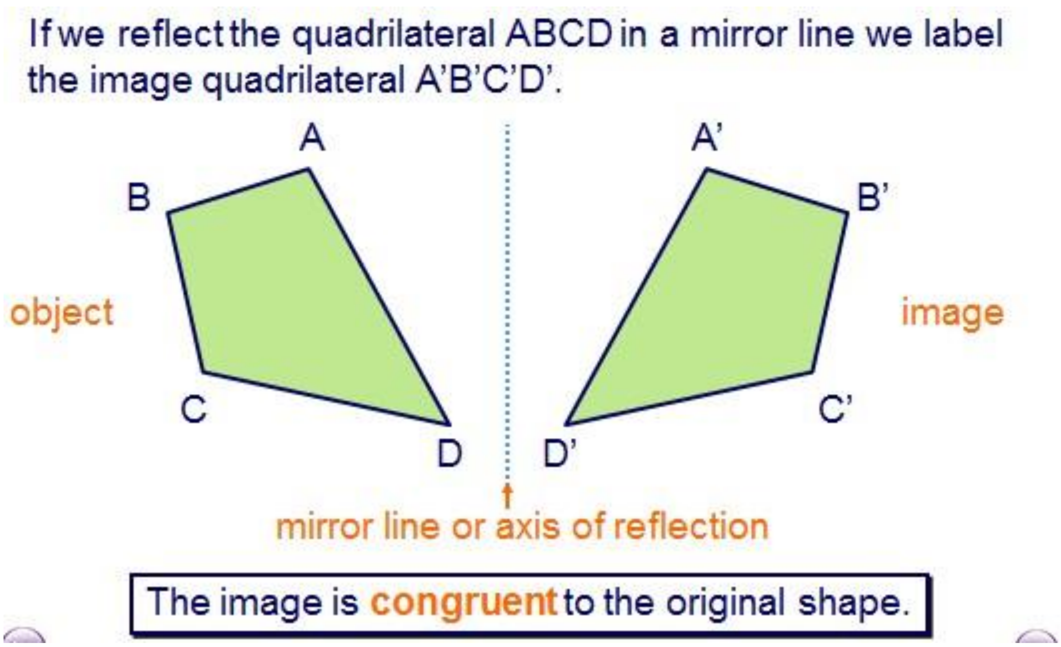


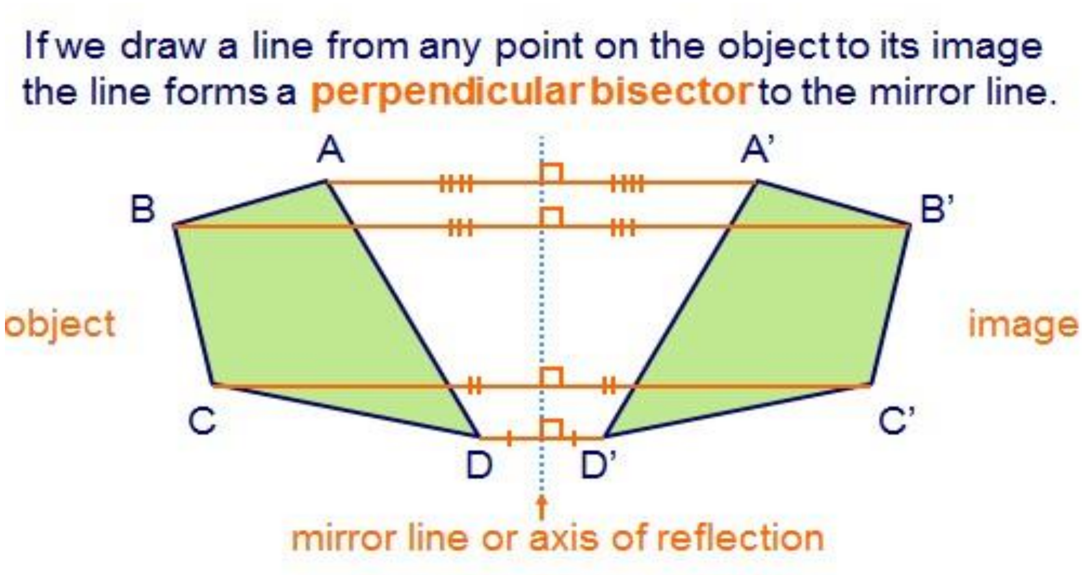
Class Activity:

1. What is the image of P(-2, -5) under the translation $\binom{6}{2}$?
2. The vertices of triangle ABC are represented by the coordinates A(-2,-1), B(2,0), C(2,-2). Draw this triangle on graph paper and show its image under the translation $\binom{4}{3}$.

Reflection of points and shapes on the Cartesian plane

A reflection is the image you see when you look in a mirror. The line of the mirror is a line of symmetry between the object shape and its image. In a Cartesian plane, there are infinitely many lines of reflection. The following describes some of the important ones.





Reflection in the x-axis:

The point P(4,2) is reflected in the x-axis. Its image P'(4,-2) is the same distance from the x-axis as the point P. if the position vector of a point is $\binom{x}{y}$, the position vector of its image under reflection in the x-axis is $\binom{x}{-y}$. This gives the mapping $\binom{x}{y} \rightarrow \binom{x}{-y}$

Reflection in the y-axis:

If a point is reflected in the y-axis, its image P'(-2,1) is the same distance from the y-axis. If the position vector of a point is $\binom{x}{y}$, the position vector of its image under reflection in the y-axis is $\binom{-x}{y}$. This gives the mapping $\binom{x}{y} \rightarrow \binom{-x}{y}$

Reflection in the line y = x:

The image of the vector P(2,5) is P'(5,2) after reflection in the line y = x, this mapping is equivalent to $\binom{x}{y} \rightarrow \binom{y}{x}$

Reflection in the line y = -x:

The image of P(1,3) is P'(-3,-1) after reflection in the line y = -x. This mapping is equivalent to $\binom{x}{y} \rightarrow \binom{-y}{-x}$

Example:

1. If a point P has the coordinates (5,-2), find its reflection in the;

(a) x-axis

(b) y-axis

(c) line y = x

(d) line y = -x

Solution:

Let the image of P be P' after reflection.

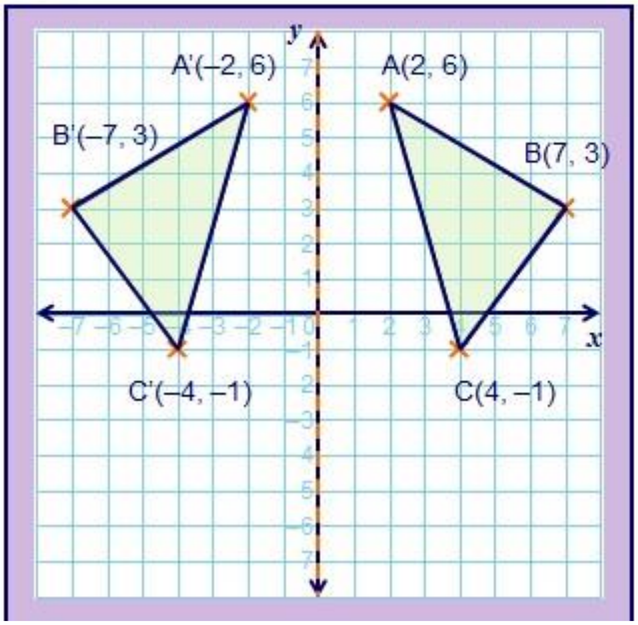
(a) In the x-axis, $\binom{5}{-2} \rightarrow \binom{5}{2}$, the coordinate of P', the image of P, are (5,2)

(b) In the y-axis, $\begin{pmatrix} 5 \\ -2 \end{pmatrix} \rightarrow \begin{pmatrix} -5 \\ -2 \end{pmatrix}$, the coordinate of P', the image of P, are (-5, -2)

(c) In the line y = x, $\begin{pmatrix} 5 \\ -2 \end{pmatrix} \rightarrow \begin{pmatrix} -2 \\ 5 \end{pmatrix}$, the coordinate of P', the image of P are (-2,5)

(d) In the line y = -x, $\begin{pmatrix} 5 \\ -2 \end{pmatrix} \rightarrow \begin{pmatrix} 2 \\ -5 \end{pmatrix}$, the coordinate of P', the image of P are (2, -5)

2.

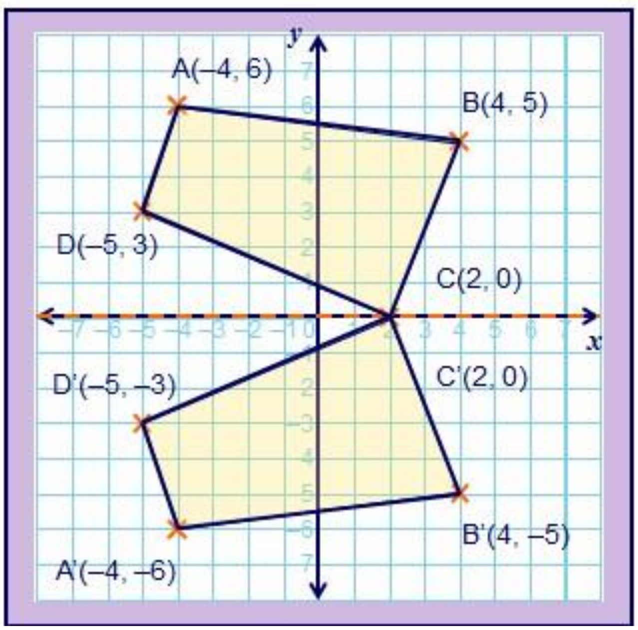


The vertices of a triangle lie on the points A(2, 6), B(7, 3) and C(4, –1).

Reflect the triangle in the *y*-axis and label each point on the image.

What do you notice about each point and its image?

3.

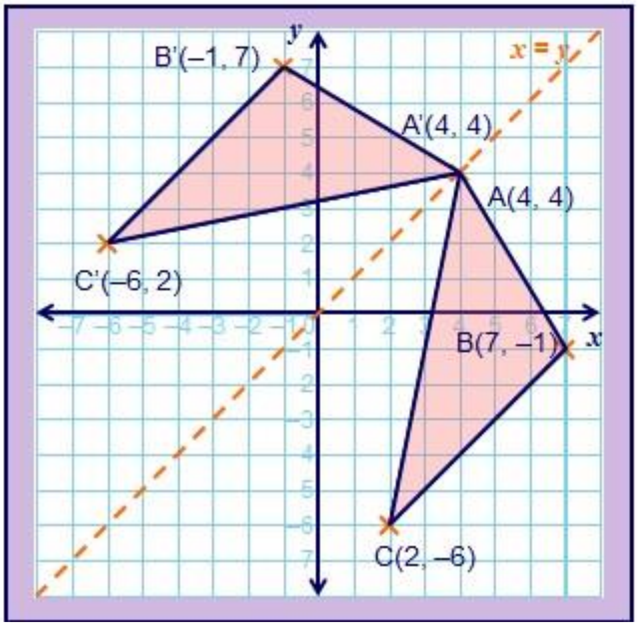


The vertices of a quadrilateral lie on the points A(–4, 6), B(4, 5), C(2, 0) and D(–5, 3).

Reflect the quadrilateral in the *x*-axis and label each point on the image.

What do you notice about each point and its image?

4.

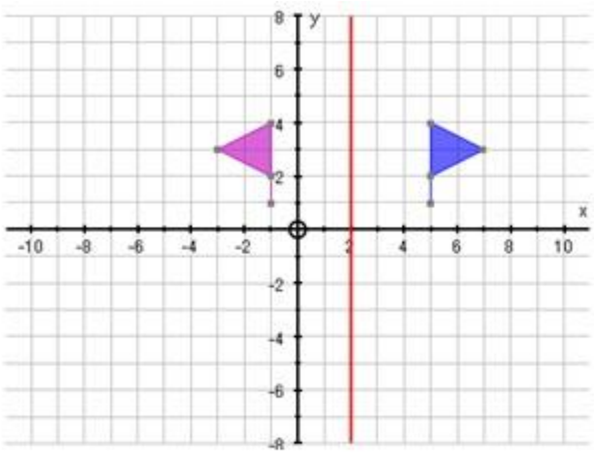


The vertices of a triangle lie on the points A(4, 4), B(7, –1) and C(2, –6).

Reflect the triangle in the line $y = x$ and label each point on the image.

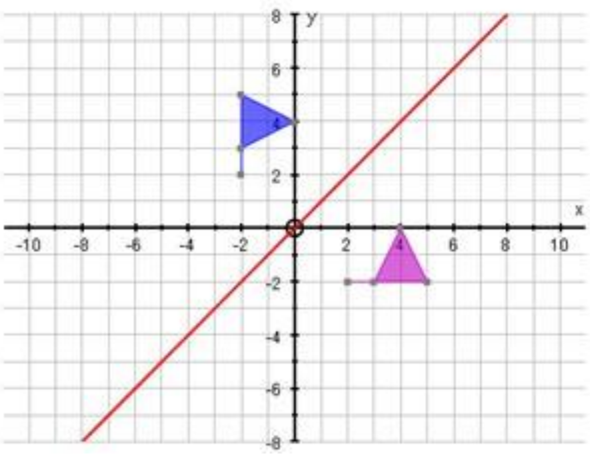
What do you notice about each point and its image?

5.



If we reflect the blue object in the red line (equation: x = 2), we end up with the purple object

Notice: Every point on the purple object (the image) is the exact same distance from the line of reflection as the matching point on the blue object



If we reflect the blue object in the red line (equation: y = x), we end up with the purple object

Notice: I find it much harder to reflect when the mirror line is diagonal, but notice how every point on the image is still the same distance away from the mirror line as the matching point on the original object.

Class Activity:

1. State the coordinate of the image of point A(3,2) after reflection in the;
   (a) x-axis
   (b) y-axis
   (c) line y = x
   (d) line y = -x
2. The coordinates of triangle ABC are A(1,6), B(4,6), C(2,5). Find the coordinate of the image of triangle ABC after reflection in (a) the line y = x (b) the line y = -x

Rotation of points and shapes on the Cartesian plane.

If a point P, whose position vector is $\begin{pmatrix} x \\ y \end{pmatrix}$, is rotated through $\theta°$ in the anticlockwise sense about the origin, by construction, the position vector of the image P' is;

(a) $\begin{pmatrix} -y \\ x \end{pmatrix}$ for $\theta = 90°$

(b) $\begin{pmatrix} -x \\ -y \end{pmatrix}$ for $\theta = 180°$

(c) $\begin{pmatrix} y \\ -x \end{pmatrix}$ for $\theta = 270°$

If the rotation is clockwise, the position vector of the image, P' is;

(a) $\begin{pmatrix} y \\ -x \end{pmatrix}$ for $\theta = 90°$

(b) $\begin{pmatrix} -x \\ -y \end{pmatrix}$ for $\theta = 180°$

(c) $\begin{pmatrix} -y \\ x \end{pmatrix}$ for $\theta = 270°$

A **rotation** occurs when an object is turned around a fixed point.

To describe a rotation we need to know three things:

- The **angle** of rotation.

  For example:

  ½ turn = 180°   ¼ turn = 90°   ¾ turn = 270°

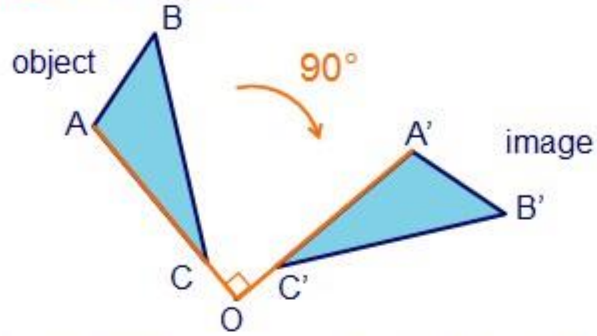
- The **direction** of rotation.

  Clockwise or anticlockwise?

- The **centre** of rotation.

  This is the fixed point about which an object moves.

If we rotate triangle ABC 90° clockwise about point O the following **image** is produced:



A is mapped onto A', B is mapped onto B' and C is mapped onto C'.

The image triangle A'B'C' is **congruent** to triangle ABC.

Sometimes the direction of the rotation is not given.

If this is the case then we use the following rules:

A **positive** rotation is an **anticlockwise** rotation.

A **negative** rotation is an **clockwise** rotation.

Here are two examples:

A rotation of 60° = an anticlockwise rotation of 60°.

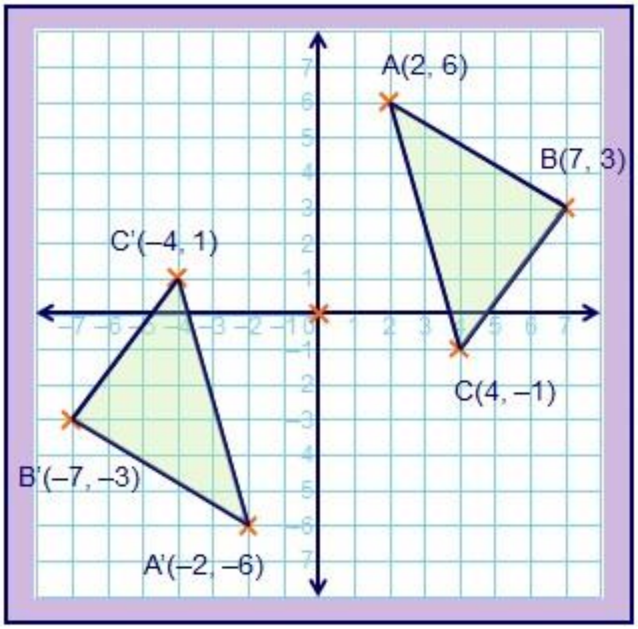
A rotation of –90° = an clockwise rotation of 90°.

> Explain why a rotation of 120° is equivalent to a rotation of –240°.

Examples:

1. If the point P(2,4) is rotated anticlockwise through 90^0 about the origin, determine the coordinates of the image.
   Solution:
   Under rotation through 90^0 anticlockwise; $\binom{x}{y} \rightarrow \binom{-y}{x}$,
   Therefore, $\binom{2}{4} \rightarrow \binom{-4}{2}$, the coordinate of the image are $(-4,2)$
2. The point T$(-3,2)$ is rotated anticlockwise through a half turn (that is 180^0) about the origin. Determine the coordinates of the image.
   Solution:
   Under rotation through 180^0 anticlockwise; $\binom{x}{y} \rightarrow \binom{-x}{-y}$,
   Therefore, $\binom{-3}{2} \rightarrow -\binom{-3}{2} \rightarrow \binom{3}{-2}$, the coordinate of the image are $(3, -2)$

3.

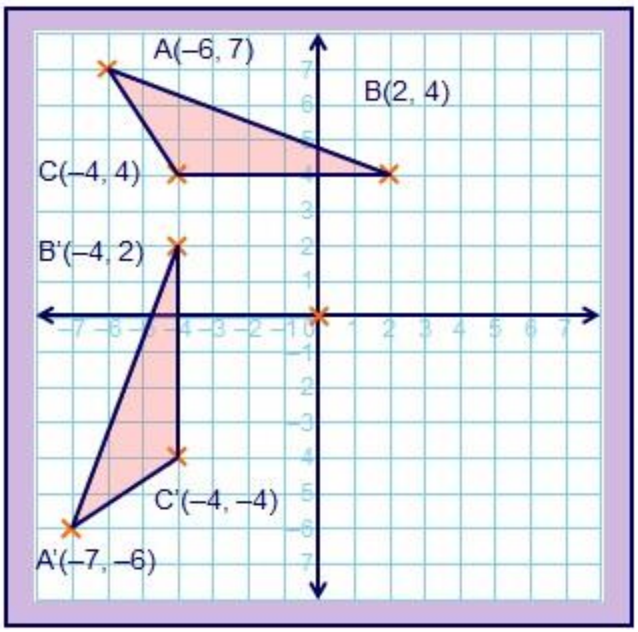


The vertices of a triangle lie on the points A(2, 6), B(7, 3) and C(4, –1).

Rotate the triangle 180° clockwise about the origin and label each point on the image.

What do you notice about each point and its image?

4.

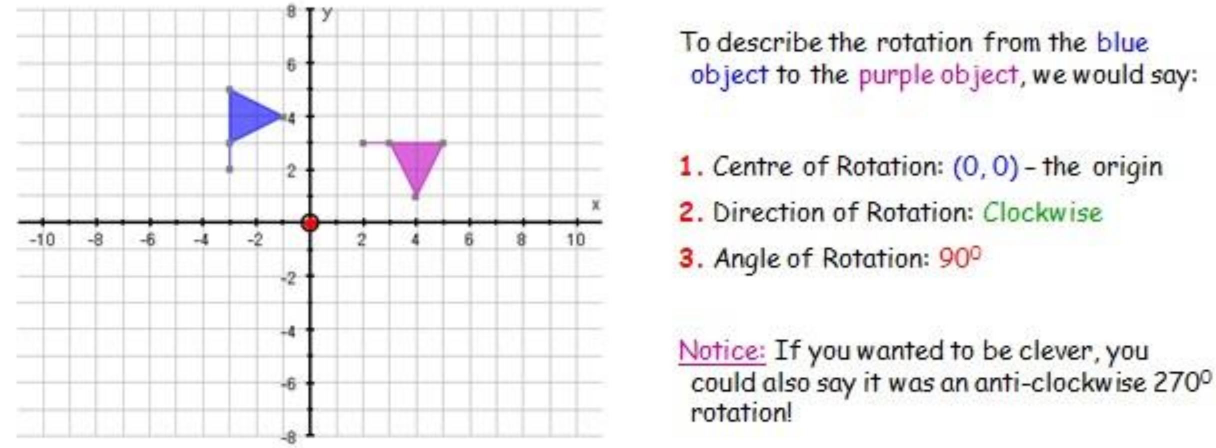


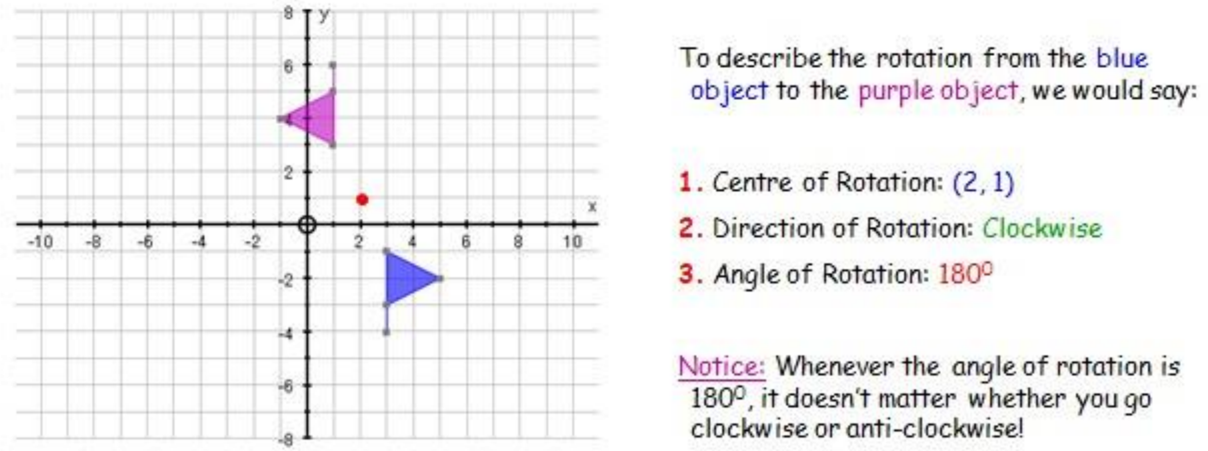
The vertices of a triangle lie on the points A(–6, 7), B(2, 4) and C(–4, 4).

Rotate the triangle 90° anticlockwise about the origin and label each point in the image.

What do you notice about each point and its image?

5.





Class Activity:

1. Determine the coordinates of the image of the point P(-4,3) if it is rotated anticlockwise through 90^0 about the origin.
2. The point B(6, -2) is rotated through a half turn about the origin. Find R(B), the image of B under rotation if
   (a) the rotation is clockwise
   (b) the rotation is anticlockwise

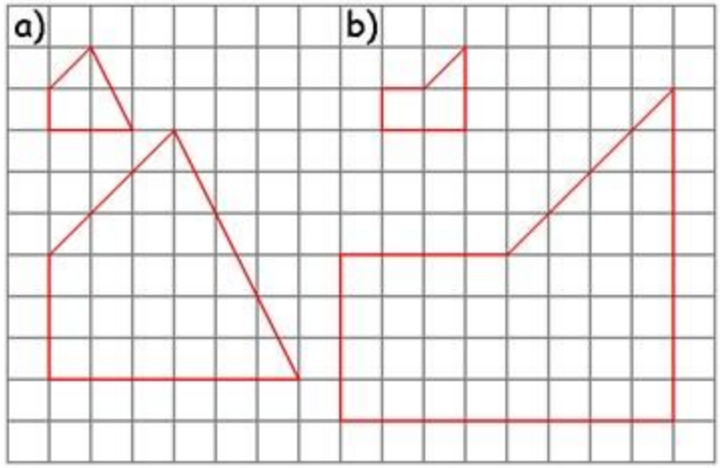
Enlargement of points and shapes on the Cartesian plane.

An enlargement is a transformation in which a shape is made bigger or smaller according to a given scale factor and a centre of enlargement which does not change.

Enlargement is the only one of the four transformations which changes the size of the object
Key Point: Enlargements can make objects bigger as well as smaller!
Each length is increased or decreased by the same scale factor

(a) Scale Factor = 3
(b) Scale Factor = 4

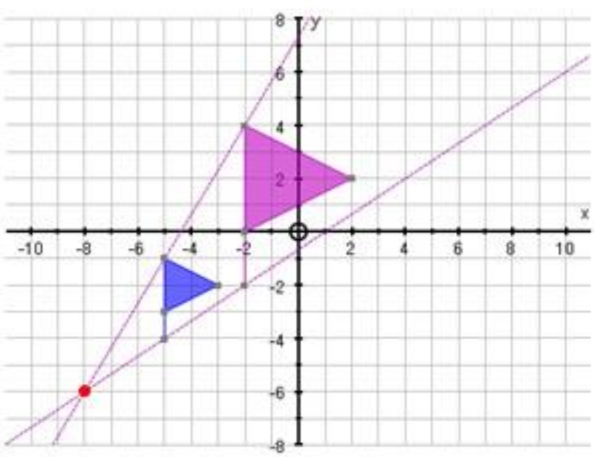
And going from big to small...
(a) Scale Factor = $\frac{1}{3}$
(b) Scale Factor = $\frac{1}{4}$



Describing Enlargements

To fully describe an enlargement, you must give:

1. The centre of enlargement (give as a co-ordinate if you can)
2. The scale factor of the enlargement



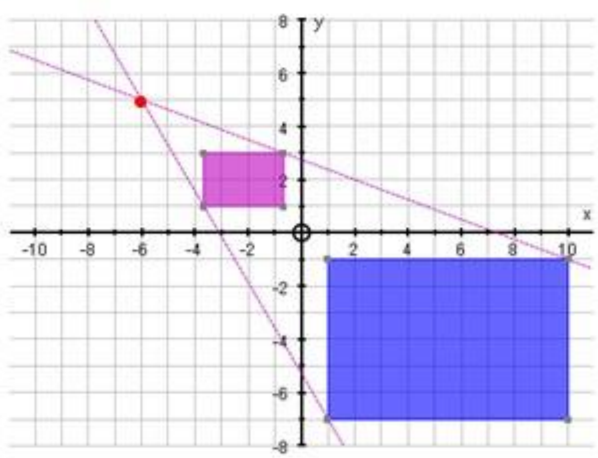
To describe the enlargement from the blue object to the purple object, we would say:

1. Centre of Enlargement: (-8, -6)
2. Scale Factor of Enlargement: 2

Notice:

(1) To find the centre of enlargement you must draw line through matching points on both objects and see where they cross

(2) Each point on the purple object if twice as far away from the centre of enlargement than the matching point on the blue!



To describe the enlargement from the blue object to the purple object, we would say:

1. Centre of Enlargement: (-6, 5)
2. Scale Factor of Enlargement: $\frac{1}{3}$

Notice:

(1) The object has gone smaller, so it must be a fractional scale factor!

(2) Each point on the purple object if one-third as far away from the centre of enlargement than the matching point on the blue!

PRACTICE EXERCISE:

1. T is a translation which moves the origin to the point (3,2). R is a anticlockwise rotation of 90^0about the origin. A is the point (2,-5), B is (-1,4) and C is (-4,4). Find the coordinates of the image of:
   (a) A after translation T
   (b) B after rotation R
   (c) C if it is first translated by T and then rotated by R.
2. A'(5,5), B'(-5,10), C'(0,20) are the images of A(2,2), B(-2,4), C(0,8) after a transformation F.
   (a) Using a scale of 1cm to 2units, draw the triangles ABC and A'B'C' on the same Cartesian plane.
   (b) Describe fully the transformation F
   (c) Find the coordinates of the image of triangle ABC after rotation 270^0 clockwise about the point (3,2)
3. Triangle A(0,2), B(1,0), C(2,1) is first enlarged about point (1,-2) with scale factor 2. It is then reflected in the line x = -1. Find the vertices of its final image.
4. Quadrilateral Q is rotated through 180^0 about the point (0,2). The result is then enlarged by a scale factor of -2 with the origin as centre. Find the coordinates of the vertices of the final image of Q.
5. (a) Using a scale of 1cm to represent 1 unit on each axis, draw x and y-axes for $-4 \leq x \leq 10 \ \ and \ -4 \leq y \leq 12$. Draw a triangle with vertices (-1,1), (-1,10), (-4,7) and label it F.
   (b) A transformation R maps triangle F on to the triangle R(F) which has vertices (0,-2), (9,-2), (6,1). Draw triangle R(F) and fully describe the transformation R.
   (c) M is a reflection in the line y = x. Find by drawing, the coordinates of the vertices of the triangle M(F).