E-LEARNING NOTES

S S 2 SCHEME SECOND TERM

WEEK	TOPIC	CONTENT
1	***LOGICAL REASONING**	(a) Simple and Compound statements. (b) Logical operation and the truth table. (c) Validity of argument.
2	**LINEAR INEQUALITIES 1**	(a) Revise linear inequalities in one variable. (b) Solutions of inequalities in two variables. (c) Range of values of combined inequalities.
3	**LINEAR INEQUALITIES 2**	(a) Graphs of linear inequalities in two variables. (b) Maximum and minimum values of simultaneous linear inequalities. (c) Application of linear inequalities in real life. (d) Introduction to linear programming.
4	**ALGEBRAIC FRACTIONS**	(a) Simplification of fractions. (b) Operation in algebraic fractions. (c) Equation involving fraction. (d) Substitution in fractions. (e) Simultaneous equation involving fractions. (f) Undefined value of a fraction.
5	**CIRCLE GEOMETRY 1**	(a) Lines and regions of a circle. (b) Circle theorems including: (i) Angles subtended by chords in circle; (ii) Angles subtended by chords at the centre; (iii) Perpendicular bisectors of chords; (iv) Angles in alternate segments. (v) Cyclic quadrilaterals
6	**CIRCLE GEOMETRY 2**	Circle Theorems: (a) The angle which an arc subtends at the centre is twice the angle it subtends at the circumference. (b) Angles in the same segment of a circle are equal. (ii) Angle in a semi-circle. (c) Tangent to a circle.
7	**MID-TERM BREAK**	
8	**TRIGONOMETRY**	(a) Derivation and application of sine rule. (b) Derivation and application of cosine rule.
9	**BEARINGS**	(a) Revision of; Trigonometric ratios; Angles of elevation and depression. (b) Notation for bearings: (i) Cardinal notations $N30^0E$ (ii) $S45^0W$ (iii) 3-digits notation. E.g. 075^0, 350^0. (c) Practical problems on bearing.
10	**REVISION**	
11	**EXAMINATION**	

WEEK 1

SUBJECT: MATHEMATICS

CLASS: SS2

TOPIC: LOGICAL REASONING

CONTENT:

- Simple and Compound statements.
- Logical operation and the truth table.
- Validity of argument

SIMPLE AND COMPOUND STATEMENTS

Mathematical logic can be defined as the study of the relationship between certain objects such as numbers, functions, geometric figures etc. Statements are verbal or written declarations or assertions. The fundamental (i.e logical) property of a statement is that it is either true or false but not both. So logical statements are statements that are either reasonably true or false but not both.

Example: The following are logical statements;

1. Nigeria is in Africa
2. The river Niger is in Enugu
3. 2 + 5 = 3
4. 3 < 7

P	Q	p^q
T	T	T
T	F	F
F	F	F

N.B The educator should ask the students to give their examples

Example: The following are not logical statements because they are neither true nor false.

1. What is your name?
2. Oh what a lovely day

3. Take her away
4. Who is he?
5. Mathematics is a simple subject (note that this statements is true or false depending on each individual, so it is not logical)
   N.B educator to ask the students to give their own examples

Compound statements—

When two or more simple statements are combined, we have a compound statement. To do this, we use the words: ‘and’, ‘or’, ‘if … then’, ‘if and only if’, ‘but’. Such words are called connectives.

Conjunction (or ^) of logical reasoning: Any two simple statements p,q can be combined by the word ‘and’ to form a compound (or composite) statement ‘p and q’ called the conjunction of p,q denoted symbolically as p^q.

Example: 1. Let p be “The weather is cold” and q be “it is raining”, then the conjunction of p,q written as p^q is the statement “the weather is cold and it is raining”.

2. The symbol ‘^’ can be used to define the intersection of two sets A and B as follows;

$$A \cap B = \{x : x \in A \wedge x \in B\}$$

The truth table for p^q is given below;

Class Activity:

1. Which of the following is (are) simple statement and non statement
   - i. The ground is wet
   - ii. It is raining
   - iii. Go to the front seat
   - iv. Base ball is not a sport
   - v. Every triangle has four sides
2. In the following problems, determine if the sentence is a statement. Classify each sentence that is a statement as simple or compound. If compound , give the components
   - i. Open the door
   - ii. 5 is a prime number
   - iii. Do you like mathematics

iv. May you live long!
v. Today is Sunday and tomorrow is Monday
vi. Rebecca is studying in class eleven and she has to offer 5 object
vii. 20 is a prime number and 20 is less than 21
viii. Abuja is a city and it is the capital of Nigeria
ix. The earth revolves around the moon
x. Every rectangle is square

LOGICAL OPERATION AND THE TRUTH TABLE

The word 'not' and the four connectives 'and', 'or', 'if … then', 'if and only if' are called logic operators. They are also referred to as logical constants. The symbols adopted for the logic operators are given below.

Logic Operators	Symbols
'not'	$-$ *or* $\sim$
'and'	$\wedge$
'or'	$\vee$
'if … then'	$\rightarrow$
'if and only if'	$\leftrightarrow$

When the symbols above are applied to propositions p and q, we obtain the representations in the table below:

Logic operation	Representation
'not p'	$\sim p$ or $\bar{p}$
'P and q'	$p \wedge q$
'p or q'	$p \vee q$
'if p then q'	$p \rightarrow q$
'p if and only if q'	$p \leftrightarrow q$

CONDITIONAL STATEMENTS AND INDIRECT PROOFS

Many statements especially in mathematics are of the form “if p then q”, such statements are called conditional statements or implications. The statement ‘if p then q’ means p implies q. The p part is called the antecedent (ante means before) whereas the q part is the consequent

Examples:

1. The student can solve the problem only if he goes through the worked examples thoroughly.

Antecedent: The student can solve the problem

Consequent: He goes through the worked examples thoroughly

2. If Dayo is humble and prayerful then he will meet with God’s favour.
   Antecedent: Dayo is humble and prayerful
   Consequent: He will meet with God’s favour

Class Activity:

Identify the antecedent and the consequent in these implicative statements

(a) If I travel then you must teach my lesson
(b) If you person well in your examinations then you will go on holidays
(c) If London is in Britain then 12 is an even number
(d) If the bus come late then I will take a motorcycle
(e) If a & b are integers then ab is a rational number

Converse statements: The converse of the conditional statement “if p then q” is the conditional statement “if q then p” i.e the converse of $p \rightarrow q$ is $q \rightarrow p$

Example;

Let p be ‘Obi is a boy’ and q be ‘3 + 3 = 4’ and so $p \rightarrow q$ is the statement ‘if Obi is a boy then 3 + 3 = 4’. The converse of the statement ($q \rightarrow p$) is the statement ‘if 3 + 3 = 4 then Obi is a boy’

(students should give more examples)

Inverse statements: The inverse of the conditional statement “if p then q” is the conditional statement “if not p then not q”. i.e the inverse of $p \rightarrow q$ is $\sim p \rightarrow \sim q$

Class Activity:

1. Write down the inverse of each of the following statements
   (a) If Mary is a model then she is beautiful
   (b) If Ibadan is the largest city in the west Africa then it is the largest city in Nigeria
   (c) If the army misbehaves again he will be demoted
2. Write down the converse of each of the following
   (a) If he sets a good, he will get a good fellowship
   (b) If it rains sufficiently then the harvest will be good
   (c) If the triangles are congruent then the ratios of their corresponding lengths are equal

VALIDITY OF ARGUMENT

A logical argument is a relationship between a sequence of statements X_1, X_2,X_3,..X_n called **premises** and another statement Y called the conclusion. Usually, an argument is denoted by X_1, X_2,X_3,..X_n; Y .One of the major application of logic is the determination of validity (correctness) or otherwise of arguments. An argument is valid if its truth T; if the truth value is false F, it is called a fallacy

Examples

1. Test the validity of the following argument with premises X_1, and X_2 and conclusion Y.

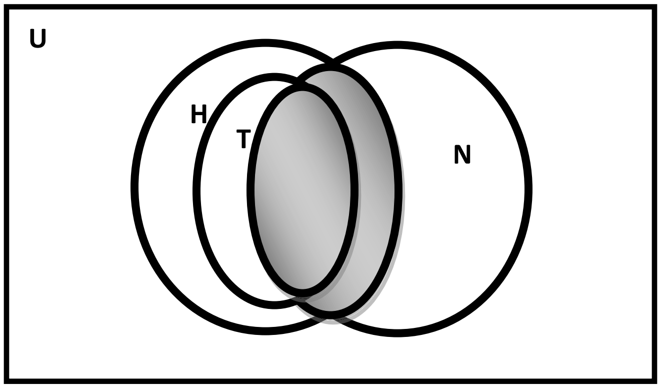
X_1 : All teacher are hardworking.
X_2 : Some young people are teachers
Y : There, some young people are hardworking

Solution

Let

U = { all people}
H = { hardworking people}
N = { young people}
T = { teachers}

The Venn diagrams in the diagram below illustrate this argument



The shaded region of the Venn diagram represent H n N , i.e young people who are hardworking

Since the conclusion follows from the premises, the argument is valid

2. Determine the validity of the following argument

   X_1 : if Bola studies hard he passes his examination

   X_2 : if Tina fails her examination, Bola passes his examination

   X_3 : Bola fails his examination

   Y : therefore, Tina passes her examination

   Solution

   First start by identifying the statement (propositional) variables in argument as follows

   P : Bola studies hard

   Q : Bola passes his examination

   R : Tina passes her examination

   Thus, using the argument form ,X1,X2,X3,.Y i.e (P→q), (r → q) ˜q ; ..r

   Second Construct the relevant truth table

P	**Q**	**R**	**˜q**	**˜r**	**p→q**	**˜r→q**
T	**T**	**T**	**F**	**F**	**T**	**T**
T	**T**	**F**	**F**	**T**	**T**	**T**

T	**F**	**T**	**T**	**F**	**F**	**T**
T	**F**	**F**	**T**	**T**	**F**	**F**
T	**F**	**F**	**T**	**T**	**F**	**F**
F	**T**	**T**	**F**	**F**	**T**	**T**
F	**T**	**F**	**F**	**T**	**T**	**T**
F	**F**	**T**	**T**	**F**	**T**	**T**
F	**F**	**F**	**T**	**T**	**T**	**F**

Class Activity

Determine the validity of the following arguments

1. Bankers are rich.
   Rich people are house owner
   Therefore, bankers are house owners
2. Idle men are never rich
   Wanderers are idle men
   Therefore, a rich man is never wanderer

PRACTICE EXERCISE

1. Determine the validity of the following argument
   i. All reptiles are intelligent animals
   A tortoise is a reptile
   Therefore, a tortoise is an intelligent animal
   ii. No doctor is dirty person
   All friends are clean person
   Therefore, all my friend s are doctors
   iii. Nurses are hospitable people
   My neighbours are hostile to one another
   Therefore, none of my neighbours is a nurse
2. Given the positive intergers x,y,z. prove that if $x<y$ and $y< z$ [Hint: you may use Venn diagram]
3. Prove that if two angles are alternate then the angles are equal

ASSIGNMENT

1. Prove that if a triangle is isosceles, then two of it angles are equal
2. Given two integers m and n. Prove that if m and n are even. Then their products also even
3. Prove that the conditional statement, if $x^2 = 16$, then $x = 4$, is a fallacy
4. Which of the following is the correct interpretation of p v q ?

   A : it will rain tomorrow and the field will be wet

   B: Either it will rain tomorrow or the field will be wet

   C: Either it will rain tomorrow or the field will be wet or it will rain tomorrow and the field will be wet

   D: It will not rain tomorrow but the field will be wet.
5. Determine the validity of the following argument

   X1 : No farmer is lazy

   X2: No non farmer wears gold wrist-watch

   Y: therefore, a lazy person does not wear a gold wrist watch

   KEYWORD: VALID, NEGATION, ARGUMENT, COMPOUND, SIMPLE STATEMENT, PROPOSITION ETC

WEEK 2

SUBJECT: MATHEMATICS

CLASS: SS 2

TOPIC: LINEAR INEQUALITIES

CONTENT:

- Revision of linear inequalities in one variable.
- Solutions of inequalities in two variables.
- Range of values of combined inequalities

INTRODUCTION

Number line can be used to show the graph of inequalities in one variable.
Symbols commonly used for inequalities include;

$$< \textit{means less than}$$

$$> \textit{means greater than}$$

$$\geq \textit{greater than or equal to}$$

$$\leq \textit{less than or equal to}$$

Steps taken in solving inequalities is similar to that of equations with few exceptions such as

(i) Reversing the inequality sign when both sides are multiplied (or divided) by negative quantity. i.e if 2 < 5 then -2 > -5

(ii) Reversing the inequality sign when reciprocals are taken

i.e if $\frac{2}{3} > \frac{1}{2}$ $\quad then\ \frac{3}{2} < \frac{2}{1}$

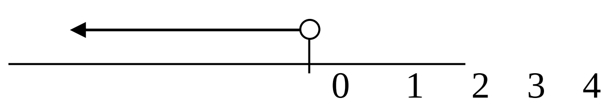
Examples;

1. solve $2x + 1 < x + 5$

solution;

$$2x - x < 5 - 1$$

$$x < 4$$



Notice that the right end point x=4 is not part of the solution so the circle above is not shaded.

(b) solve the inequality ; $\frac{2x}{3} - \frac{1}{6} \leq \frac{3x}{4}$

Solution; To clear the fraction, multiply through by the LCM of the denominators i.e 12

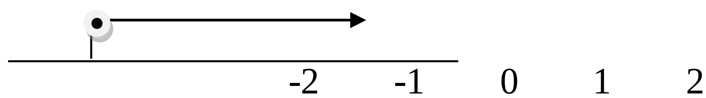
$$12\left(\frac{2x}{3}\right) - 12\left(\frac{1}{6}\right) \leq 12\left(\frac{3x}{4}\right)$$

$$8x - 2 \leq 9x$$

$$8x - 9x \leq 2$$

$$-x \leq 2$$

$\therefore x \geq -2$



Notice that the left end point, $x = -2$ is part of the solution, so small circle above is shaded.

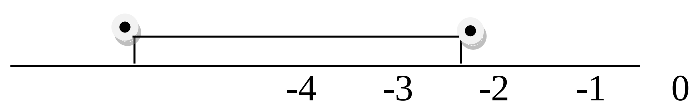
2. Find the range of values of x which satisfy $3 + x \leq 5$ *and* $8 + x \geq 5$ (WAEC)

Solution; $3 + x \leq 5$ *and* $8 + x \geq 5$

$$x \leq 5 - 3 \quad and \quad x \geq 5 - 8$$

$$x \leq 2 \; and \; x \geq -3$$

$\therefore -3 \leq x \leq 2$



Class Activity:

1. Solve the inequality and represent your result on a number line;
$$\frac{2(x+3)}{5} + \frac{3(x-1)}{4} \leq \frac{x+1}{2}$$
2. Find the three highest whole number that satisfy $2(3x+1) \leq \frac{1}{2}(2x-5)$

3. Solve and show on number line the values of x which satisfy $2x - 1 \geq 3\ and\ x - 3 < 5$

Solution of inequalities in two variables

For linear inequalities in two variables, first draw the corresponding straight line. Inequalities in two variables are usually plotted on x, y plane (the Cartesian coordinate plane)

Example: Show the expression $2y - x > 1$ on a graph.

Solution; first put y on one side of the inequality i.e $y > \frac{x+1}{2}$

Then draw the corresponding line $y = \frac{x+1}{2}$

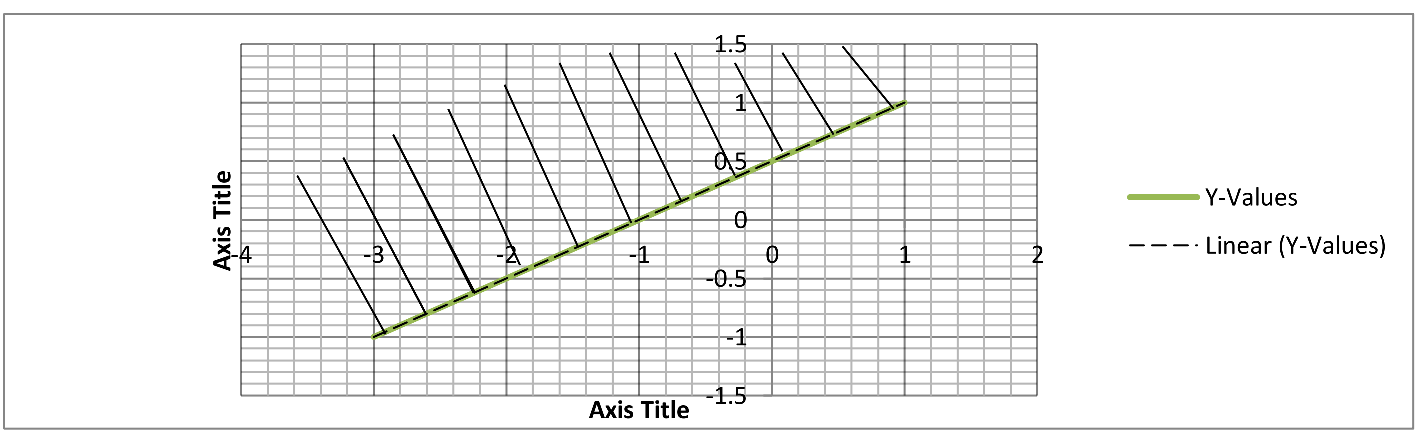
x	-3	-1	0	3
y	-1	0	1	2

This line divides the plane into two. To find the side with the solution, we select and try out a pair of points. E.g $p_1(0,0) p_2(1,1)$

For $p_1(0,0)$, $0 > \frac{0+1}{2}$ $(false)$

$p_2(1,1)$ $1 > \frac{1+1}{2}$ $(false)$

Hence, the solution set is in the region above the line $y = \frac{x+1}{2}$



The upper part of the graph shaded satisfies the inequality $2y - x > 1$

Class Activity:

1. Shade the region common to $2x \geq 3 \quad and \;\; y - 1 > 0$
2. Show on a graph the region that contains the set of points for which $2x + y < 5$
3. Shade the region that satisfy the following
   (a) $y < 3x$
   (b) $x + y \leq 4$
   (c) $2x + y + 2 \geq 0$
   (d) $4x + 3y > 0$

Now we shall consider range of values of combined inequalities.

Examples:

1. if $5x < 42 - x \quad and \;\; x + 5 < 2x$ what range of x satisfies both inequalities?

   Solution:

   solving the inequalities separately we obtain $x < 7 \quad and \quad x > 5$ respectively.

$$\therefore 5 < x < 7$$

2. The integral values of z which satisfy the inequality $-1 < 2z - 5 \leq 5$ are
   solution

$-1 < 2z - 5 \leq 5$ $\qquad$ $2z - 5 \leq 5$

$-1 + 5 < 2z$ $\qquad$ $2z \leq 5 + 5$

$\frac{4}{2} < z$ $\qquad$ $\frac{2z}{2} \leq \frac{10}{2}$

$2 < z$ $\qquad$ $z \leq 5$

$2 < z \leq 5$

The values are 3,4,5

Class Activity:

1. What range of p satisfy both $1 - p > 1 \quad and \quad 3(1 + p) \geq 0$
2. Find the range of values of x such that $6x - 7 \leq 5x \quad and \quad x \leq 3x + 8$
3. Express the inequality $-1 - x < 5 < 6 - x$ in the form $a < x < b$ where a & b are both integers

PRACTICE EXERCISE

1. Illustrate the following on graph paper and shade the region which satisfies all the three inequalities at the same time: $-x + 5y \leq 10$, $3x - 4y \leq 8$ and $x > -1$. (SSCE 1988)

2. Illustrate graphically and shade the region in which inequalities $y - 2x < 5$, $2y + x \geq 4$. $y + 2x \leq 10$ (SSCE 1993)

3. If $4x < 2 + 3x$ and $x – 8 < 3x$, what range of values of x satisfies both inequalities. Represent your result on a number line. (SSCE 2006)

4. Solve the inequality : $\frac{2}{5}(x – 2) - \frac{1}{6}(x + 5) \leq 0$ (SSCE 2008)

5. Given that x is an interger,find the three greatest values of x which satisfy the inequality
   $7x < 2x - 13$ (SSCE 2009)

ASSIGNMENT

1. What is the range of values of x for which $2x + 5 > 1$ and $x - 4 < 1$ are both satisfied.

2. Solve $3x – 5 < 5x – 3$. Represent your result on a number line.

3. Solve $2x + 6 < 5(x – 3)$. Represent your result on a number line.

Solve $\frac{5x-1}{3} - \frac{1-2x}{5} \leq 8 + x$. Represent your result on a number lin

4. Show on a graph ,the area which gives the solution set of the inequalities:
   $y - 2x \leq 4$, $3y + x \geq 6$,$y \geq 7x$ (SSCE 1990)

5. Find the range of values of x for which $\frac{x+2}{4} - \frac{x+1}{3} > \frac{1}{2}$

A. x > 4 B. x > - 4 C. x < 4 D. x < -4 (SSCE 2004)

KEYWORDS: INEQUALITY, GREATER THAN, LESS THAN,VARIABLES,LINEAR,

WEEK 3

SUBJECT: MATHEMATICS

CLASS: SS 2

INEQUALITIES

CONTENT:

- Graphs of linear inequalities in two variables.
- Maximum and minimum values of simultaneous linear inequalities.
- Application of linear inequalities in real life.
- Introduction to linear programming

Graph of linear inequalities in two variables: We shall consider simultaneous inequalities.

Examples:

Show on a graph the region that contains the solution of the simultaneous inequalities

$$2x + 3y < 6, \quad y - 2x \leq 2, \quad y \geq 0$$

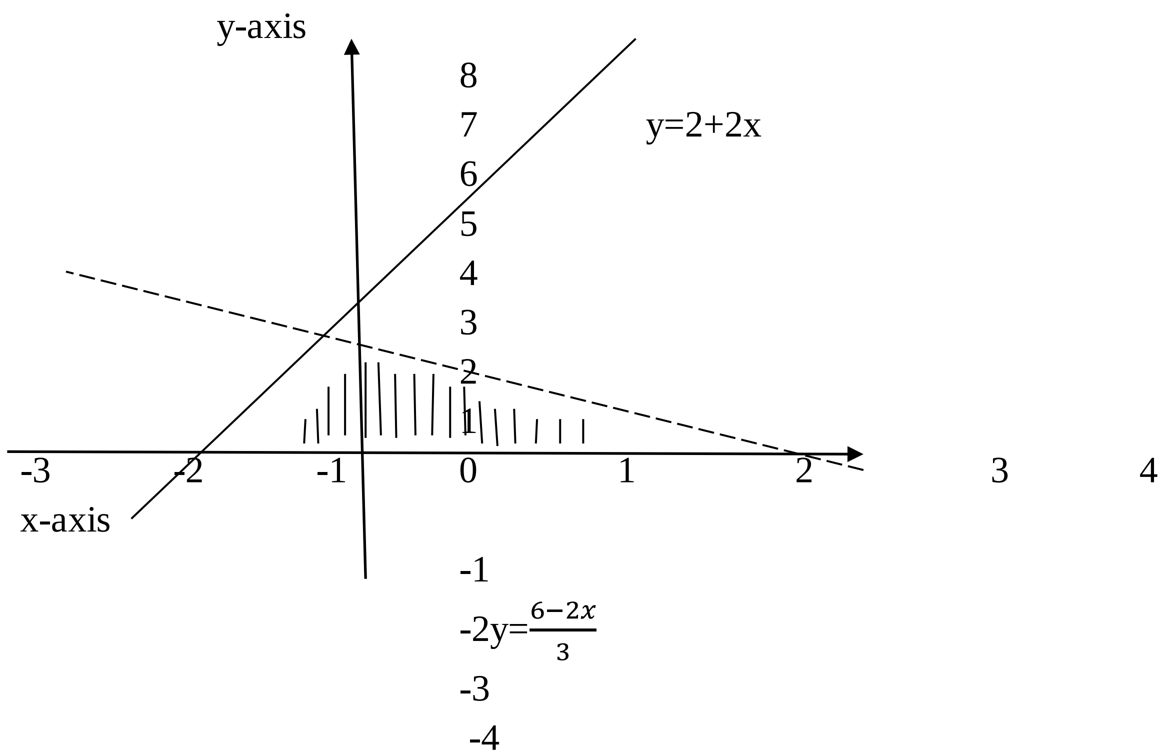
Solution: In each case put y on one side of the inequality

$$y < \frac{6 - 2x}{3} \qquad y \leq 2 + 2x \quad and \quad y \geq 0$$

We shall draw the lines $y = \frac{6-2x}{3}$, $\quad y = 2 + 2x \quad and \quad y = 0$

$\boldsymbol{x}$	-2	0	2	3
$\boldsymbol{y_1} = \frac{6-2x}{3}$	3.3	2	0.7	0

$\boldsymbol{y_2} = 2 + 2x$	-2	2	6	8



Points $p_1(0,0)$ *and* $p_2(1,1)$ are in the solution set for the three inequalities.

The shaded portion is the required region. The integral values of x & y that satisfy the inequalities simultaneously are (-1,0), (0,0), (0,1), (0,2), (1,0), (1,1), (2,0), (3,0)

Class Activity:

1. Shade the region defined by;
   (a) $y > -1,\ y < 3x\ ,\ x + y \le 4$
   (b) $y < 4,\ x < 3.5,\ 2x + y \ge -2\ ,\ x \ge y + 2$
2. Show on a graph the region which contains the solutions of the simultaneous inequalities

$$x - y \le 2\ \ ,\ \ 3x + 2y > 6$$

3. Find the region common to $x + y \le 3$ *and* $3x + 2y \ge 0.$ show the region on a graph

MAXIMUM AND MINIMUM VALUES OF SIMULTANEOUS LINEAR INEQUALITIES; APPLICATION OF LINEAR INEQUALITIES IN REAL LIFE.

In solving simultaneous inequalities involving variables x & y, the expression x+y=n is called the objective function. Linear programming usually involves either maximizing or minimizing the function x+y=n. These problems are sometimes called minimax problems.

Example: A manufacturer has 120kg and 100kg of wood and plastic respectively. A product A requires 2kg of wood and 3kg of plastic. Product B requires 3kg of wood and 2kg of plastic. If A sells for #3500 and B for #5000. How many must be made to obtain the maximum gross income?

Solution:

	Wood (kg/unit)	Plastic (kg/unit)	# per kg
Product A	2	3	3500
Product B	3	2	5000

Suppose there is x number of product A and suppose there is y number of product B

$$2x + 3y \leq 120$$

$$3x + 2y \leq 100$$

For the first inequality we shall draw line 2x + 3y = 120

If x=0, y=40. This line passes through (0,40)

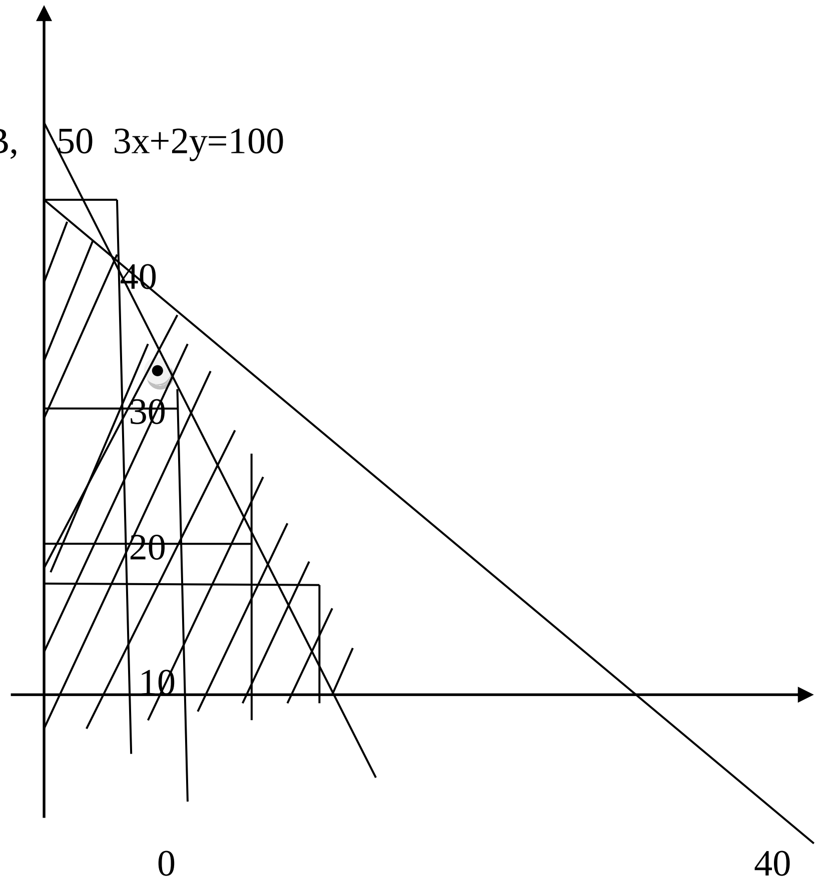
If y=0, x=60. The line passes through (60,0)

For the second inequality, consider line 3x+2y=100

If x=0, y=50. This other line passes through (0,50)

If y=0,x=$\frac{100}{3}$=33$\frac{1}{3}$. This line passes through point (33$\frac{1}{3}$,0)

Number of product B,



Number of A

60 2x+3y=120

From the shaded region, we can get the integral values of A & B and at the given price that will give maximum income.

At the point of intersection we have approximately 12kg of A and 32kg of B.

The income from item A is 12 x #3500 = #42000

The income from item B is 32 x #5000 = #160000

Total income = #202,000

Class Activity:

1. The number of units of protein and carbohydrate in food type F1 and F2 are recorded in the table below

Food	Protein (units/kg)	Carbohydrate (units/kg)	Cost per Kg
F1	**5**	**8**	**N200**
F2	**6**	**3**	**N300**
Minimum daily requirement	**15**	**12**	

i. What are the restrictions on the type of food eaten daily?
ii. Draw the graph to illustrate the region of possible solutions
iii. How much food should be bought to satisfy the minimum daily requirement

INTRODUCTION TO LINEAR PROGRAMMING

In some real life situations in business there are some constraints or restrictions. This may be in form of constraint in amount of money available for a project, constraint in number of skilled workers available. Such restriction problems can be solved using graphs of linear inequalities. This method is called linear programming. Now we shall use linear programming to solve a problem.

Example: A man has #2000. He buys shirts at #500 each and belt at #200 each. He gets at least 2 shirts and at least one belt. If he spent over #400 more on shirts than on belts, find

(a) How many ways the money can be spent
(b) The greatest number of shirts that can be bought
(c) The greatest number of belt that can be bought
Solution;

Let the man buy x shirts at #500 each and y belts at #200. From the first two sentences, we have

$$500x + 200y \leq 2000$$

Divide through by 100 to get; $5x + 2y \leq 20 \quad \dots \dots (i)$

At least 2 shirts implies $x \geq 2 \quad \dots \dots \dots . (ii)$

At least 1 belt implies $y \geq 1 \quad \dots \dots \dots . . (iii)$

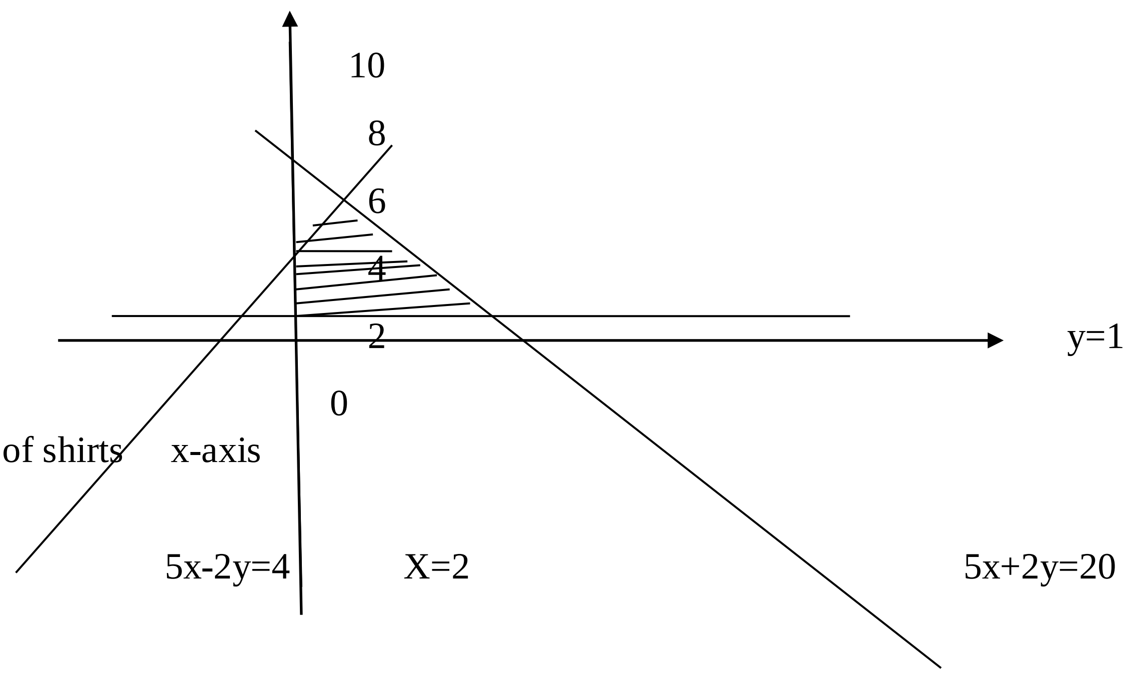
Spending over #400 on shirts than belt implies $500x - 200y \geq 400$

i.e $5x - 2y \geq 4$ ………..(iv)

we shall now draw the graph for four inequalities. For the first inequality i.e $5x + 2y \leq 20$, we need line $5x + 2y = 20$. if x=0, y=10 the line passes through (0,10). If y=0, x=4. It passes through (4,0). For $5x - 2y \geq 4$,consider line $5x - 2y = 4$ if x=0, y= -2,this line passes through point (0,-2), if y=0 x=0.8

so this line passes through (0.8,0)

number of belt, y-axis



Ans: (a) there are five ways the money can be spent i.e (2,1),(2,2),(2,3),(3,1) and (3,2)

(b) The greatest number of shirts that can be bought is 3

(c.) The greatest number of belts that can be bought is 3

Notice that the points to maximize the number of items that can be bought are (2,3) i.e 2shirts and 3belts. In the two situations, five items can be bought.

The maximum expenses occurs when we have 3shirts and 2belts,
i.e $3 \times \#500 + 2 \times \#200 = \#1900$

Class Activity:

A business man needs at least 5 buses and 12 cars. He is not able to run more than 25 vehicles altogether. A bus takes up 3units of the parking space, a car takes 2units and there are only 60 units available. Find the greatest number (a) buses (b) cars

PRACTICE EXERCISE

1. Show by shading the region S of all the points (x,y) which satisfies simultaneously the following four inequalities: $2y \geq 2 - x, x + y \geq 5, y \leq 2x - 3$
   Use your diagram to find
   i. The maximum and the minimum values of x and y
   ii. The minimum value of x^2
   iii. The maximum and minimum value of x-y
2. A dietician wishes to combine two foods, A and B, to make a mixture that contains at least 50g of protein, at least 130mg of calcium, And not than 559 calories. The nutrient values of foods A and B are give in the table

Food	A	A
Protein (g/cup)	20	10
Calcium (mg/cup)	20	50
Calories (cup)	100	150

   How many cups of each of the foods should the dietician use?

3. Solve the following integer programming problem
   Maximize $4x + 3y$
   Subject to $2x + 3y \geq 150$
   $3x + y \geq 100$
   $x < 4y$

ASSIGNMENT

1. When twice a certain number is added to 7. The result is more than 10. What is the number
2. You need to buy some filling cabinets. You know that cabinet X costs N10 per unit, requires six square feet of floor space, and holds eight files. Cabinet Y costs N20 per unit, requires eight square feet of floor space, and ho floor space, and holds twelve cubic feet of files. You have been given N140 for this purchase, though you don’t have to spend that much. The office has room for not than 72 square feet of cabinets. How many of which model should you buy, in order to maximize storage volume?

WEEK 4

CLASS: SS 2

TOPIC: ALGEBRAIC FRACTIONS

CONTENT:

- Simplification of fractions.
- Operation in algebraic fractions.
- Equation involving fraction.
- Substitution in fractions.
- Simultaneous equation involving fractions.
- Undefined value of a fraction.

SIMPLIFICATION OF FRACTIONS

An algebraic fraction is a part of a whole, represented mathematically by a pair of algebraic terms. The upper part is called the numerator while the lower part the denominator. To simplify algebraic fractions, we need to factorize both the numerator and the denominator.

Examples:

1. Reduce the following to their lowest term

(a) $\frac{3x^2+9x^2y^2}{3x^2y}$

(b) $\frac{x^2-y^2+3x+3y}{x-y+3}$

(c) $\frac{x^2-9}{x^2+x-6}$

(d) $\frac{5xy-10x+y-2}{8-2y^2}$

Solution:

(a) $\frac{3x^2+9x^2y^2}{3x^2y} = \frac{3x^2(1+3y^2)}{3x^2 \times y}$

Cancel the common factors i.e. $3x^2$

$\therefore \; Ans = \frac{1+3y^2}{y}$

(b.) $\frac{x^2-y^2+3x+3y}{x-y+3} = \frac{(x+y)(x-y)+3(x+y)}{x-y+3}$

$$= \frac{(x+y)(x-y+3)}{x-y+3}$$

$$\therefore Ans = x + y$$

(c.) $\frac{x^2-9}{x^2+x-6} = \frac{(x+3)(x-3)}{(x+3)(x-2)}$

$$= \frac{x-3}{x-2}$$

(d.) $\frac{5xy-10x+y-2}{8-2y^2} = \frac{5x(y-2)+(y-2)}{2(4-y^2)}$

$$= \frac{(y-2)(5x+1)}{2(2-y)(2+y)}$$

$$= \frac{-(2-y)(5x+1)}{2(2-y)(2+y)}$$

$$= \frac{-(5x+1)}{2(2+y)}$$

Class Activity:

Simplify the following fractions

(a) $\frac{x^2+9x+8}{x^2+6x+5}$

(b) $\frac{p^2+pq-6q^2}{p^2-3pq+2q^2}$

OPERATIONS IN ALGEBRAIC FRACTIONS ARE THE PROCESS OF ADDING AND SUBTRACTING ALGEBRAIC FRACTIONS

Addition and Subtraction algebraic fractions

Examples;

Simplify the following

(a) $\frac{4}{a}+\frac{6}{a+2}$

(b) $\frac{5}{x-4}-\frac{2}{x+4}$

(c) $\frac{1}{2}-\frac{1}{x-y}+\frac{2}{x+y}$

(d) $\frac{3mn}{2m^2+2n^2}+\frac{5mn}{3m^2+3n^2}$

Solution:

(a) $\frac{4}{a}+\frac{6}{a+2}$

Express the two fractions as a single fraction by taking LCM

$$=\frac{4(a+2)+6a}{a(a+2)}$$

$$=\frac{4a+8+6a}{a(a+2)}$$

$$=\frac{10a+8}{a^2+2a}$$

(b.) $\frac{5}{x-4}-\frac{2}{x+4}$

Take the LCM and then express a single fraction

$$=\frac{5(x+4)-2(x-4)}{(x-4)(x+4)}$$

$$= \frac{5x + 20 - 2x + 8}{(x-4)(x+4)}$$

$$= \frac{5x - 2x + 20 + 8}{(x-4)(x+4)}$$

$$= \frac{3x + 28}{x^2 - 16}$$

(c.) $\frac{1}{2} - \frac{1}{x-y} + \frac{2}{x+y}$

The LCM is the product of the denominator of the three terms

$$= \frac{(x-y)(x+y) - 2(x+y) + [2(x-y)]}{2(x-y)(x+y)}$$

$$= \frac{x^2 + xy - xy - y^2 - 2x - 2y + 4x - 4y}{2(2x-y)(x+y)}$$

$$= \frac{x^2 - y^2 + 4x - 2x - 2y - 4y}{2(x-y)(x+y)}$$

$$= \frac{x^2 - y^2 + 2x - 6y}{2(x^2 - y^2)}$$

(d.) $\frac{3mn}{2m^2+2n^2} + \frac{5mn}{3m^2+3n^2} = \frac{3mn}{2(m^2+n^2)} + \frac{5mn}{3(m^2+n^2)}$

$$= \frac{9mn + 10mn}{6(m^2 + n^2)}$$

$$= \frac{19mn}{6(m^2 + n^2)}$$

Class Activity:

Simplify the following expressions to its lowest terms

(a) $\frac{4}{x} - \frac{6}{x+2}$

(b) $\frac{1}{4(u-v)} - \frac{1}{5(v-u)}$

MULTIPLICATION AND DIVISION OF FRACTIONS

In multiplication and division of algebraic fractions, we need to factorize both the numerator and the denominator fully and then divide both the numerator and denominator by common factor(s)

Examples:

(a) $\left[\frac{2}{x} - \frac{5}{y}\right] \div \frac{4}{xy}$

(b) $\frac{x^2-y^2}{xy+x^2} \times \frac{2x^3}{xy-x^2}$

(c) $\frac{a^2+ab-2b^2}{a^2-2ab-3b^2} \times \frac{a^2-b^2}{ab+2b^2} \div \frac{a^2-2ab+b^2}{a^2-3ab}$

Solution:

(a) $\left[\frac{2}{x} - \frac{5}{y}\right] \div \frac{4}{xy}$

.$= \frac{2y-5x}{xy} \div \frac{4}{xy}$

$$= \frac{2y-5x}{xy} \times \frac{xy}{4}$$

.$= \frac{2y-5x}{4}$

(b.) $\frac{x^2-y^2}{xy+x^2} \times \frac{2x^3}{xy-x^2} = \frac{(x+y)(x-y)}{x(y+x)} \times \frac{2x^3}{x(y-x)}$

$$= \frac{-(x+y)(y-x)}{x(y+x)} \times \frac{2x^3}{x(y-x)}$$

$$= \frac{-2x^3}{x^2}$$

$$= -2x$$

(c.) $\frac{a^2+ab-2b^2}{a^2-2ab-3b^2} \times \frac{a^2-b^2}{ab+2b^2} \div \frac{a^2-2ab+b^2}{a^2-3ab}$

Re-writing the question and factorise each fraction fully, we have

$$= \frac{a^2 + ab - 2b^2}{a^2 - 2ab - 3b^2} \times \frac{a^2 - b^2}{ab + 2b^2} \times \frac{a^2 - 3ab}{a^2 - 2ab + b^2}$$

$$= \frac{(a + 2b)(a - b)}{(a - 3b)(a + b)} \times \frac{(a + b)(a - b)}{b(a + 2b)} \times \frac{a(a - 3b)}{(a - b)(a - b)}$$

After thorough and correct factorization we then cancel factors accordingly

$$= \frac{a}{b}$$

Class Activity: Simplify the following to its lowest term

1. $\frac{18ab}{15bc} \times \frac{20cd}{24de}$

2. $\frac{uv}{3u-6v} \times \frac{4u-8v}{u^2v}$

SUBSTITUTION IN FRACTION

Examples:

Given $\frac{x}{y} = \frac{2}{7}$, evaluate $\frac{7x+y}{x-\frac{1}{7}y}$

Solution: divide both numerator and denominator by y

$\Rightarrow \frac{7\left(\frac{x}{y}\right)+\frac{y}{y}}{\frac{x}{y}-\frac{1}{7}\left(\frac{y}{y}\right)} = \frac{7\left(\frac{x}{y}\right)+1}{\frac{x}{y}-\frac{1}{7}}$

Substitute $\frac{2}{7}$ for $\frac{x}{y}$in the algebraic expression

$.= \frac{7\times\frac{2}{7}+1}{\frac{2}{7}-\frac{1}{7}} = \frac{3}{\frac{1}{7}} = 3 \times 7 = 21$

Or we can also check by putting $x = 2$ and $y = 7$

$\Rightarrow \frac{7(2)+7}{2-\frac{1}{7}(7)} = \frac{14+7}{2-1} = \frac{21}{1} = 21$

Examples:

If $a = \frac{d+1}{d-1}, express\ \frac{a+1}{a-1}$ in terms of d

Solution:

Substitute $\frac{d+1}{d-1}$ for a in the given expression

$\Rightarrow\ .\frac{\frac{d+1}{d-1}+1}{\frac{d+1}{d-1}-1}$

Multiply the numerator and the denominator by $d - 1$, we obtain

$.= \frac{d+1+d-1}{d+1-d+1}$

$.= \frac{2d}{d}$

$.= d$

Class Activity:

1. Given p:q = 9:5, evaluate $\frac{15p-2q}{5p+16q}$

2. If $X = \frac{2a+3}{3a-2}$, express $\frac{X-1}{2X+1}$in terms of a

3. If $\frac{x}{y} = \frac{3}{4}$, evaluate $\frac{2x-y}{2x+y}$

4. If $x = \frac{a+2}{a-1}$, then express $\frac{x-3}{x+1}$ in terms of a.

EQUATION INVOLVING FRACTION

Examples:

1. Solve the equation; $\frac{4r-3}{6r-1} = \frac{2r-1}{3r+4}$

Solution: on cross multiplying, we have

.$(4r-3)(3r+4)=(2r-1)(6r-1)$

.$12r^2+16r-9r-12=12r^2-2r-6r+1$

Collecting like terms

,$12r^2-12r^2+7r+8r-12-1=0$

,$15r-13=0$

,$\therefore r=\frac{13}{15}$

2. $\frac{2}{x-4}=\frac{3}{x-1}+\frac{2}{3}$

   The LCM of the denominators is 3(x -4)(x-1)

   Multiply each term by 3(x -4)(x-1) to clear the fractions

   $3(\mathrm{x}\ -4)(\mathrm{x}-1)\times\frac{2}{x-4}=3(\mathrm{x}\ -4)(\mathrm{x}-1)\times\frac{3}{x-1}+$
   $3(\mathrm{x}\ -4)(\mathrm{x}-1)\times\frac{2}{3}$

   6(x - 1) = 9(x-4) + 2(x-4)(x-1)

   6x-6 = 9x – 36 + 2[x^2-5x+4]

   6m – 6 = 9x -36 +2x^2-10x+8

   6m – 6 = 2x^2 - x -28

   0 = 2x^2 – 7x -22

   2x^2 – 11x + 4x -22 = 0

   (x-2)(2x-11) = 0

   X + 2 = 0 or 2x -11 = 0

   X =-2 or x = 11/2

Class Activity:

Solve the following equation

1. $x+1=\frac{5}{x+2}$
2. $\frac{4}{x}=x-3$
3. $\frac{3p-4}{2p-1}=\frac{6p-1}{4p-3}$

SIMULTANEOUS EQUATION INVOLVING FRACTIONS

Examples;

1. Solve the simultaneous equation

$$\frac{2x}{5} - \frac{y}{2} = 2$$
$$x - 2y = 2$$

Solution;

$$\frac{2x}{5} - \frac{y}{2} = 2 \quad \dots\dots\dots.(i)$$

$x - 2y = 2$ ....(ii)

Multiply each term of equation (i) by 10 and multiply each term of equation (ii) by 4

$$\left(\frac{2}{5} \times \frac{10}{1}\right)x - \left(\frac{1}{2} \times \frac{10}{1}\right)y = 2 \times 10 \qquad \dots\dots\dots\dots.(iii)$$

$\Rightarrow 4x - 5y = 20 \dots\dots\dots\dots.(iii)$

$$4x - 8y = 8 \qquad \dots\dots\dots\dots.(iv)$$

Subtracting equation (iv) from (iii), we have

, $3y = 12$

Divide both sides by 3

, $y = 4$

Substitute 4 for y in equation (iv)

$$4x - 8(4) = 8$$
$$4x - 32 = 8$$
$$4x = 40$$
$$x = 10$$
$$\therefore \quad x = 10, \ \ y = 4$$

2. Solve the equation; $\frac{x+1}{3} + \frac{y-1}{2} = 5$

$$\frac{2x+5}{3} - \frac{y+1}{4} = 3$$

Solution: $\frac{x+1}{3}+\frac{y-1}{2}=5 \quad \ldots\ldots\ldots.(i)$

$\frac{2x+5}{3}-\frac{y+1}{4}=3$ ....(ii)

Multiply each term in equation (i) by 6 and equation (ii) by 12

$$2(x+1)+3(y-1)=30$$

$$2x+3y=31 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(iii)$$

Multiply each term in equation (2) by 12

$$4(2x+5)-3(y+1)=36$$

$$8x+20-3y-3=36$$

$$8x-3y=19 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(iv)$$

Adding equations (iii) & (iv)

$$10x=50$$

$$x=5$$

Substitute 5 for x in equation (iv)

$$8(5)-3y=19$$

$$40-3y=19$$

$$-3y=19-40$$

$$-3y=-21$$

Divide both sides by $-3, we\ have$

$$y=7$$

$$\therefore x=5 \,\&\, y=7$$

Class Activity:

Solve the following pairs of equations

1. $\frac{x+1}{3}-\frac{3y-1}{2}=1$

$$\frac{3-8y}{5}-\frac{7-3x}{4}=1$$

2. $\frac{a}{2}+b=1$

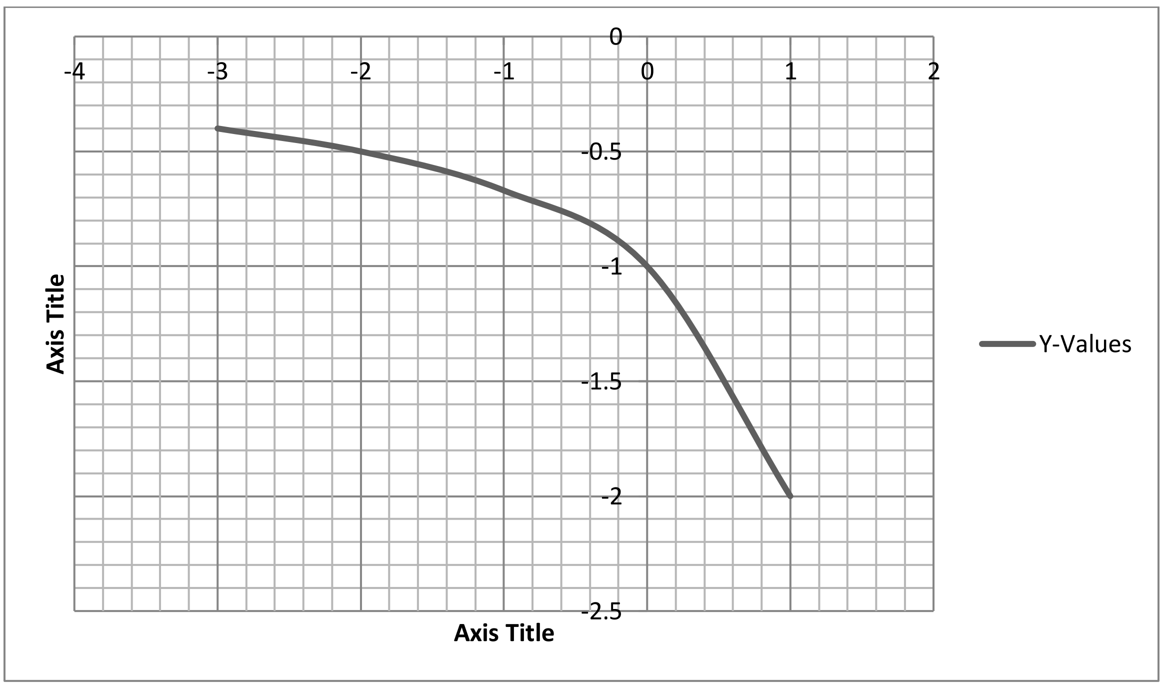
$$3a-\frac{b}{3}=\frac{31}{2}$$

UNDEFINED VALUE OF A FUNCTION

An algebraic fraction whose denominator is equal to zero is said to be undefined. If an expression contains an undefined fraction, the whole expression is undefined. For instance, $\frac{1}{x+2}$ will be undefined if the value of x is -2

When $x = -2$, then; $\frac{1}{x+2} = \frac{1}{-2+2} = \frac{1}{0}$, but division by zero is impossible. Therefore the fraction is **undefined.** Below is the table of values and corresponding graph of the function $\frac{2}{x-2}$, values of x ranges from -3 to 5

x	-3	-2	-1	0	1	2	3	4	5
y	-0.4	-0.5	-0.67	-1	-2	0	2	1	0.67



Notice that: (i) As the values of x approaches 2 from below the value of $\frac{2}{x-2}$ decreases rapidly.

(ii) As the value of x approaches 2 from above, the value of $\frac{2}{x-2}$ increases rapidly.

When $x = 2, y = \frac{2}{2-2} = \frac{1}{0}$

Division by zero is impossible. The fraction $\frac{2}{x-2}$ is said to be undefined when $x = 2$.

The table of values and the graph clearly shows that $\frac{2}{x-2}$ is undefined when $x = 2$

Examples;

Find the values of x for which the following fractions are not defined.

1. $\frac{5}{x+2}$
2. $\frac{x+3}{3x+2}$

Solution:

1. $\frac{5}{x+2}$ is undefined when $x + 2 = 0$, if $x + 2 = 0$ then $x = -2$
   The fraction is not defined when $x = -2$
2. $\frac{x+3}{3x+2}$ is undefined when $3x + 2 = 0$

   Which implies that , $x = \frac{-2}{3}$

Class Activity:

1. If k is a constant not equal to zero. Find the value(s) of x for which the expression is undefined $\frac{k}{x} + \frac{b}{x-3} + \frac{c}{x(x-3)}$
2. Find the values of x for which the following expressions are undefined.
   (a) $\frac{3x+2}{x+7}$
   (b) $\frac{2a}{x(x+2)}$

PRACTICE EXERCISE

1. Simplify this expression to its lowest terms

$$i.\ \frac{(m+n)^2}{m^2-n^2} + \frac{m^2+mn}{n^2-mn}$$

ii. $\frac{x^2-4xy+4y^2}{x^2+xy-6y^2} \div \frac{x^2+3xy}{x^2+6xy+9y^2}$

iii. $\frac{2}{2u+3} \times \frac{3}{2u+3} \div \frac{2}{4u^2-9}$

$$iv. \frac{18m^2u}{16n^3v^2} \div \frac{24m}{15nu^3} \times \frac{8n^2v^3}{30m^3v}$$

2. Find the values of x for which this expression are undefined.

$$\frac{x^2 - 3x - 10}{x^2 + 12x + 36}$$

ASSIGNMENT

1. Find the values of x which the following expressions are undefined:

$$\boldsymbol{i}.\ \frac{\mathbf{5a}}{(\mathbf{3} - \mathbf{2x})\boldsymbol{x}}$$

ii. $\frac{\mathbf{2x-3}}{(\boldsymbol{x}+\mathbf{2})(\boldsymbol{x}+\mathbf{3})}$

iii. $\frac{\mathbf{13}}{\boldsymbol{x}} = \frac{\mathbf{2x^2+15}}{\boldsymbol{x^2-4}}$

2. Given that y =$\frac{2m+3}{2m-3}$

   Express $x = \frac{3y-1}{2y+4}$ in terms of m

3. Using the substitution $p = {}^1/x, q = {}^1/y$. Solve the simultaneous equations

$\frac{2}{x} + \frac{1}{y} = 3, \frac{1}{x} - \frac{5}{y} = 7$

(SSCE 1991)

WEEK 5

CLASS: SS 2

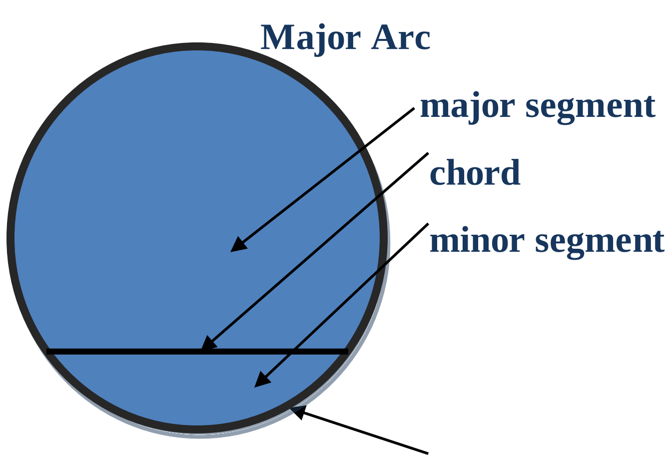
TOPIC: CIRCLE GEOMETRY

CONTENT:

- Lines and regions of a circle.
- Circle theorems including:
  - Angles subtended by chords in circle;
  - Angles subtended by chords at the centre;
  - Perpendicular bisectors of chords;
  - Angles in alternate segments.
  - Cyclic quadrilaterals

ANGLES SUBTENDED BY CHORDS IN CIRCLE

The word chord is a straight line joining any two points such as A and B on the circumference of a circle. The chord divides the circle into two parts called the segments (minor and major)



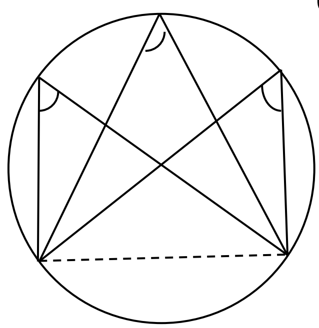
Minor Arc

The larger part of the circle is called the major segment while the smaller part --- the minor segment. Each of these parts is called the alternate segment of the other.

Note: A major segment has a major arc while a minor segment a minor arc.

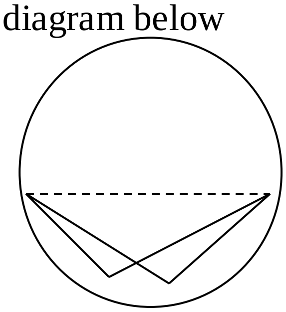
A circle is the set of all points at a constant distance from a fixed point in a plane. The fixed point is the centre of the circle, the distance from the fixed point (is constant), is called the radius.

It will be noted that it is the chord that subtends (project out) angles viz:



From the diagram, P,Q and R are points on the circumference of a circle. $A\hat{P}B, A\hat{Q}B, A\hat{R}B$ are angles subtended at the circumference by the chord AB or by the minor arc AB. $A\hat{P}B, A\hat{Q}B, A\hat{R}B$ are all angles in the same major segment APQRB.

Similarly, from the diagram below

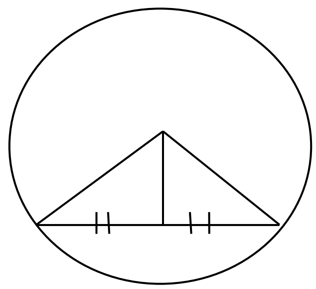


.A$\hat{X}B$ & $A\hat{Y}B$ are angles subtended by the chord AB or by the major arc AB in the minor segment AXYB or the alternate segment.

ANGLES SUBTENDED BY CHORDS AT THE CENTRE

Examples:

Theorem*: A straight line drawn from the centre of the circle to the middle point of a chord which is not a diameter, is at right angle*



D B

Given: A chord AB of a circle with centre O, is the mid-point of AB such that AD = DB

To prove: $A\hat{D}O = B\hat{D}O = 90^{\circ}$

Construction: join OA and OB

Proof: $\overline{OA} = \overline{OB}$ (radii of the circle)

$\overline{AD} = \overline{DB}$ (Given)

$\overline{OD}$ is common

Hence $\Delta AOD \equiv \Delta BOD \quad (SSS)$

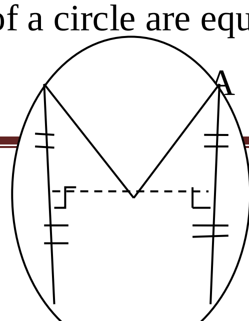
$$A\hat{D}O = B\hat{D}O$$

But $\quad A\hat{D}O + B\hat{D}O = 180° \ (angles\ on\ a\ straight\ line)$

$$\therefore \ A\hat{D}O = 180 \div 2 = 90°$$

$\Rightarrow A\hat{D}O = B\hat{D}O = 90°$

THEOREM: Equal chords of a circle are equidistant from the centre of the circle.



D

M N

B C

Given: chord AB = chord DC

To prove: $\overline{OM} = \overline{ON}$

Construction: join $\overline{OA}\ and\ \overline{OD}$

Proof: In $\Delta s\ OMA\ and\ OND$

$$O\widehat{M}A = O\widehat{N}D = 90°$$

OA = OD (radii)

$$\overline{AM} = \frac{1}{2}\overline{AB} = \frac{1}{2}\overline{DC} = \overline{DN}(chord\ property)$$

$$\therefore\ \Delta OMA \equiv \Delta OND \quad (R.H.S)$$

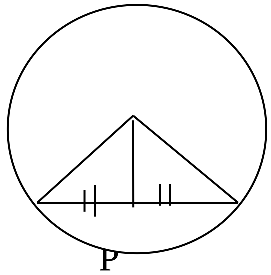
$$\therefore \overline{OM} = \overline{ON}$$

Converse: chords that have the same distance (i.e equidistant) from the centre of the circle are of the same length. If $\overline{OM} = \overline{ON}$, then $\overline{AB} = \overline{DC}$

Examples:

A chord of length 24cm is 13cm from the centre of the circle. Calculate the radius of the circle

Solution:



P Q

From the diagram, $\overline{PQ} = 24cm, \overline{PM} = \overline{MQ} = 12cm \quad (\overline{OM}\ is\ midpoint\ of \overline{PQ})$

In $\Delta POM\ or\ OMQ,$

$.r^2 = 13^2 + 12^2 (pythagoras'rule)$

= 169 + 144

= 313

, r = $\sqrt{313}$ = 17.69cm

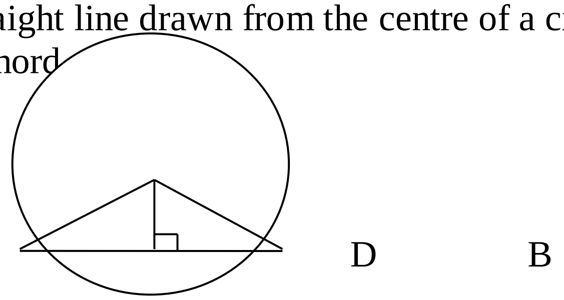
Class Activity:

1. A chord is 5cm from the centre of a circle of diameter 26cm.Find the length of the chord. (WAEC)
2. Calculate the length of a chord which is 6cm from the centre of the circle of radius 10cm

PERPENDICULAR BISECTORS OF CHORDS

This talks of line(s) that divides another line into two equal parts.

THEOREM: A straight line drawn from the centre of a circle perpendicular to a chord bisects the chord



Given: A chord AB of a circle with centre O and $\overline{OD} \perp \overline{AB}$

To prove: $\overline{AD} = \overline{BD}$

Construction: join OA and OB

Proof: In $\Delta s\ AOD\ and\ BOD$

$A\widehat{D}O = B\widehat{D}O$ (given)

$\overline{OA} = \overline{OB} \quad (radii)$

OD is common

$\therefore\ \Delta AOD \equiv \Delta BOD \quad (R.H.S)$

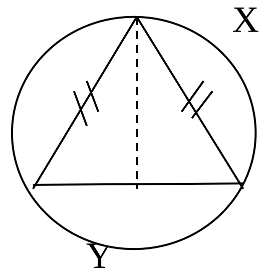
$\therefore\ \overline{AD} = \overline{BD}$

Examples;

1. XYZ is an isosceles triangle inscribed in a circle centre O. XY = XZ = 20cm and YZ = 18cm. calculate to 3s.f

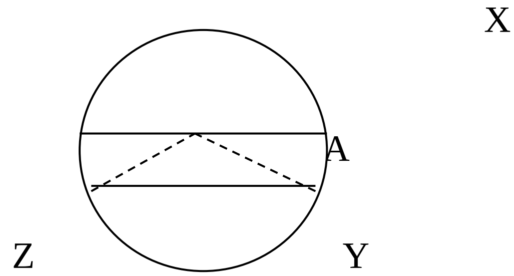
   (a) The altitude of ΔXYZ
   (b) The diameter of the circle

Solution:



B

Z



In $\Delta XYQ,$

.$(XQ)^2 = (XY)^2 + (YQ)^2$

$$= 20^2 - 9^2$$

$$= 400 - 81$$

$$= 319$$

(XQ) = $\sqrt{319}$

= 17.9cm

(b.) $\overline{AB}$ is the diameter of the circle $\overline{AC} = \overline{CB}$, radii = $\overline{YC}\ or\ \overline{ZC}$

In $\Delta XYQ,$

$$\text{Sin}\,\theta = \frac{9}{20}$$

$$\text{Sin}\,\theta = 0.45$$

$$\theta = \sin^{-1}(0.45)$$

$$\theta = 26.7°$$

$$\therefore 2\theta = 2 \times 26.7° = 53.4°$$

In $\Delta CYQ,$

$$\sin 53.4 = \frac{9}{r}$$

$$r = \frac{9}{\sin 53.4}$$

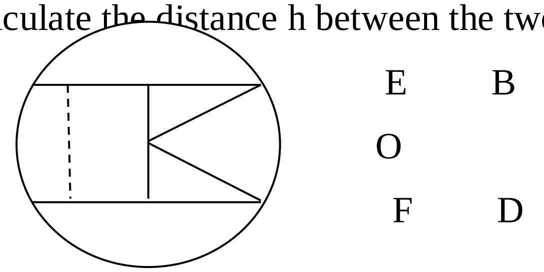
$$r = 11.21cm$$

But diameter, $d = 2r$

$$= 2 \times 11.21cm$$

$$= 22.42cm$$

2.The diagram below shows two parallel chords AB and CD that lie on opposite sides of the centre O of the circle. AB = 40cm, CD = 30cm and the radius of the circle is 25cm. Calculate the distance h between the two chords



Solution:

$$\overline{FE} = \overline{EB}(\overline{OE}\ \ bisects\ \overline{AB})$$

$$\overline{AE} = \overline{EB} = 40cm \div 2 = 20cm$$

Similarly, $\overline{CF} = \overline{FD} = 30cm \div 2 = 15cm$

In $\Delta OEB,$ by Pythagoras' theorem,

.$(EO)^2 = (OB)^2 - (EB)^2$

$$= 25^2 - 20^2$$

$$= 625 - 400$$

$$= 225$$

$\therefore EO = \sqrt{225}$

$$= 15cm$$

In $\Delta OFD,$

$$(OF)^2 = 25^2 - 15^2$$

$$= 625 - 225$$

$$= 400$$

$$OF = \sqrt{400}$$

$$\therefore OF = 20cm$$

But, $h = EO + OF$

$$= 15 + 20$$

$$= 35cm$$

Class Activity

1. A chord 26cm long is 10cm away from the centre of a circle. Find the radius of the circle.
2. The diameter of a circle is 12cm if a chord is 4cmfrom the centre, calculate the length of the chord.

ANGLES IN ALTERNATE SEGMENTS

Recall: The chord that passes through the centre of the circle is called diameter and is the largest chord in a circle.

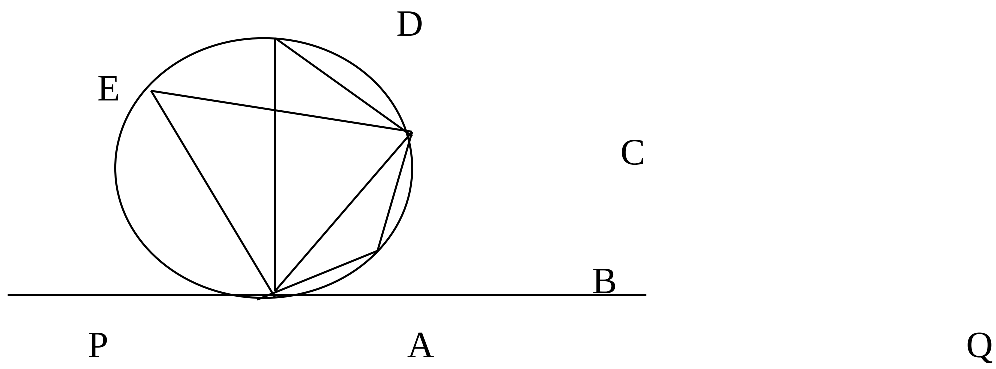
A segment is a region bounded by a chord and an arc lying between the chord's end point.

The chord that is not a diameter divides the circle into two segments -- a major and a minor segment.

But, a tangent to a circle is a straight line that touches the circle at a point.

Thus;

Theorem*: An angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment*



Given: A circle with tangent PAQ at A and chord AC dividing the circle into two segments AEC and ABC. Segments AEC is alternate to $Q\hat{A}C$

To prove: <QAC = <AEC and <PAC

= <ABC

Construction: Draw the diameter AD. Join CD

Proof: From the lettering in the above,

$$X_1 + X_2 = 90° \quad \ldots\ldots\ldots(i) \quad (DA \perp AQ)$$

Also, <ACD = 90 (angle in a semi-circle)

In $\Delta ACD,$

$X_2 + X_3$ +<ACD = 180 (sum of angles in a Δ)

$$X_2 + X_3 + 90 = 180$$

$$\therefore X_2 + X_3 = 90 \quad \ldots\ldots\ldots\ldots(ii)$$

Subtracting X_2 from equations (i) and (ii)

$$\therefore X_1 = X_3 = X_4$$

$$\therefore < QAC = < AEC$$

Also, B is a point in the minor segment.

< PAC + < CAQ = 180 (angles on a straight line)

< PAC + X_1= 180

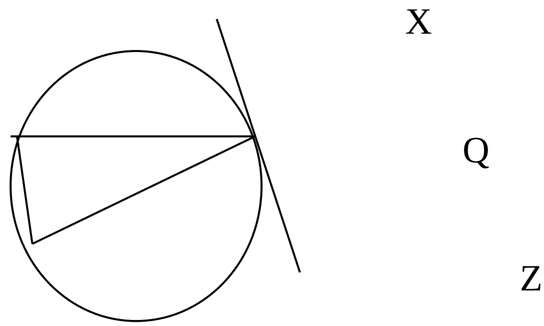
< PAC = 180 – X_1

$= 180 - X_4$ (proved $X_1 = X_4$)

< PAC = < ABC (opposite angles of a cyclic quadrilateral)

Example:

$\overline{ZQX}$ is a tangent to circle QPS. Calculate < SQX



Solution:

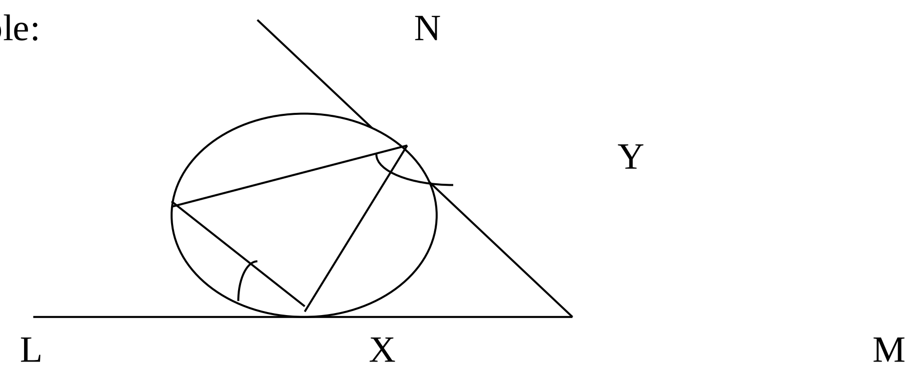
In ΔPQS,< SPQ = 180 – (55 + 48)

= 180 – 103

= 77

∴ < SQX = 77 (angles in alternate segment)

Example:



From the above, $\overline{LXM}$ *and* $\overline{NYM}$ are tangents to the circle with centre O. Find X

Solution:

<XYZ = $2x°$ (angles in alternate segment)

<ZYN + 100 = 180 (angles on a straight line)

$$\therefore < ZYN = 80°$$

<ZXY = 80° (angles in alternate segment)

<XZY = 100 – 2x (angles in alternate segment)

<MXY = <XZY = 100 – 2x (angles in alternate segment)

.∴ <MXY = <MYX =100 – 2x

.ΔMXY is an isosceles triangle

.∴ 35 + 2(100 –2x) = 180 (sum of angles in a Δ)

2(100 –2x) = 180 – 35

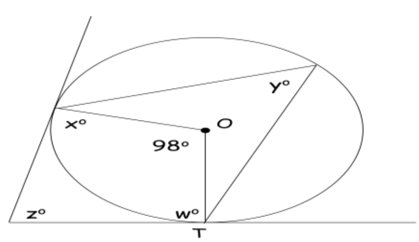
200 – 4x = 145

4x = 55

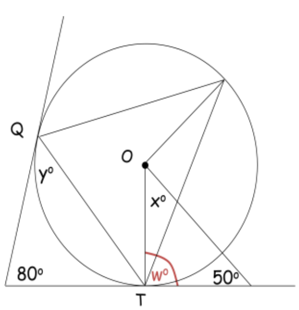
.∴ $x = 13.75°$

Class Activity:

1. PQ and PT are tangents to a circle with centre O. Find the unknown angles giving reasons.



2. PQ and PT are tangents to a circle with centre O. Find the unknown angles giving reasons.

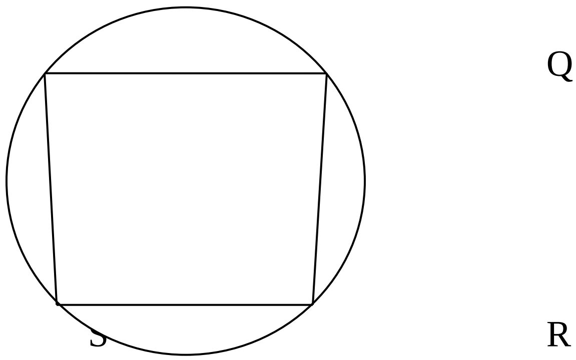


Cyclic Quadrilateral

(i) Quadrilateral is a four sided plane shape

(ii) A cyclic quadrilateral is a quadrilateral that is enclosed in a circle such that the four vertices touch the circumference of the circle.

Note: the four points where the vertices touch are referred to as concyclic points.



Theorem:

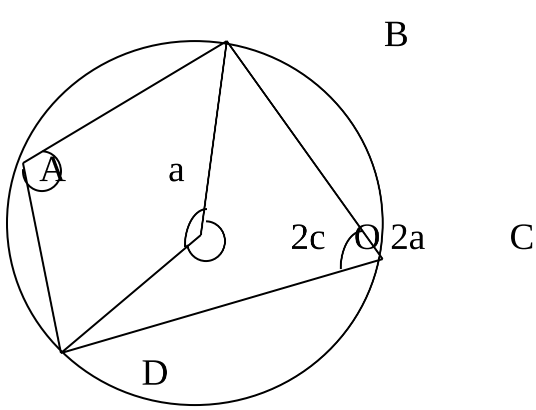
The opposite angles in a cyclic quadrilateral are supplementary.

Note: Two angles are supplementary if their sum is 180 and complementary if their sum is 90.

Given: A cyclic quadrilateral ABCD in a circle with centre O.

To prove: <BAD + < BCD = 180

Construction: Join OB, OD



Proof: Using letters in the diagram, Let <BAD = a

Reflex BOD = 2a (angle at the centre is twice the angle at the circumference)

Let < BCD = c

Obtuse BOD = 2c (angle at the centre is twice the angle at the circumference)

But 2a + 2c = 360 (angle at a point)

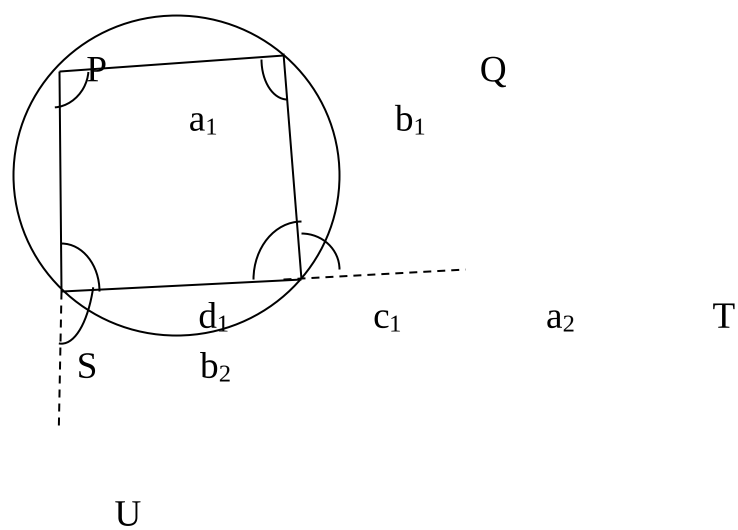
$\Rightarrow$ 2(a + c) = 360

$\Rightarrow$ a + c = $\frac{360}{2}$

$\therefore$ a + c = 180^0

Theorem

The exterior angle of a cyclic quadrilateral is equal to the interior opposite angles. Using the letters in the diagram,



Given: A Cyclic quadrilateral PQRS

To prove: $a_1 = a_2\ and\ \ b_1 = b_2$

Construction: Produce SR to T and PS to U.

Proof:

$$a_1 + c_1 = 180 \quad \text{(opposite angles of a cyclic quadrilateral)}$$

$$a_2 + c_1 = 180 \quad \text{(angles on a straight line)}$$

$$\Rightarrow a_1 = a_2$$

Similarly;

$$b_1 + d_1 = 180 \quad \text{(opposite angles of a cyclic quadrilateral)}$$

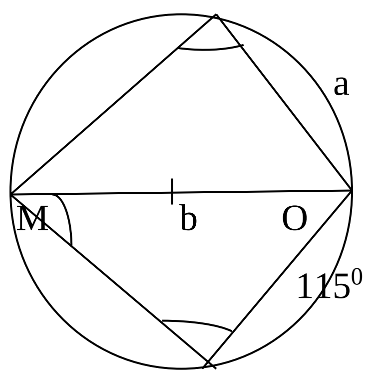
$$b_2 + d_1 = 180 \quad \text{(angles on a straight line)}$$

$$\Rightarrow \quad b_1 = b_2$$

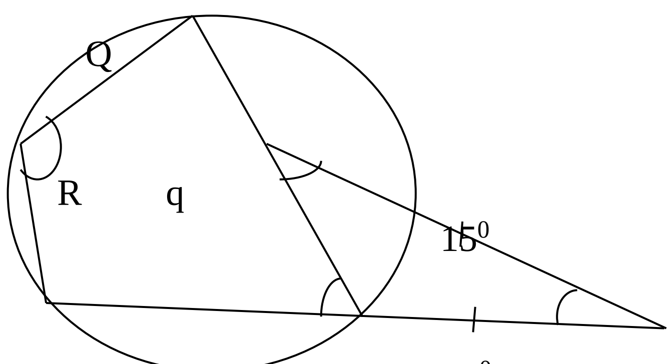
Class Activity

(1) Find the lettered angles in each of the figures below;

(a)

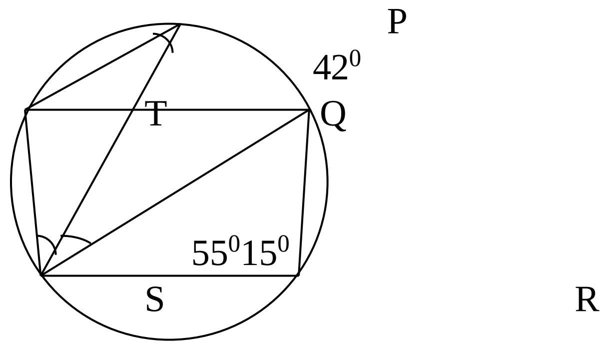


(b)

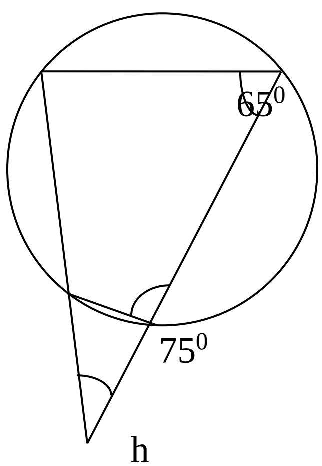


PRACTICE EXERCISE

(1) O is the centre of the circle PQRST. If <SPT = 42^0, <PST = 55^0 and <PSQ = 15^0, Find <QRS.



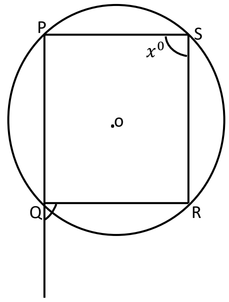
(2) Find angle h in the diagram below;



(3) In the diagram, O is the centre of the circle and PQRS is a cyclic quadrilateral. Find the value of x.

A. 25^0 B. 65^0 C. 115^0 D. 130^0

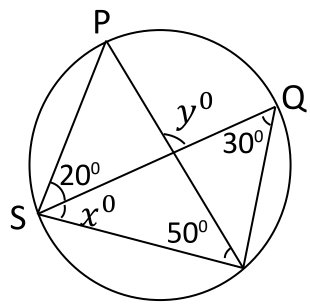
(SSCE 2008)



(4) . In the diagram, P, Q, R, S are points on the circle, PQS = 30^0, PRS = 50^0 and PSQ = 20^0. What is the value of $x^0 + y^0$?

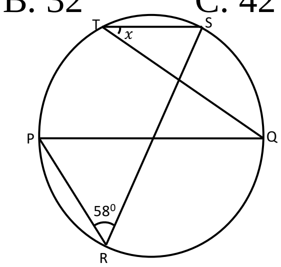
A. 260^0 B. 130^0 C. 100^0 D.80^0

(SSCE 2006)



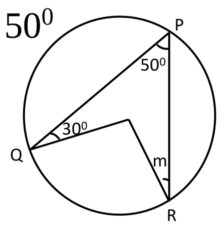
(5) In the diagram, PQ is a diameter of the circle and <PRS = 58^0. Find <STQ.

A. 29^0 B. 32^0 C. 42^0 D. 53^0 (SSCE 2001)



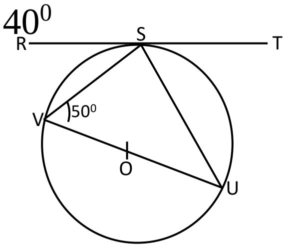
ASSIGNMENT

1. In the diagram, PQR is a circle with centre O. <QRP = 50^0, <PQO = 30^0 and <ORP = m. Find m.
   A. 20^0 B. 25^0C. 30^0D. 50^0 (SSCE 1999)



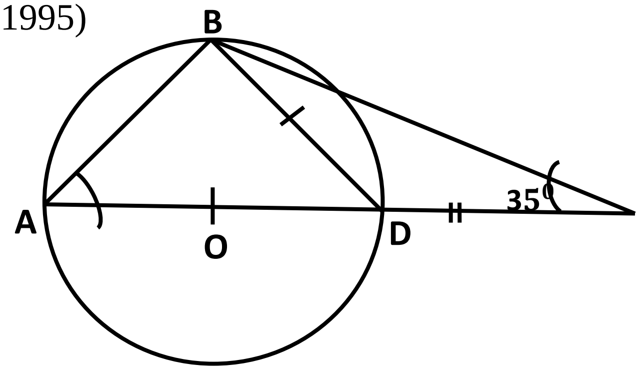
2. In the diagram, PST is a tangent to circle VSU centre O. <SVU = 50^0 and UV is a diameter. Calculate <RSV.

A. 90^0 B. 50^0 C. 45^0 D. 40^0 (SSCE 1999)



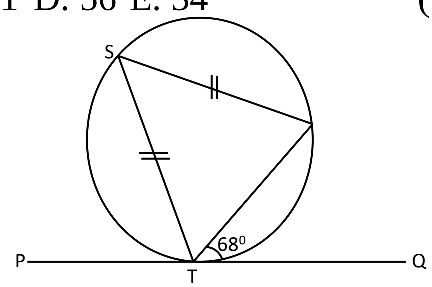
3. In the diagram below, O is the centre of the circle and $|BD| = |DC|$. If $\angle DBC = 35^0$ find the ∠BAO.

A. 20^0 B. 25^0 C. 30^0 D. 35^0 E. 40^0
(SSCE 1995)



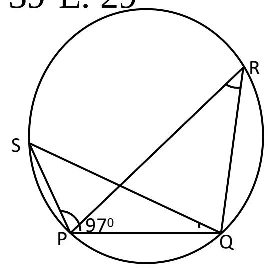
4. In the diagram, PQ is the tangent to the circle RST at T. /ST/ = /SR/ and <RTQ = 68^0. Find <PST.

A. 68^0 B. 62^0 C. 61^0 D. 56^0 E. 34^0 (SSCE 1994)



5. The diagram shows a circle PQRS in which <PQR = 54^0 and <SPQ = 97^0. Find <PQS.

A. 61^0 B. 51^0 C. 43^0 D. 39^0 E. 29^0 (SSCE 1994)



KEYWORDS: THEOREM, PROVE, CYCLIC, QUADRILLATERAL, SUBTENDS,SUPPLIMENTARY, RIGHT ANGLE, ETC

WEEK 6

CLASS: SS 2

TOPIC: CIRCLE THEOREM

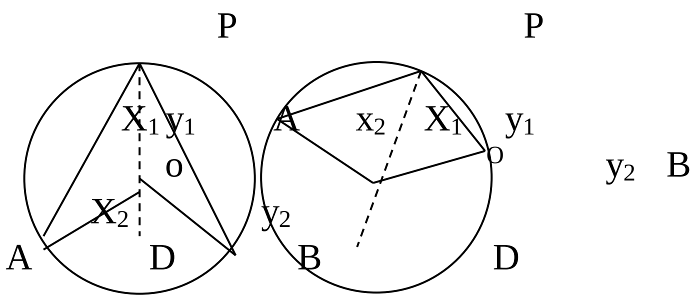
- The angle which an arc subtends at the centre is twice the angle it subtends at the circumference.
- Angles in the same segment of a circle are equal.
- Angle in a semi-circle.
- Tangent to a circle.

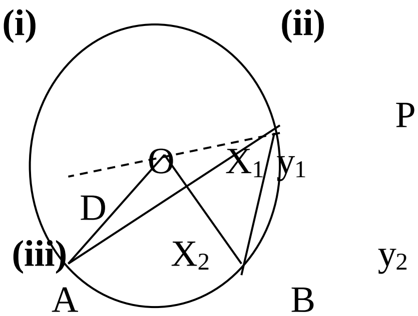
PROOF OF (i) ***The angle which an arc subtends at the centre is twice the angle it subtends at the circumference.***

The angle which an arc (or a chord) of a circle subtends at the centre of the circle is twice the angle which it subtends at any point on the remaining part of the circumference.

Note: An arc of a circle is any connected part of the circle's circumference.
A chord which is not a diameter divides the circle into two arcs- a major and a minor arc.

Given: An arc AB of a circle with 'O' and a point 'P' on the circumference.
To Prove: $A\hat{O}B = 2A\hat{P}B$
Construction: Join $\overline{PO}$ and produce the line to a point D
Sketch:





Proof: since $\overline{AO} = \overline{OP}$ (radii in the same circle)

$X_1 = X_2$ (base angles of isosceles A$\hat{O}$P)

A$\hat{O}$D = $X_1 + X_2$ (exterior angle of A$\hat{O}$P)

A$\hat{O}$D = $2X_1$ (since $X_1 = X_2$)

Similarly, BOD = $2Y_1$

In (a) acute/obtuse AOB = AOD + BOD

In (b) reflex AOB = AOD + BOD

= $2X_1$ + $2Y_1$

= $2(X_1 + Y_1)$

= 2APB

In (c) AOB = AOD – BOD

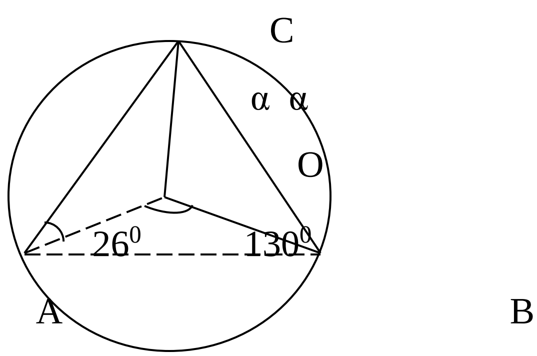
= $2Y_1 - 2X_1$

= $2(Y_1 - X_1)$

= 2APB

∴ AOB = 2APB (in all cases)

(2) in the diagram below, O is the centre of the circle ACB. If <CAO = 26^0 and <AOB = 130^0, calculate (a) <OBC and (b) <COB (WAEC)



Solution:

ACB = $\frac{130}{2}$ $(proved\ theorem)$

$= 65^0$

$= \alpha + \alpha$

$\alpha = \frac{65}{2}$

$= 32.5^0$

AOC = 180 – (26 + 32.5)

= 180 – 58.5

$= 121.5^0$

COB = 360 – (130 + 121.5) (angle at a point)

= 360 – 251.5

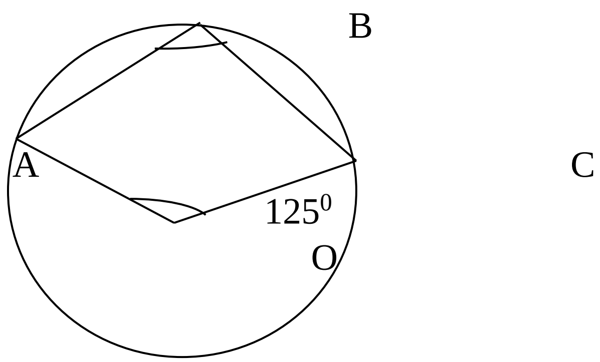
$= 108.5^0$

∴ OBC = 180 – (108.5 + 32.5)

= 180 – 141

$= 39^0$

(3) Given a circle with centre O while A,B and C are points on the circumference. Find <ABC, if the obtuse <AOC = 125^0



Solution:

Reflex AOC = 360 – 125 (angle at a point)

$= 235^0$

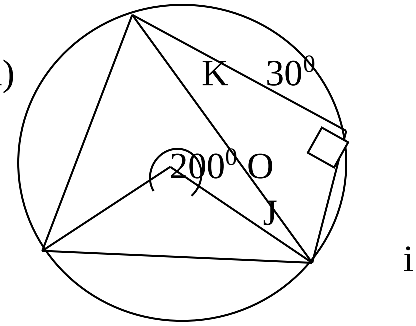
∴ ABC = $\frac{235}{2}$ (angle at the centre is twice the angle at the circumference)

$= 117.5^0$

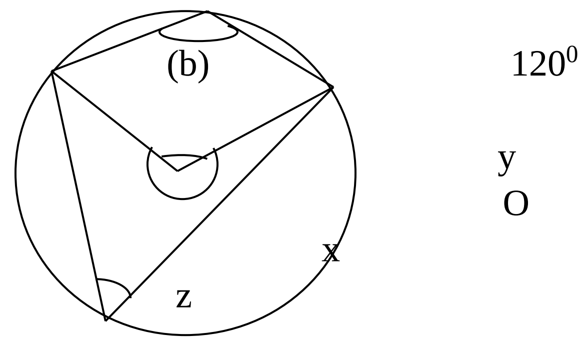
Class Activity

1. Find the lettered angles in each of the figures below;

(a)



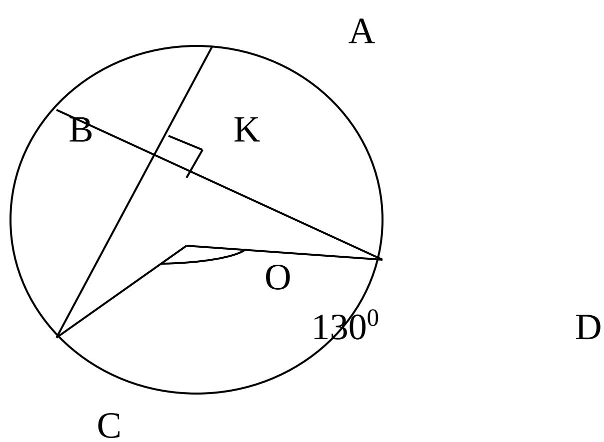
(b)



2. In the diagram, ABCD is a circle centre O. AC and BD intersect at right angles at K. Angle COD is 130^0, calculate angles (i) DAC

   (ii) ADB

   (iii) AOB (WAEC)

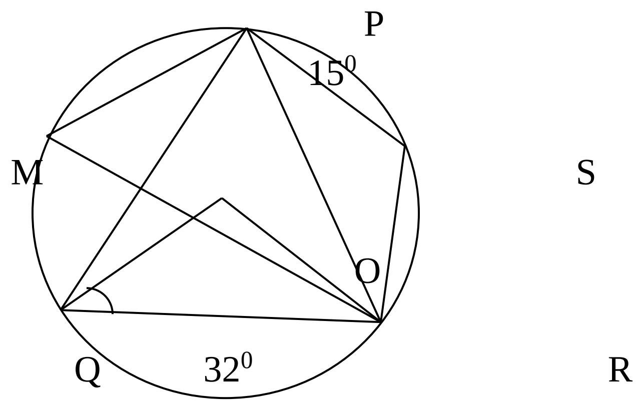


3. (a) Prove that the angle which an arc of a circle subtends at the centre is twice that which it subtends at any point on the remaining part of the circumference.

   (a) In the diagram below, O is the centre of the circle <OQR = 32^0 and <MPQ = 15^0

   Calculate: (i) <QPR

(ii) <MQO (WAEC)

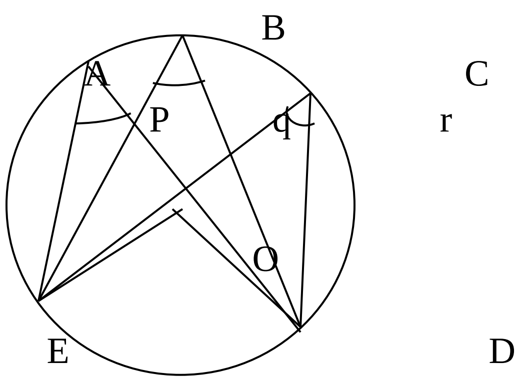


PROOF OF :Angles in the same segment of a circle are equal.

Given: points A,B and C on the major segment of a circle ABCDE with centre O.

To Prove: <EAD = <EBD = <ECD

Construction: Join EO; DO



Proof:

EOD = 2p (angle at the centre is twice angle at the circumference)

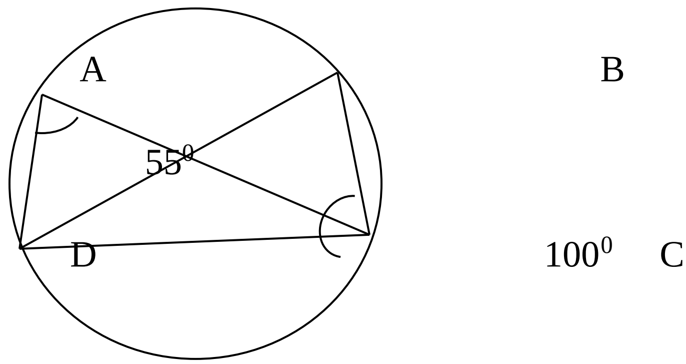
EOD = 2q (angle at the centre is twice angle at the circumference)

EOD = 2r (angle at the centre is twice angle at the circumference)

$\Rightarrow$ p = q = r

$\therefore$ EAD = EBD = ECD

(2) The diagram below shows a circle ABCD in which <DAC = 55^0 and <BCD = 100^0, find BDC.



Solution:

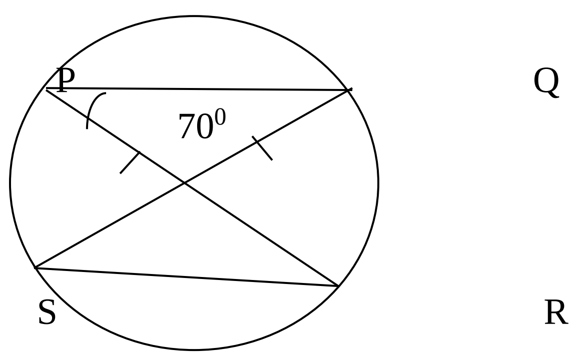
<CAD = <CBD = 55^0 (angles on the same segment)

∴ <BDC + <CBD + <BCD = 180^0 (sum of the angles of a triangle)

⇒ <BDC + 55^0 + 100^0 = 180^0

⇒ <BDC = 25^0

(3) In the diagram below, PQRS is a circle if /PT/ = /QT/ and <QPT = 70^0, calculate < PRS? (WAEC)



In△ PQT, PT = TQ (isosceles triangle)

∴ QPT = PQT = 70^0

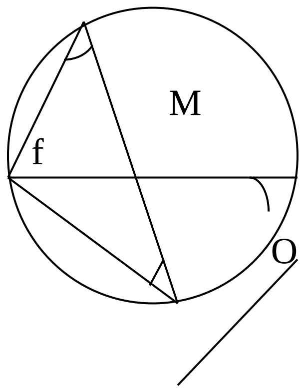
But PQ = SR common chord

SRT = QPT = 70^0 (alternate angle)

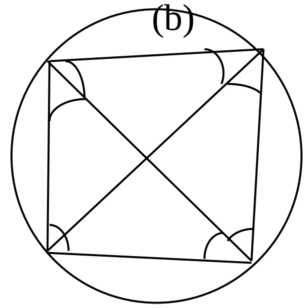
Class Activity

1. Find the lettered angles in each of the figures below;

(a)



(b)

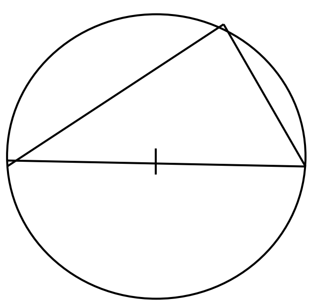


PROOF OF:Angle in a semi-circle

Given: PQ is the diameter of a circle with c entre O and R is any point on the circumference.

To Prove: PRQ = 90^0

Construction: PR, RQ



Proof:

<POQ = 2PRQ (angle at the centre is twice that at the circumference)

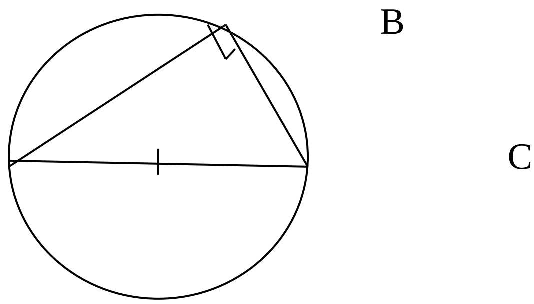
But POQ = 180^0 (angle on a straight line)

$\therefore$ 2PRQ = 180^0

$PRQ = \frac{180}{2}$

$\therefore PRQ = 90^0$

(2) In the diagram, O is the centre of the circle. If <BAC = 55^0, find the value of <ACB



Solution:

<ABC = 90^0 (angle in a semi-circle)

<ABC + <ACB + <BAC = 180^0 (sum of angles in a triangle)

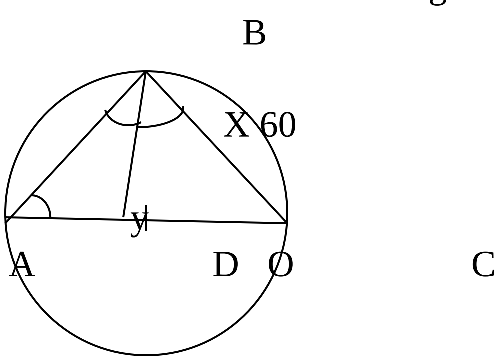
$\Rightarrow 90^0$ + <ACB + 55^0 = 180^0

<ACB + 145^0 = 180^0

<ACB = 180 – 145

< ACB = 35^0

(3) Find the values of the lettered angles in the figure below;



Solution:

<ABC = 90

$\therefore$ X = 90 – 60

= 30

In ABD, <ADB = 90 (perpendicular bisector)

∴ $x + y + 90 = 180$

$30 + y + 90 = 180$

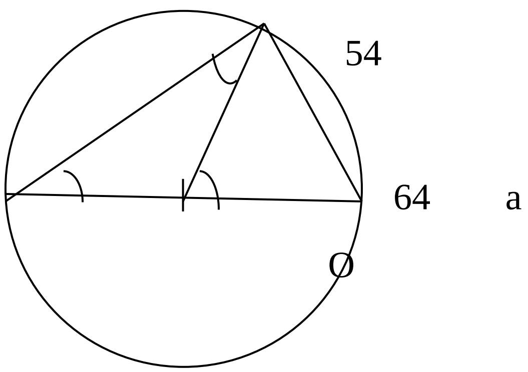
$y = 180 - 120$

$y = 60^0$

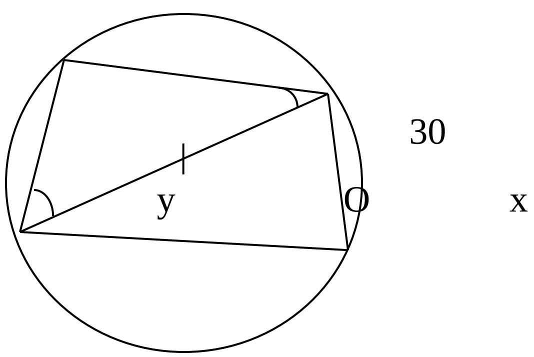
Class Activity

(1) Find the values of the lettered angles in the figures below;

(a)



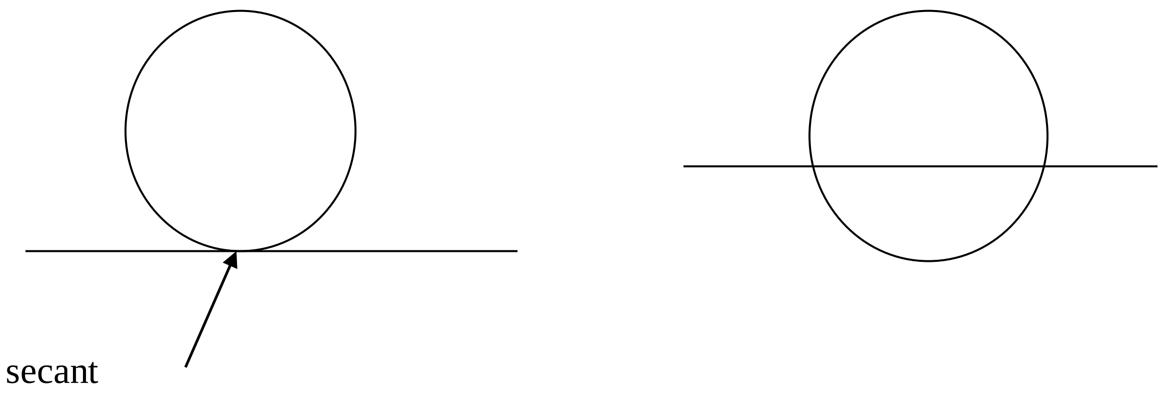
(b)



Tangent to a circle

The tangent to a circle is a straight line drawn to touch the circle at a point. The point where the line touches the circle is referred to as the point of contact.

A secant is a straight line that cuts a given circle into two clear points



Note:

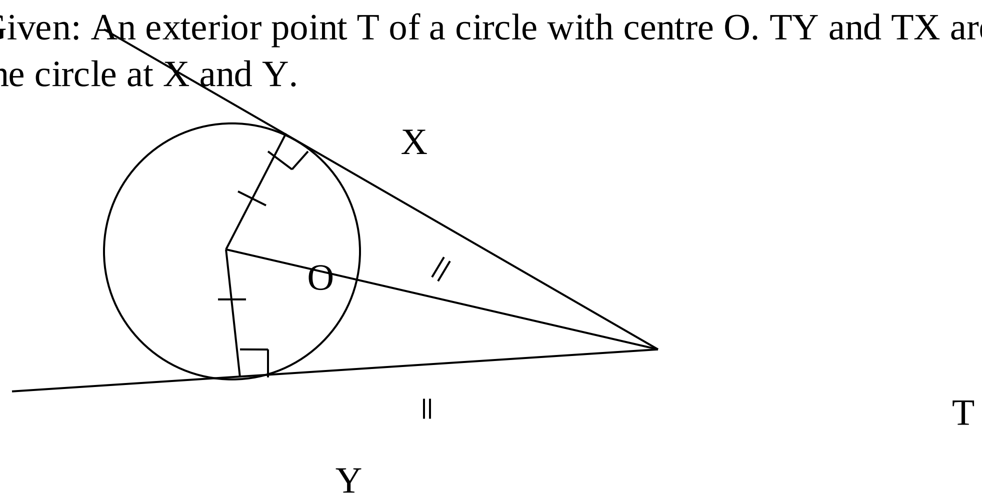
(i) A tangent to a circle is perpendicular to the radius drawn to its point of contact.

(ii) The perpendicular to a tangent at its point of contact passes through the centre of the circle.

Theorem:

Two tangents drawn to a circle from an external point are equal in length.

Given: An exterior point T of a circle with centre O. TY and TX are tangents to the circle at X and Y.



To Prove: /TX/ = /TY/

Construction: Join TX, TO and TY

Proof: In triangles TXO and TYO

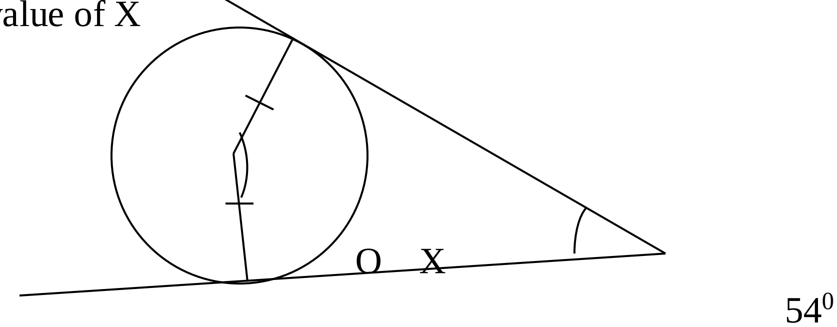
TXO = TYO = 90 (tangent perpendicular to radius)

/XO/ = /YO/ (radius)

/TO/ = /TO/ (common)

∴ /TX/ = /TY/

(1) AB and AC are tangents from a point A to a circle centre O. If <BAC = 54^0, find the value of X



Solution:

ABO = ACO (tangents to a circle from an external point are equal)

ABO = ACO = 90 (tangents perpendicular to radius)

∴ ABO + ACO + BAC + X = 360 (sum of angles in a quadrilateral)

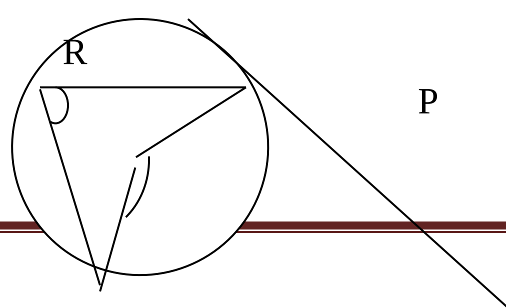
⇒ 90 + 90 + 54 + X = 360

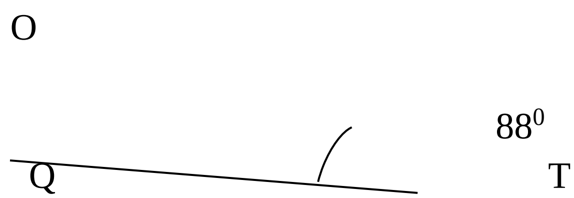
⇒ 234 + X = 360

⇒ X = 360 – 234

∴ X = 126^0

(2) Calculate PRQ





Solution:

PTQ = 88^0

Join PO and QO

OP and OQ are radii

TQO = TPO = 90 (radii perpendicular to tangent)

∴ OPT + OQT = 180

PTQ + QTP = 180

QOP = 180 – 88

= 92^0

But QRP = ½ (QOP) (angle at centre is twice angle at the circumference)

= ½ (92)

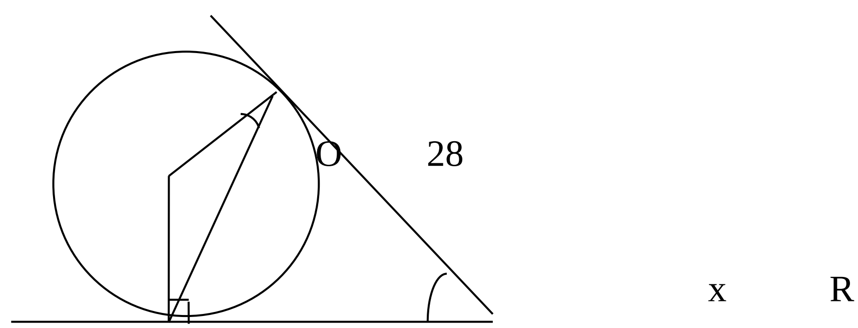
= 46

∴ PRQ = 46^0

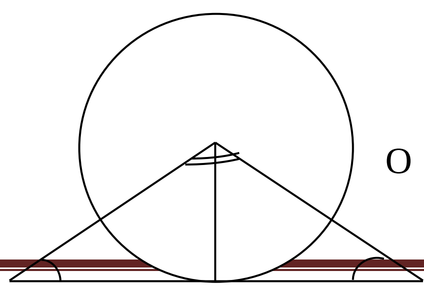
Class Activity

1. Calculate the values of the marked angles below;

(a)



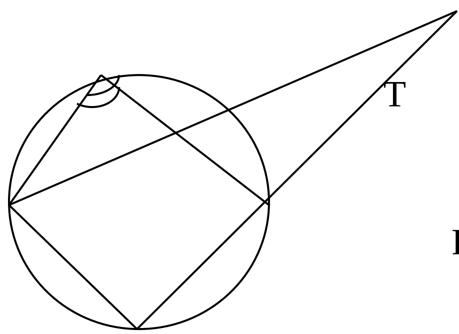
(b)



45 45

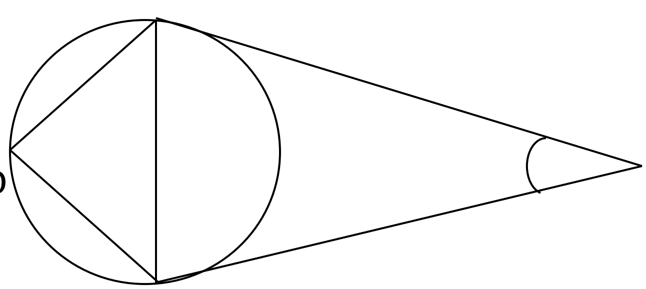
PRACTICE EXERCISE

(1) PQRT is a circle. /ST/ = /RS/ and TSR = 51^{0}, find POR (JAMB)

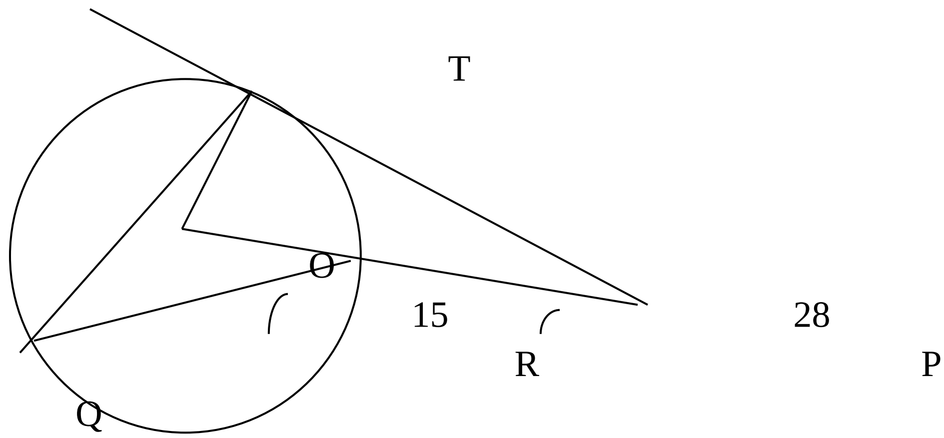


(2) AB and CB are tangents to the circle. Given that CBA = 54^{0}, calculate <ADC

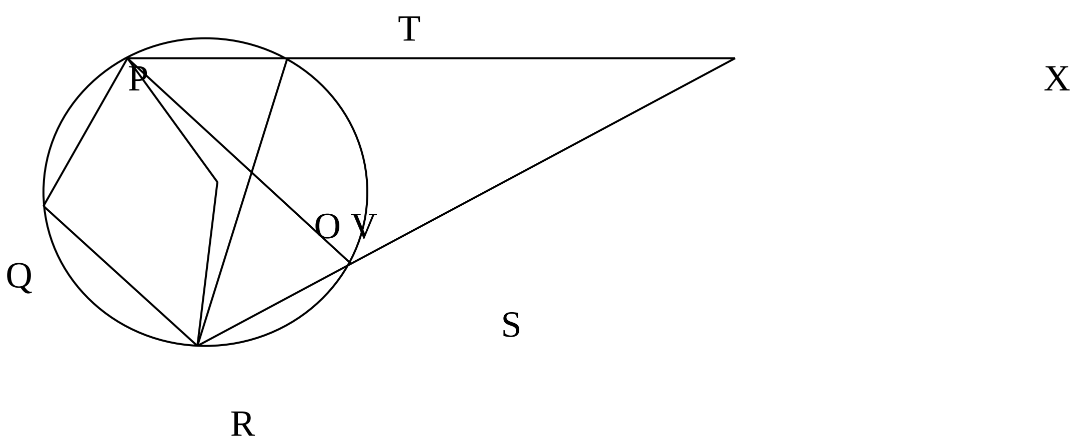
(NECO)



(3) TP is a tangent to the circle TRQ with centre O. if <TPO = 28^0 and <ORQ = 15^0. Find (a) <RQT (b) <QTO
(NECO)

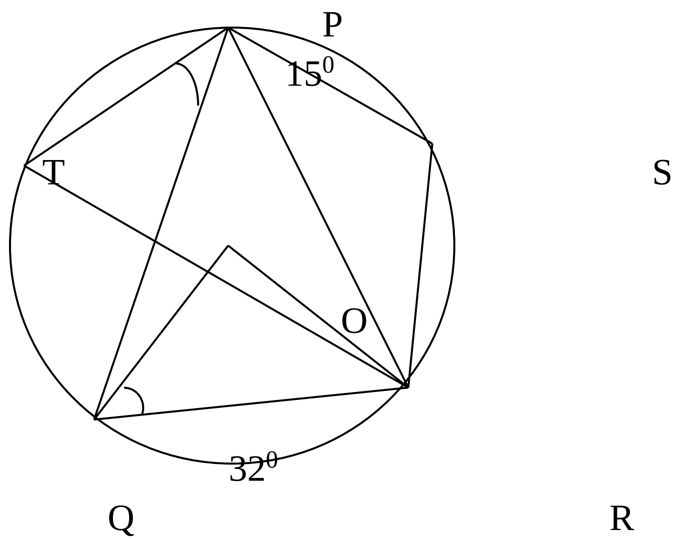


(4) PQRST lie on the circumference of the circle with centre O. The chords PS and RT intersect at V and the chords PT and RS produced meet at X as shown below;



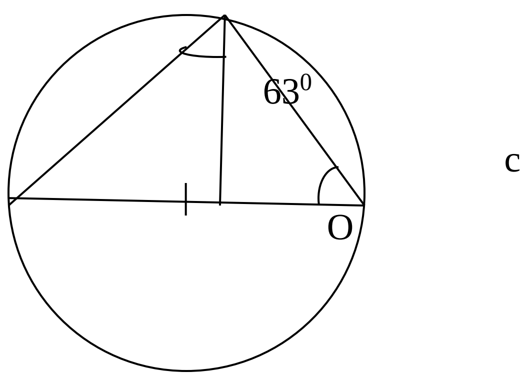
Given that the obtuse POR = 4 PXR
Prove that: (a) SVT = 3 PXR , (b) PSR = PQR
(London G.C.E)

(5) O is the centre of the circle. <OQR = 32^0 and <TPQ = 15^0, Calculate (a) <QPR (b) <TQO (WAEC)

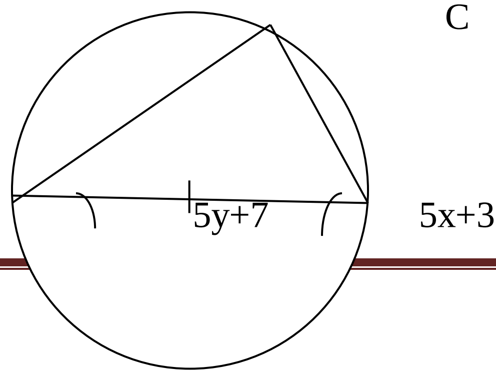


ASSIGNMENT

(1) Find the values of the lettered angles in the figure below;

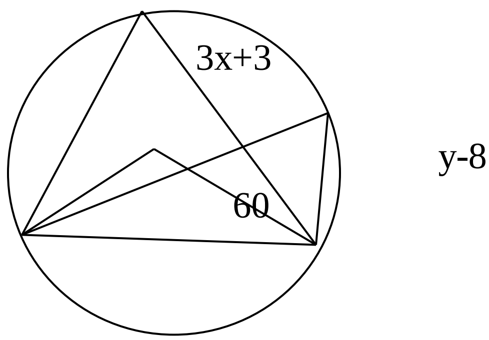


(2) In the diagram, AB is the diameter. <ABC = $(5x + 3)^0$ and <BAC = $(5y + 7)^0$. Express y in terms of x

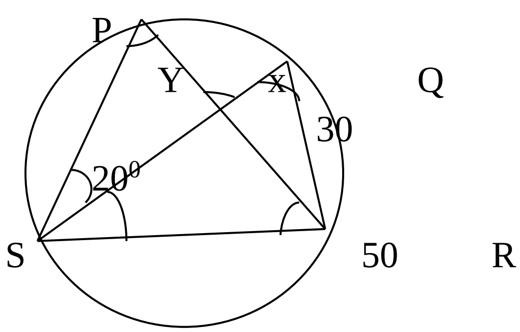


A B

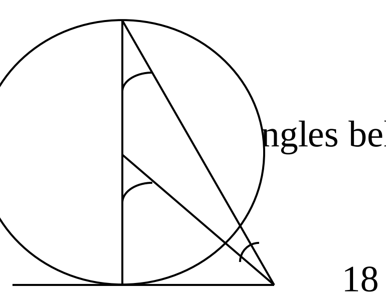
(3) The diagram below is a circle with its centre at O. Find the value of (a) x and (b) y



(4) P,Q,R and S are points on the circle. If <PSQ = 30^0, <PRS = 50^0 and <PSQ = 20^0, what is the value of $x + y$?

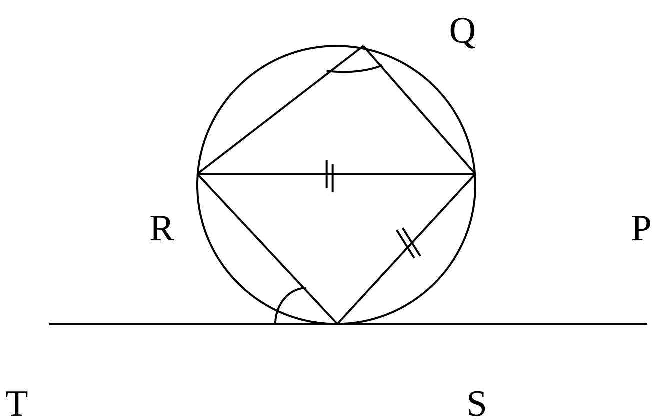


(5) Calculate the valu ngles below;



30 y

1. TS is a tangent to a circle PQRS. If /PR/ = /PS/ and PQR = 117^0, calculate RST (WAEC)



KEYWORDS: THEOREM, PROVE, CYCLIC, QUADRILLATERAL, SUBTENDS,SUPPLIMENTARY, RIGHT ANGLE, ETC

WEEK 7

MID TERM BREAK

WEEK 8

Subject: Mathematics

Class: SS 2

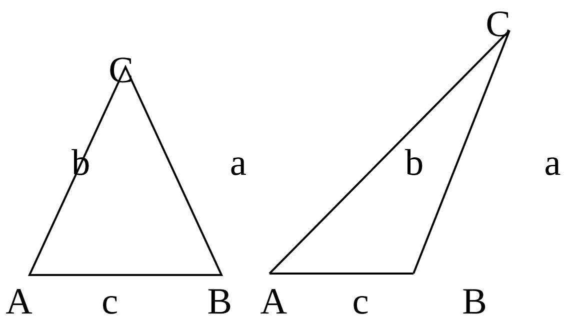
TOPIC: TRIGONOMETRY (Sine and Cosine Rule)

CONTENT:

- Derivation and application of sine rule.
- Derivation and application of cosine rule.

SINE RULE

Given any triangle ABC (acute or obtuse), with the angles labelled with capital letters A, B, C and the sides opposite these angles labelled with the corresponding small letters a, b, and c respectively as shown below.



The sine rule states that;

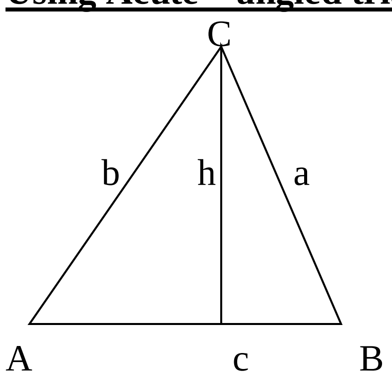
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

OR

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

<u>PROOF OF THE RULE</u>

<u>Using Acute – angled triangle</u>



Given: Any ΔABC with B acute.

To prove: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Construction: Draw the perpendicular from C to AB.

Proof: *Using the lettering in the diagram above.*

$\sin A = \frac{h}{b}$

$h = b\sin A$ --------------------- (1)

$\sin B = \frac{h}{a}$

$h = a\sin B$ ---------------------- (2)

From equation (1) and (2)

$b\sin A = a\sin B$

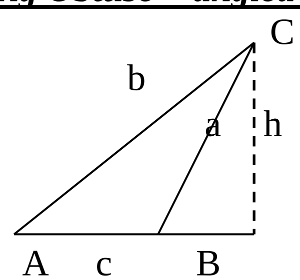
$\therefore \frac{a}{\sin A} = \frac{b}{\sin B}$

Similarly, by drawing a perpendicular from B to AC

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ ***D***

Using Obtuse – angled triangle



Given: any ΔABC with B obtuse

To Prove: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Construction: Draw the perpendicular from C to AB produced.

Proof: *With the lettering in the diagram.*

$$\sin A = \frac{h}{b}$$

$$h = b\sin A \text{ -----------------(1)}$$

$$\sin(180 - B) = \frac{h}{a} \text{ but } \sin(180-\theta) = \sin\theta$$

$$\therefore \sin B = \frac{h}{a}$$

$$h = a\sin B \text{ ----------------(2)}$$

From equation (1) and (2)

$$b\sin A = a\sin B$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Similarly, by drawing a perpendicular from A to CB produced.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\text{Sin}A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Q.E.D

APPLICATION OF SINE RULE

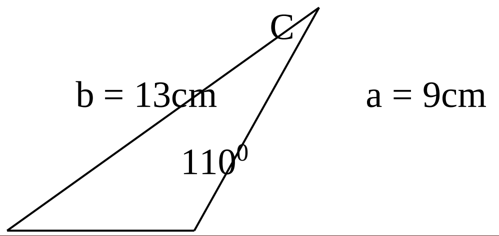
The sine rule is used for solving problems of triangle, which are NOT right – angled, and in which either two sides and the angle opposite one of them are given or two angles and any side are given.

Example 1:

In ΔABC, $a = 9cm$, $B = 110^0$, $b = 13cm$. Solve the triangle completely.

Solution:

The diagram representing the information above is given below as



A c B

Using sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{9}{\sin A} = \frac{13}{\sin 110^0}$$

$$9\sin 110^0 = 13\sin A$$

$$\sin A = \frac{9\sin 70^0}{13}$$

$$\sin A = 0.6506$$

$$A = \sin^{-1} 0.6506$$

$$A = 40.6^0$$

$\therefore$ $A \approx 41^0$ (*nearest degree*)

To find angle C

$A + B + C = 180^0$ *[sum of <s in a Δ]*

$$41^0 + 110^0 + C = 180^0$$

$$C = 180^0 - 151^0$$

$$\therefore C = 29^0$$

To find side c, use sine rule

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{9}{\sin 41} = \frac{c}{\sin 29}$$

$$c = \frac{9\sin 29}{\sin 41}$$

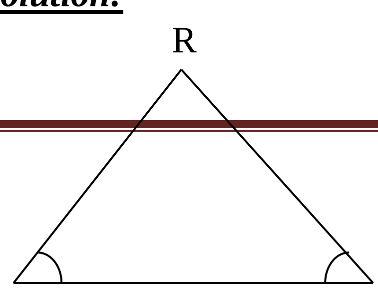
$$c = 6.65\text{cm}$$

$$\therefore c = 6.7\text{cm}$$

Example 2:

In ΔPQR, given that P = 50^0, Q = 60^0,
r = 7.5cm. Find (i) p (ii) q

Solution:



q p

50^0 60^0

P 7.5cm Q

(i) $P + Q + R = 180^0$ *[sum of <s in a Δ]*

$50^0 + 60^0 + R = 180^0$

$R = 180^0 - 110^0$

$R = 70^0$

Using sine rule

$$\frac{r}{\sin R} = \frac{p}{\sin P}$$

$$\frac{7.5}{\sin 70^0} = \frac{p}{\sin 50^0}$$

$$p = \frac{7.5 \sin 50^0}{\sin 70^0}$$

$$p = 6.11\text{cm}$$

$$\therefore p \approx 6\text{cm}$$

(ii) ***Using sine rule***

$$\frac{r}{\sin R} = \frac{q}{\sin Q}$$

$$\frac{7.5}{\sin 70^0} = \frac{q}{\sin 60^0}$$

$$q = \frac{7.5 \sin 60^0}{\sin 70^0}$$

$$q = 6.9\text{cm}$$

$$\therefore q \approx 7\text{cm}$$

Class Activity:

Find the missing sides and angles of the following triangles. Calculate all angles to the nearest degree and all sides to 1 decimal place.

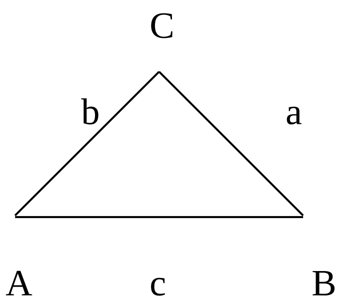
(1) ΔABC, given that $B = 68^0$, b = 27m and a = 22m.

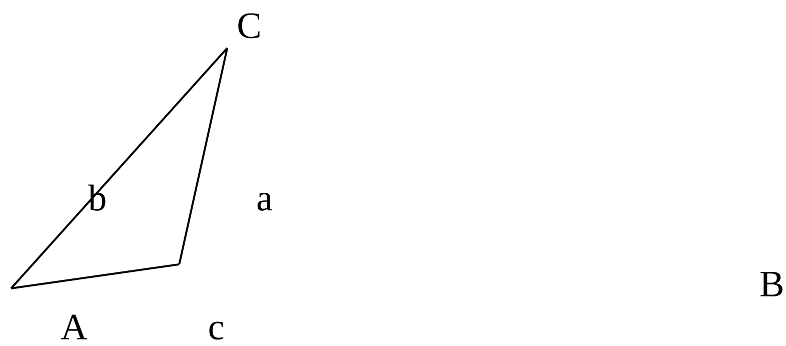
(2) ΔPQR, given that $Q = 121^0$, q = 57km and r = 17km.

(3) ΔABC, given that $C = 27^0$, c = 7cm and b = 13cm.

COSINE RULE

Given any triangle ABC (acute or obtuse), with the angles labeled with the capital letters A, B, C and the sides opposite these angles labeled with the corresponding small letters a, b, and c respectively as shown below





The cosine rule states that

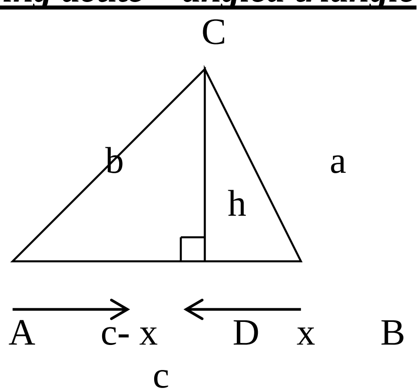
$a^2 = b^2 + c^2 – 2bc \cos A$

$b^2 = a^2 + c^2 – 2ac \cos B$

$c^2 = a^2 + b^2 – 2ab \cos C$

PROOF OF THE RULE

Using acute – angled triangle



Given: Any ΔABC with B acute.

To prove: $b^2 = a^2 + c^2 – 2ac \cos B$

Construction: Draw a perpendicular from C to AB.

Proof: *With the lettering in the diagram.*

$b^2 = (c - x)^2 + h^2$ *(Pythagoras)*

$= c^2 – 2cx + x^2 + h^2$

But in Δ BCD, $a^2 = x^2 + h^2$

$\therefore\ b^2 = c^2 - 2cx + a^2$ ----------(1)

In ΔBCD,

$\cos B = \frac{x}{a}$

$\therefore\ x = a \cos B$

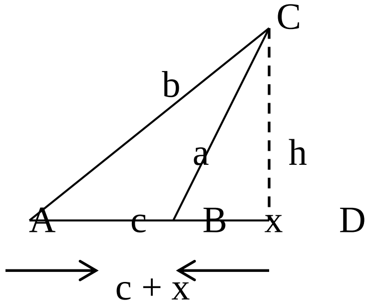
From Eqn (1)

$b^2 = c^2 + a^2 - 2cx$

$b^2 = c^2 + a^2 - 2ca \cos B$

Q.E.D

<u>Using obtuse – angled triangle</u>



<u>Given:</u> Any ΔABC with B obtuse

<u>To prove:</u> $b^2 = a^2 + c^2 - 2ac \cos B$

<u>Construction:</u> Draw the perpendicular from C to AB produced.

<u>Proof:</u> *With the lettering in the diagram.*

$b^2 = (c + x)^2 + h^2$

$= c^2 + 2cx + x^2 + h^2$

But in ΔBCD

$a^2 = x^2 + h^2$ *(by Pythagoras)*

$\therefore\ b^2 = c^2 + 2cx + a^2$

ie $b^2 = a^2 + c^2 + 2cx$ -------- (1)

In ΔBCD, $\cos B = \frac{x}{a}$

$\cos (180 - B) = \frac{x}{a}$

$-\cos B = \frac{x}{a}$

$\therefore\ x = -a\cos B$

From Eqn (1)

$b^2 = a^2 + c^2 + 2c(-a\cos B)$

$\therefore\ b^2 = a^2 + c^2 - 2ac\cos B$

Q.E.D

Similarly, $a^2 = b^2 + c^2 - 2bc\cos A$

$c^2 = a^2 + b^2 - 2ab\cos C$

<u>APPLICATIONS OF COSINE RULE</u>

Cosine rule can be used for solving problems involving triangles, which are not right–angled, in which two sides and the angle between the two sides are given i.e. two sides and the included angle.

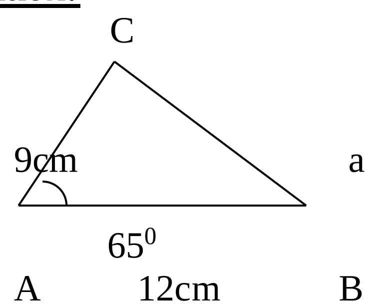
Secondly, the formula can be used to find the angles of a triangle when the three sides of the triangle are given.

<u>USING COSINE RULE TO FIND THE MISSING SIDE OF A TRIANGLE</u>

<u>Examples:</u>

(1) In Δ ABC, given that $A = 65^0$, $b = 9cm$ and $c = 12cm$, Find a.

<u>Solution:</u>



Using cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$

$= 9^2 + 12^2 - 2 \times 9 \times 12\cos 65$

$= 81 + 144 - 216\cos 65$

$= 225 - 216 \times 0.4226$

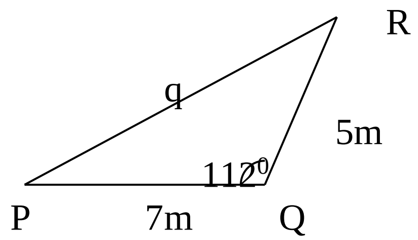
$= 225 - 91.28$

$= 133.72$

$a = \sqrt{133.72}$

$\therefore\ a = 11.56cm.$

(2) Find the value of q in the figure below.



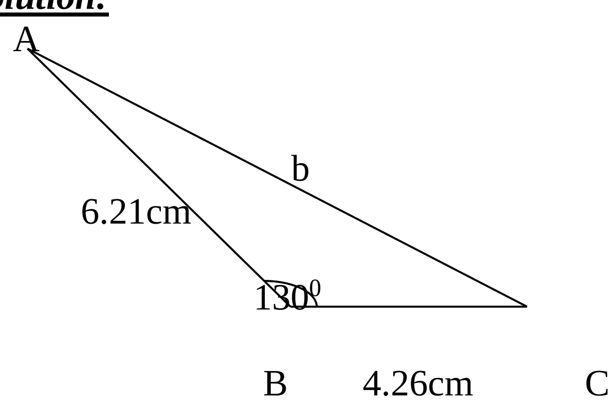
Solution:

Using cosine rule

$q^2 = p^2 + r^2 – 2prcosQ$
$= 5^2 + 7^2 – 2x5x7cos112^0$
$= 25 + 49 – 70[-cos(180 – 112)]$
$= 74 – 70(-cos\ 68)$
$= 74 + 70cos68$
$= 74 + 70\ x\ 0.3746$
$= 74 + 26.222$
$= 100.222$
$q = \sqrt{100.22}$
$\therefore\ q = 10.01$
$\therefore\ q \approx 10m$

(3) In $\Delta\ ABC$, $B = 130^0$, $a = 4.62cm$ and $c = 6.21cm$, Calculate b.

Solution:



Using cosine rule

$b^2 = a^2 + c^2 – 2ac\ cos\ B$
$= 4.62^2 + 6.21^2 – 2x4.62x6.21cos130^0$

$= 21.34+38.56-57.38[-\cos 180-130]$
$= 59.9 - 57.38 [-\cos 50]$
$= 59.9 + 57.38 \times 0.6428$
$= 59.9 + 36.88$
$= 96.78$
$b^2 = \sqrt{96.78}$
$\therefore b = 9.8$cm.

Class Activity:

Solve the following questions and approximate all answers to 1 decimal place.

(1) In ΔABC, $B = 53^0$, $c = 45$km and $a = 63$km. Find b.
(2) In ΔPQR, $Q = 111^0$, $r = 47$km and $p = 39$km. Find q.
(3) In ΔABC, $B = 87^0$, $a = 25$m and $c = 19$m. Find b.
(4) In ΔABC, $B = 142^0$, $a = 33$km and $c = 27$km. Find b.

USING COSINE RULE TO CALCULATE ANGLES

Cosine rule can also be used to calculate the angles of a triangle when the three sides are given. This is done by making the cosine of the desired angle the subject of the formula.

E.g. If

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$2bc\cos A = b^2 + c^2 - a^2$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Similarly, $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

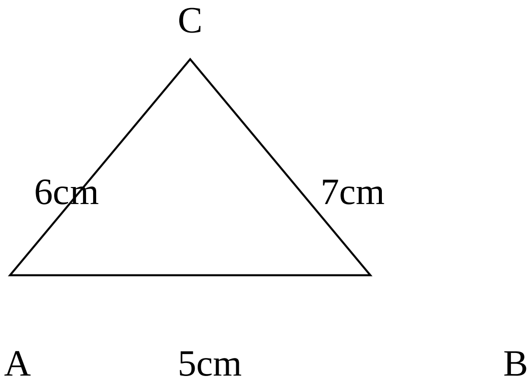
and $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

This formula is used to calculate the angles of a triangle when all the three sides of the triangle are given.

Examples:

Find the angles of the ΔABC given that a = 7cm, b = 6cm and c = 5cm.

Solution:



To find angle A,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{6^2 + 5^2 - 7^2}{2 x 6 x 5}$$

$$= \frac{36 + 25 - 49}{60}$$

$$\cos A = 0.2000$$

$$A = \cos^{-1} 0.2000$$

$$\therefore A = 78.5^0 \text{ ----------------- (1)}$$

To find angle B,

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{7^2 + 5^2 - 6^2}{2 \text{ x } 7 \text{ x } 5}$$

$$= \frac{49 + 25 - 36}{70}$$

$$\cos B = 0.5429$$

$$B = \cos^{-1} 0.5429$$

$$\therefore B = 57.1^0 \text{ ------------------(2)}$$

To find angle C,

$$\cos C = \frac{7^2 + 6^2 - 5^2}{2 \text{ x } 7 \text{ x } 5}$$

$$= \frac{49 + 36 - 25}{84}$$

$$\cos C = 0.7143$$

$$C = \cos^{-1} 0.7143$$

$$\therefore C = 44.4^0 \text{ ---------------- (3)}$$

Check: From Eqn (1), (2) and (3).

$A + B + C = 78.5^0 + 57.1^0 + 44.4^0$

$= 180^0$

Class Activity

Using cosine rule, calculate the three angles of the following triangles whose sides are given below. Approximate all your answer to the nearest degree.

(1) Δ XYZ, x = 10m, y = 16m and
z = 13m.

(2) Δ PQR, p = 25km, q = 30km, and
r = 8km.

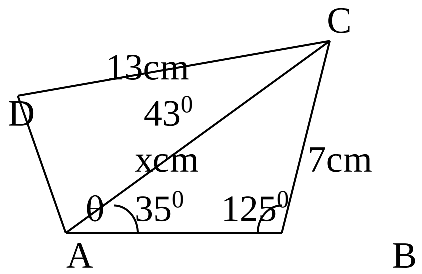
(3) ΔABC, a = 5.7cm, b = 3.5cm and
c = 4.3cm.

GENERAL PROBLEM SOLVING USING SINE AND COSINE RULE.

A combination of sine and cosine rule can be used to solve a given problem, as we shall see subsequently.

Example 8:

Find the value of the following from the diagram below (i) x (ii) θ (iii) $|BD|$.



Solution:

(i) ***Using sine rule***

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{7}{\sin 35^0} = \frac{x}{\sin 125^0}$$

$X = \frac{7 \sin 125^0}{\sin 35^0}$

$X = \frac{7 \sin 55^0}{\sin 35^0}$

$x = 9.99cm$

$\therefore\ x \approx 10cm$

(ii) ***Using sine rule***

$\frac{10}{\sin 43^0} = \frac{13}{\sin\theta}$

$10\sin\theta = 13\sin 43^0$

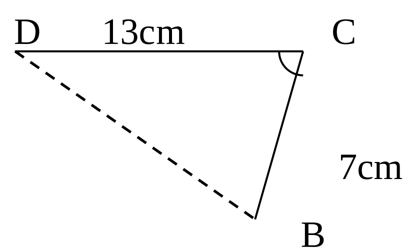
$\sin\theta = \frac{13\sin 43^0}{10}$

$\sin\theta = 0.8866$

$\theta = \sin^{-1} 0.8866$

$\theta = 62^0$

(iii) To find /BD/



BCD = BCA + ACD ------------------ (1)

BCA = $180^0 - (125^0 + 35^0)$ *(sum of ∆s in ∆ABC)*

$= 180^0 - 160^0$

$= 20^0$

ACD = $180 - (43^0 + \theta^0)$

$= 180 - (43^0 + 62^0)$

$= 180 - 105^0$

$= 75^0$

From (1)

BCD = $20^0 + 75^0$

$= 95^0$

Using cosine rule to find /BD/

$/BD/^2 = b^2 + d^2 - 2bd\cos C$

$= 13^2 + 7^2 - 2 \times 13 \times 7\cos 95^0$

$= 169 + 49 - 182[-\cos 180 - 95]$

$= 218 - 182 [-\cos 85]$

$= 218 + 182 \times 0.0872$

$= 218 + 15.87$

$/BD/^2 = 233.87$

$/BD/ = \sqrt{233.87}$

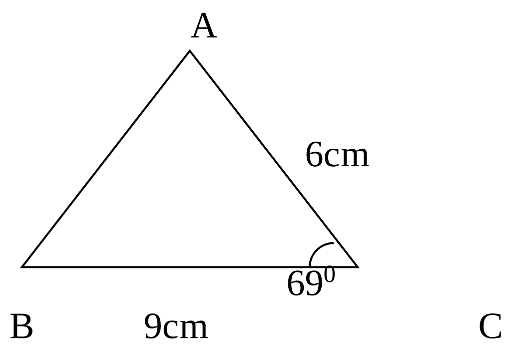
/BD/ = 15.29cm

/BD/ = 15.3cm *(1. d.p)*

<u>Example 9:</u>

Find the unknown sides and angles of a triangle ABC given that $C = 69^0$, a = 9cm and b = 6cm. Give answer to 3 significant figure.

<u>Solution:</u>



Using cosine rule

$c^2 = a^2 + b^2 - 2ab\cos C$

$= 81 + 36 - 108 \cos 69^0$

$= 117 - 108 \times 0.3584$

$= 118 - 38.71$

$= 79.29$

$C = \sqrt{79.29}$

C = 8.90cm

To get angle B, we shall use sine rule

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{6}{\sin B} = \frac{8.9}{\sin 69^0}$$

$$6\sin 69^0 = 8.9\sin B$$

$$\sin B = \frac{6\sin 69^0}{8.9}$$

$$\sin B = 0.6294$$

$$B = \sin^{-1} 0.6294$$

$$\therefore B = 39^0$$

To get angle A,

$A + B + C = 180^0$ *[sum of Ls in a Δ]*

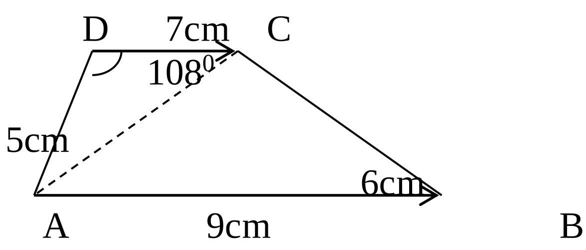
$$A + 39^0 + 69^0 = 180^0$$

$$A = 180^0 - 108^0$$

$$\therefore A = 72^0$$

Class Activity:

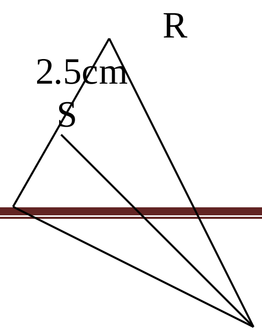
(1) The figure below is a trapezium ABCD, in which /AB/ is parallel to /DC/, and the lengths of the sides are as shown below.



Calculate the value of the following

(i) /AC/ (ii) ABC

(2)



3cm 8.3cm

P

6.4cm

Q

The figure above is a triangle PQR with the dimension as shown above. Calculate the following (i) RPQ (ii) /QS/

(3) In ΔPQR p:q:r = √3:1:1. Calculate the ratio P:Q:R in its simplest form.

(WAEC).

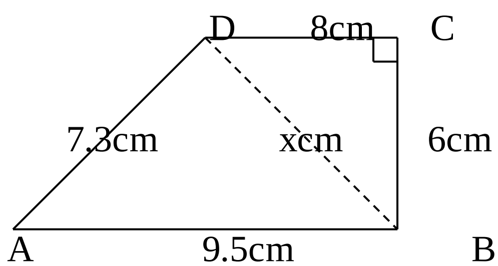
(4) Calculate the angles of the triangles whose sides are in the ratio 4:5:3.

(5) Given a triangle PQR, in which /PQ/ = 13cm, /QR/ = 9cm, /PR/ = 7cm and QR is produced to S so that /RS/ = 6cm. Calculate the following. (i) cos PRS (ii) /PS/

(6) Find the value of the following from the diagram below.

(i) x

(ii) DAB

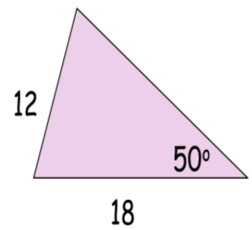


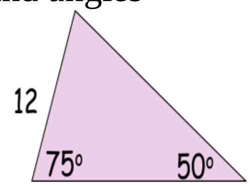
PRACTICE EXERCISE

(1) The angle of elevation of the top of a building measured from point A is 25^{o}. At point D which is 15m closer to the building, the angle of elevation is 35^{o} Calculate the height of the building.(Hint: use sine rule)

(2) The angle of elevation of the top of a column measured from point A, is 20°. The angle of elevation of the top of the statue is 25°. Find the height of the statue when the measurements are taken 50 m from its base(Hint: use sine rule)

(3) Find the values of the unknown sides and angles





ASSIGNMENT

(1) A fishing boat leaves a harbour (H) and travels due East for 40 miles to a marker buoy (B). At B the boat turns left onto a bearing of 035° and sails to a lighthouse (L) 24 miles away. It then returns to harbour.

I. Make a sketch of the journey

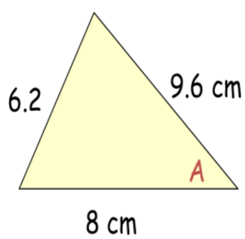
II. Find the total distance travelled by the boat. (nearest mile)

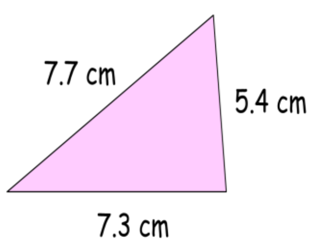
(2) A fishing boat leaves a harbour (H) and travels due East for 40 miles to a marker buoy (B). At B the boat turns left and sails for 24 miles to a lighthouse (L). It then returns to harbour, a distance of 57 miles.

I. Make a sketch of the journey.

II. Find the bearing of the lighthouse from the harbour. (nearest degree)

(3) Find the unknown sides and angles





KEYWORDS: COSINE RULE, SINE RULE, ANGLES, SIDES, DEGREE, DIRECTION ,TRIGONOMETRY, ETC

WEEK 9

Subject: Mathematic

Class: SS 2

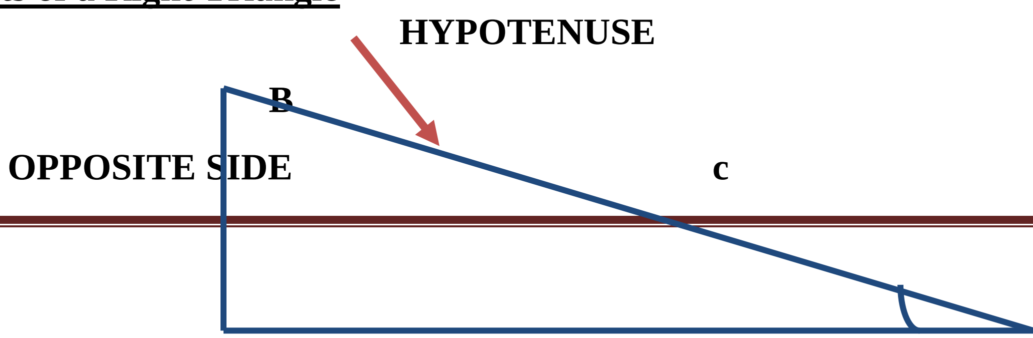
TOPIC: BEARING

CONTENT:

- Revision of;
  - Trigonometric ratios;
  - Angles of elevation and depression.
- Notation for bearings: (i) Cardinal notations $N30^0E$ (ii) $S45^0W$
- 3-digits notation. E.g. 075^0, 350^0.
- Practical problems on bearing.

REVISION OF TRIGONOMETRIC RATIOS

Parts of a Right Triangle



A

C **ADJACENT SIDE (b)**

A

The **hypotenuse** will always be the longest side, and opposite from the right angle.

(**Imagine that you are at Angle A looking into the triangle.**)

The **adjacent side** is the side next to Angle A. It's that sides that has angle 90 and unknown angle on it.(The **opposite side** is the side that is on the opposite side of the triangle from Angle A.)

Opposite side is the side facing the unknown angle

The ratios are still the same as before!!

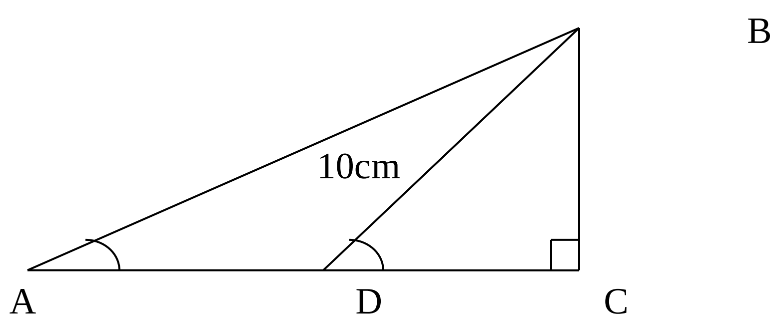
Sine A = $\frac{opposite}{hypotenuse} = \frac{a}{c}$ Cosec A = $\frac{1}{\sin A} = \frac{Hypotenuse}{Opposite} = \frac{c}{a}$

Cos A = $\frac{Adjacent}{Hypotenuse} = \frac{b}{c}$ Sec A = $\frac{1}{\cos A} = \frac{Hypotenuse}{Adjacent} = \frac{c}{b}$

Tan A = $\frac{\boldsymbol{opposite}}{\boldsymbol{adjacent}} = \frac{a}{b}$ Cot A = $\frac{1}{\tan A} = \frac{opposite}{Adjacent} = \frac{b}{a}$

Examples

(1) In the diagram below, calculate /BD/ and /AD/



Solution

BC/AB= sin 35 implies BC = Ab sin 35 = 10 sin 35

From sin tables, sin 35 = 0.5736

Hence BC = 10×0.5736

BC/BD = sin 60 implies BD = BC/sin 60 = 5.736/0.866= 6.62 cm

BC/CD = tan 60= 1.732

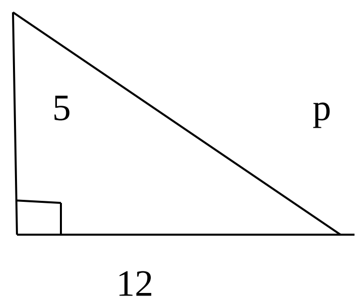
CD= BC/1.732 = 5.736/1.732 = 3.31cm

AC/AB = cos 35 implies AC=AB cos 35 = 10(0.8192) = 8.19cm

AD=AC-CD= 8.19 -3.31 = 4.88cm

(2) Given that tan x = 5/12, what is the value of Sin x + Cos 2x?

Solution



$P^2 = 12^2 + 5^2$

$P^2 = 144 + 25$

$P^2 = 169$

P = $\sqrt{169}$

P = 13

Sin x + cos 2x

5/13 + 2(12/13)

5/13 + 24/13 = $\mathbf{\frac{5+24}{13} = \frac{29}{13} = 2\frac{3}{13}}$

Class Activity:

If Cos 60^0 = ½, which of the following angles has a cosine of -½?

A. 30^0 B. 120^0 C. 150^0 D. 210^0 E. 300^0 (SSCE 1988)

2. Cos x is negative. Which of the following is true of x?

A. $0^0 < x < 90^0$ B. $90^0 < x < 180^0$ C. $180^0 < x < 270^0$ D. $270^0 < x < 360^0$ E. $-90^0 < x < 90^0$
(SSCE1988)

3. If Sin θ = 3/5, find tan θ from $\theta < \theta < 90^0$

A. ⅜ B. ⅝ C. ¾ D. ½ E. $^4/_5$
(SSCE 1988)

4. If Sin θ = ½ and Cos θ = $-\sqrt{3}/2$ what is the value of θ?

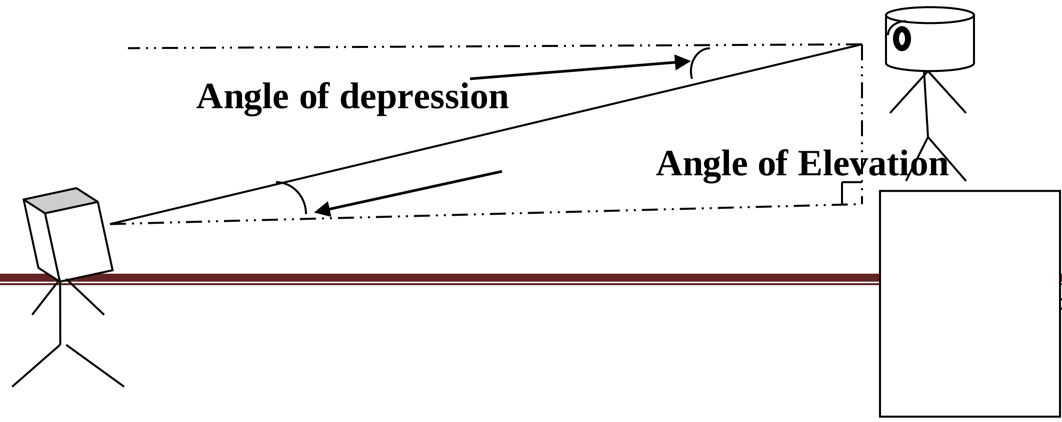
A. 30^0 B. 60^0 C. 90^0 D. 120^0 E. 150^0 (SSCE 1989)

5. Given that Sin θ = -0.9063 where $0 \le \theta \le 270^0$ find θ

A. 65^0 B. 115^0 C. 145^0 D. 245^0 E. 265^0 (SSCE 1991)

ELEVATION AND DEPRESSION

Right triangle trigonometry is often used to the height of a tall object indirectly. To solve a problem of this type, measure the angle from the horizontal to your line of sight when you look at the top or bottom of the object

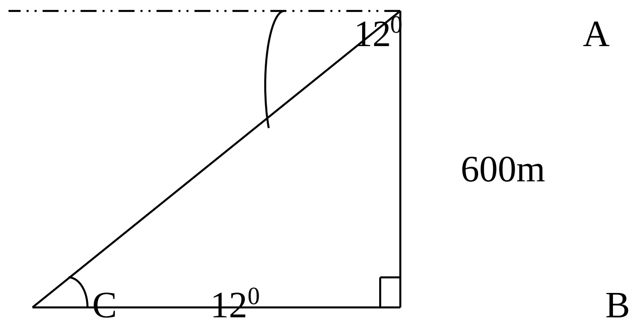


If you look up, you measure the angle of elevation. If you look down, you measure the angle of depression

Examples:

(1) An aircraft is circling an airport at a height of 600m. The navigator finds that the angle of depression of the control tower of the airport is 12^0. What is the distance between the aircraft and the control tower?

Solution



In theb above diagram, the point A represents the position of the aircraft while the point C represents the position of the control tower

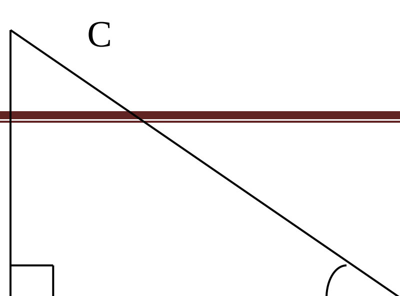
From triangle ABC

$$\frac{AB}{AC} = \sin 12^0$$

/AC/ = $\frac{AB}{SIN\ 12} = \frac{600}{0.2079} = 2886m$

The aircraft is approximately 2886m from the control tower

(2) A survey stands 60m a vertical tree. The angle of elevation of top of the tree from a point 2m above the ground is 25^0. Calculate the height of the tree to 2 sig fig



A 60m 25^0 B

E 2m D

If BC= a and AC = b

a/b = tan 25^0 a = 60 tan 25

$$60 \times 0.4663 = 27.978m$$

The height of the tree = 2m + 27.978m

29.789m≈ $30.0m\ (2.s.f)$

Class Activity

(1) A cell phone tower is supported by two guy-wires, attached on opposite sides of the tower. One guy-wire is attached to the top of the base of the tower at point A and the other is attached to the base at point E, at a height of 70 m above the ground

(2) From the top of a building 10 m high, the angle of depression of a stone lying on the horizontal ground is 69^0. Calculate, correct to 1 decimal place, the distance of the stone from the foot of the building

A. 3.8m B. 6.0m C. 9.3m D. 26.1m

(SSCE 1990)

(3) From the top of a building 10 m high, the angle of depression of a stone lying on the horizontal ground is 69^0. Calculate, correct to 1 decimal place, the distance of the stone from the foot of the building

A. 3.8m B. 6.0m C. 9.3m D. 26.1m

(SSCE 1990)

(4) From the top of a cliff, the angle of depression of a boat on the sea is 60^0. If the top of the cliff is 25m above the sea level, calculate the horizontal distance from the bottom of the cliff to the boat.

A. 50√30m B. 25√25m C. $\frac{25\sqrt{3}}{3}m$ D. $\frac{25}{3}m$ E. $\frac{\sqrt{3}}{25}m$

(SSCE 1991)

(5) The angle of elevation of the top of a tree 39m away from a point on the ground is 30^0. Find the height of the tree.

A. 39√3m B. 13√3m C. $\frac{13}{\sqrt{3}}m$ D. $\frac{13}{3\sqrt{3}}m$ E. $\frac{\sqrt{3}}{13}m$

(SSCE 1991)

BEARINGS

This is a system of measuring the location of points on the earth's surface in relation to another using the four cardinal points of the earth. i.e. the North, South, East and West.

There are two major ways of measuring the bearings of points. They are

(i) The three-digit bearing (True bearing).

(ii) The points of compass bearing.

The Three-digit bearing or True bearing

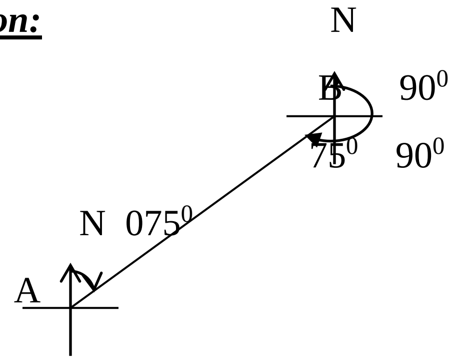
This type of bearing is normally expressed using three digits as the name implies e.g. 003^0, 007^0, 025^0, 067^0, 125^0, 218^0 e.t.c.

The bearing is normally read from the North Pole in a clockwise direction until the desired point is reached.

Example 1:

The bearing of B from A is 075^0, what is the bearing of A from B?

Solution:



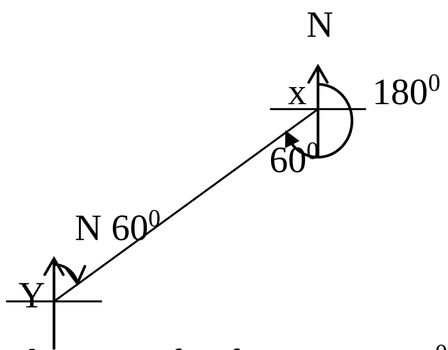
The bearing of A from B is

$90^0 + 90^0 + 75^0 = 255^0$

(This is read from the North Pole at point B)

Example 2:
The bearing of Y from X is 240^0, what is the bearing of X from Y?

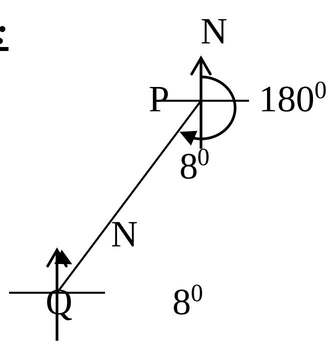
Solution:



The bearing of X from Y is 060^0
(This is read from the North Pole at point Y)
Example 3:

The bearing of Q from P is 188^0, what is the bearing of P from Q?

Solution:



The bearing of P from Q is 008^0

(This is read from the North Pole at Q)

The Points of Compass Bearing

This type of bearing is usually read either from the North or South to any of the directions specified, East or West. It is usually started with the letters N or S denoting North or South and it is normally ended with the letters E or W denoting East or West i.e. $N\theta^0W$, $N\theta^0E$, $S\theta^0W$, $S\theta^0E$ where θ lie between 0 and 90^0 ($0^0 < \theta < 90^0$).

The first letter N or S as the case may be, signifies the point we are reading from and the last letters E or W signifies the direction we are reading to.

e.g.

$N65^0E \Rightarrow$ We are reading from the North 65^0 towards the East.

$S30^0W \Rightarrow$ We are reading from the South 30^0 towards the West.

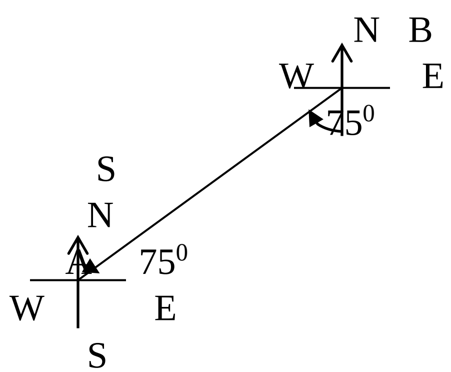
$S17^0E \Rightarrow$ We are reading from the South 17^0 towards the East.

We shall reframe the three examples under the three-digit bearing using point of compass bearing specifications.

Examples

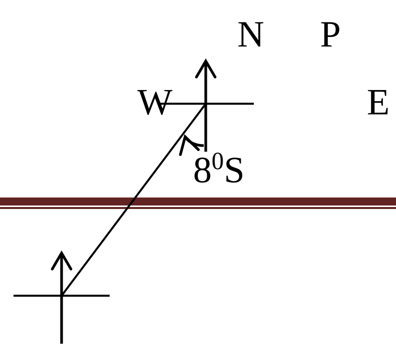
(1) *The bearing of B from A is $N75^0E$, what is the bearing of A from B?*

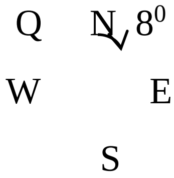
Solution:



The bearing of A from B is $S75^0W$.

(2) *The bearing of Q from P is $S8^0W$, what is the bearing of P from Q?*

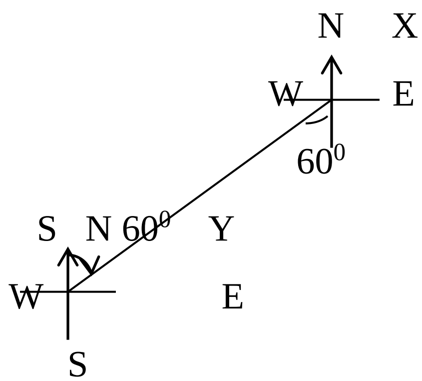




The bearing of P from Q is $N8^0E$.

(3) *The bearing of Y from X is $S60^0W$, what is the bearing of X from Y?*

Solution:

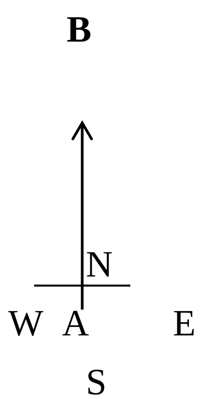


The bearing of X from Y is $N60^0E$

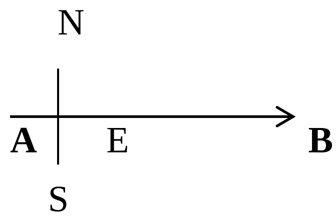
NOTE THAT:

The bearing of a place is said to be due North if it is directly to the North; due South if it is directly down South; due East if it is directly towards the East and due West if it is directly towards the West.

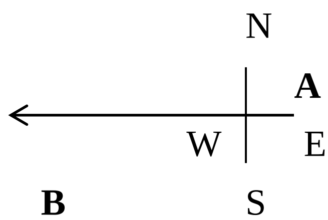
(1) B is due North of A



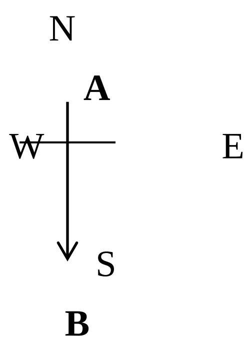
(2) B is due East of A



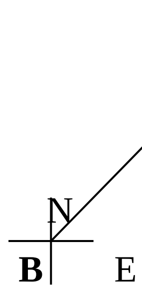
(3) B is due West of A



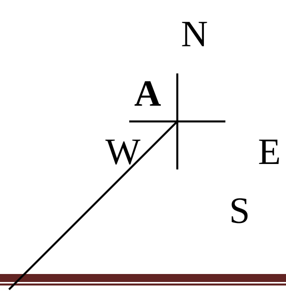
(4) B is due South of A



(5) C is North East of B



(6) B is directly South West of A



B

Class Activity

(1) What's the bearing of Q from P to the nearest whole degree?

A. 16^0 B. 17^0 C. 73^0 D. 106^0 E. 164^0

(SSCE 1988)

(2) Points X and Y are respectively 20km north and 9km east of a point O. What is the bearing of Y from X correct to the nearest degree?

A. 024^0 B. 114^0 C. 154^0 D. 204^0 E. 336^0

(SSCE 1989)

(3) Town P is on a bearing 315^0 from town Q while town R is south of town P and west of town Q. if town R is 60km away from Q, how far is R from P?

A. 30km B. 42km C. 45km D. 60km E. 120km

(SSCE 1992)

(4) Points X and Y are respectively 12m North and East of point Z. Calculate /XY/.

A. 7m B. 12m C. 13m D. 17m E. 18m

(SSCE 1992)

(5) A plane flies 90km on a bearing 030^0 and then flies 150km due east. How far east of the starting point is the plane?

A. 120km B. 165km C. 195km D. $(150 + 45\sqrt{3})$ km E. 240km

(SSCE 1993)

PRACTICAL PROBLEMS ON BEARING.

_THREE POINTS MOVEMENT WITH DISTANCE GIVEN

Examples:

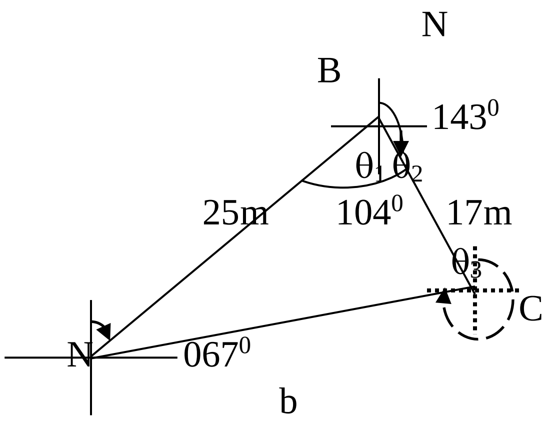
(1) A dragonfly flew from point A to point B, 25m away on a bearing of 067^0. It then flew from point B to point C 17m away on a bearing of 143^0.

(a) How far is the dragonfly from the starting point to the nearest metre?

(b) What is the bearing of the starting point from the dragonfly?

Solution:

We shall represent the movement of the dragonfly with a diagram.



HOW TO FIND THE ANGLE B

$\theta_1 = 67^0$ *[alternate angles]*

$\theta_2 + 143^0 = 180^0$ *[sum of <s on a straight line]* $\theta_2 = 180 - 143$

$\theta_2 = 37^0$

$B = \theta_1 + \theta_2$

$= 67^0 + 37^0$

$= 104^0$

(a)

Using cosine rule

$b^2 = a^2 + c^2 - 2ac \cos B$

$= 17^2 + 25^2 - 2x17x25 \cos 104^0$

$= 289 + 625 - 850 (-\cos 76^0)$

$= 914 + 850 \text{ x } 0.2419$

$= 914 + 205.6$

$= 1119.6$

$b = \sqrt{1119.6}$

$b = 33.46$

$\therefore$ $b = 33m$ *(nearest metre)*

$\therefore$ The dragonfly is approximately 33m from the starting point.

(b) ***Using sine rule***

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{33.46}{\sin 104} = \frac{25}{\sin C}$$

$$33.46 \sin C = 25 \sin 104^0$$

$$\sin C = \frac{25 \sin 104}{33.46}$$

$$\sin C = \frac{25 \sin 76^0}{33.46}$$

$$\sin C = 0.7249$$

$$C = \sin^{-1} 0.7249$$

$$C = 46.47^0$$

The bearing of the starting point from the dragonfly is

$= 360 - (\theta_3 + C)$

$= 360 - (37^0 + 46.47$

$= 360^0 - 83.47^0$

$= 276.5^0$

$\approx 277^0$

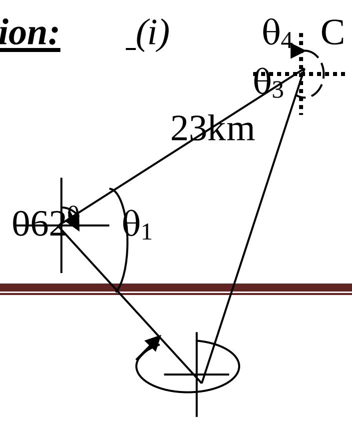
But $\theta_3 = \theta_2 = 37^0$ *(alternate <s)*

(2) A ship in an open sea sailed from a point A to another point B, 15km away on a bearing of 310^0. It then sailed from the point B to another point C, 23km away on a bearing of 062^0.

(i) *How far is the ship from the starting point?*

(ii) *What is the bearing of the starting point from the ship?*

Solution: *(i)*



B θ_2 68^0 b

15km 50^0N

θ_5

A 310^0

To find the angle B,

$\theta_1 + 62^0 = 90^0$ *[complimentary angles]*

$\theta_1 = 90^0 - 62^0$

$\theta_1 = 28^0$

$\theta_5 + 50^0 = 90^0$ *[complimentary angles]*

$\theta_5 = 90^0 - 50^0$

$\theta_5 = 40^0$

$\theta_2 = \theta_5 = 40^0$ *(alternate angles)*

$B = \theta_1 + \theta_2$

$= 28^0 + 40^0$

$= 68^0$

Using cosine Rule

$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$= 23^2 + 15^2 - 2 x 23 x 15 \cos 68^0$$
$$= 529 + 225 - 690 \text{ x } 0.3746$$
$$= 754 - 258.474$$
$$b^2 = 495.526$$
$$b = \sqrt{495.526}$$
$$b = 22.3\text{km}$$

(ii) ***Using sine Rule***

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{22.3}{\sin 68^0} = \frac{15}{\sin C}$$
$$22.3 \sin C = 15 \sin 68^0$$
$$\sin C = \frac{15 \sin 68^0}{22.3}$$
$$\sin C = 0.6237$$
$$C = \sin^{-1} 0.6237$$

$\hat{C} = 38.6^0$

The bearing of the starting point from the ship is obtained from $360^0 – (\theta_3 + \theta_4 + C)$.

$\theta_1 = \theta_3$ *[alternate <s]*
since $\theta_1 = 28^0$
$\therefore \theta_3 = 28^0$

$= 360^0 – (28^0 + 90^0 + 38.6^0)$

$= 360^0 – 156.6^0$

$= 203.4^0$

$\therefore$ The bearing of the starting point from the ship is $\approx 203^0$

THREE POINTS MOVEMENT WITH SPEEED AND TIME GIVEN

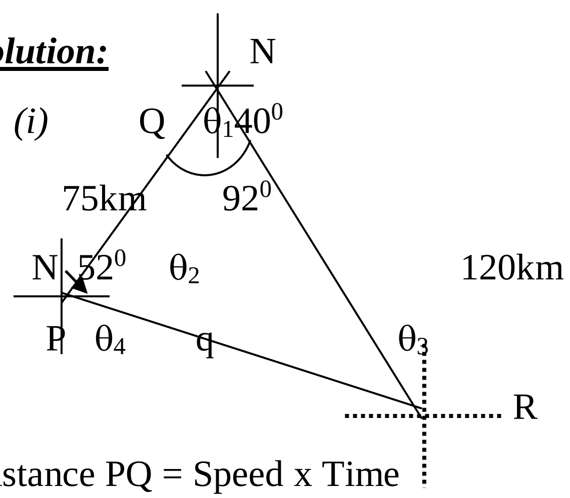
(Under this case, we shall be considering the bearing of ONE OBJECT moving to three different points with no distance given but the SPEED AND TIME OF THE VEHICLE GIVEN)

(3)A boat sails at 50km/h on a bearing of N52^{0}E for 1½ hours and then sails at 60km/h on a bearing of S40^{0}E for 2 hours.

(i) *How far is the boat from the starting point?*
(ii) *What is the bearing of the starting point from the boat?*
(iii) *What is the bearing of the boat from the starting point?*

Solution:

(i)



Distance PQ = Speed x Time

= (50 x 1½) km

$= (50 \times {}^3/_2)$ km

$= 75$km

Distance QR $= (60 \times 2)$ km

$= 120$km

To find angle Q,

$\theta_1 = 52^0$ *(alternate angles)*

$Q = \theta_1 + 40^0$

$Q = 52^0 + 40^0$

$= 92^0$

Using cosine rule

$q^2 = p^2 + r^2 - 2pr \cos Q$

$= 120^2 + 75^2 - 2x120x75 \cos 92^0$

$= 14400 + 5625 - 18000 [-\cos 180^0 - 92^0]$

$= 20025 - 18000 (-\cos 88)$

$= 20025 + 18000 \times 0.0349$

$= 20025 + 628.2$

$= 20653.2$

$q = \sqrt{20653.2}$

$\therefore$ q = 143.7km.

(ii) ***Using sine rule***

$$\frac{q}{\sin Q} = \frac{r}{\sin R}$$

$$\frac{143.7}{\sin 92} = \frac{75}{\sin R}$$

$$143.7 \sin R = 75 \sin 92^0$$

$$\sin R = \frac{75 \sin 92^0}{143.7}$$

$$\sin R = \frac{75 \sin 88^0}{143.7}$$

$$\sin R = 0.5216$$

$$R = \sin^{-1} 0.5216$$

$$R = 31.4^0$$

But $\theta_3 = 40^0$ *[alternate <s]*

The bearing of the starting point from the boat is $= N(R + \theta_3)^0 W$

$= N(31.4^0 + 40^0)W$

$= N\ 71.4^0\ W$

$\approx$ N 71^0 W

(iii)

But $\theta_2 + 92^0 + R = 180^0$ *[<s in a Δ]*

$\theta_2 + 92^0 + 31.4^0 = 180^0$

$\theta_2 = 180^0 - 123.4^0$

$\therefore\ \theta_2 = 56.6^0$

$\theta_4 = 180^0 - (52^0 + \theta_2)$

$\theta_4 = 180^0 - (52^0 + 56.6^0)$

$\theta_4 = 180^0 - 108.6^0$

$\theta_4 = 71.4^0$

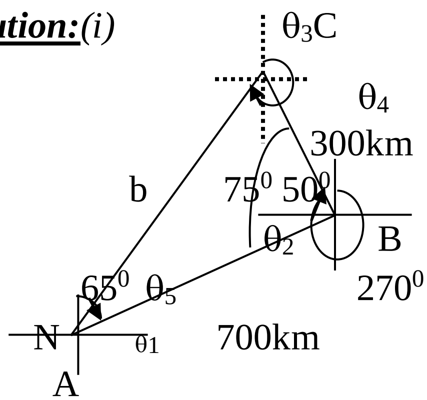
$\therefore\ \theta_4 \approx 71^0$

The bearing of the boat from the starting point is read from the point P as S71^0E.

(4)An aircraft flew from an airport A to another airport B, on a bearing of 065^0 at an average speed of 300km/h for $2^1/_3$ hrs, It then flew from the airport B to another airport C, on a bearing of 320^0 at an average speed of 450km/h for 40min.

(i) *How far is the aircraft from the starting point?*
(ii) *What is the bearing of the starting point from the aircraft?*
(iii) *What is the bearing of the aircraft from the starting point?*

Solution:*(i)*



Distance = Speed x Time

Distance AB = $(300 \times 2^1/_3)$ km

$= (300 \times {}^7/_3)$ km

$= (100 \times 7)$ km

= 700km.

Distance BC = $(450 \times \frac{40}{60})$ km

$= (450 \times {}^2/_3)$ km

$= (150 \times 2)$ km

$= 300$km.

since 60min = 1hr

∴ to get time in hrs

$= \frac{40}{60}$

$= \frac{2}{3}$ hrs

To find angle B,

$\theta_1 + 65^0 = 90^0$ *[complimentary angles]*

$\theta_1 = 90^0 - 65^0$

$\theta_1 = 25^0$

$\theta_1 = \theta_2$ *[alternate angles]*

$\therefore \theta_2 = 25^0$

$B = \theta_2 + 50^0$

$\therefore B = 25^0 + 50^0$

$\therefore B = 75^0$

U

5^0

2588

$= 580000 - 108696$

$= 471304$

$b = \sqrt{471304}$

$\therefore b = 686.5$km

∴ The aircraft is 686.5km from the starting point.

(ii) ***Using sine rule***

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{686.5}{\sin 75} = \frac{700}{\sin C}$$

$$686.5 \sin C = 700 \sin 75$$

$$\sin C = \frac{700 \sin 75}{686.5}$$

$\sin C = 0.9849$

$C = \sin^{-1} 0.9849$

$\therefore\ C = 80^0$

The bearing of the starting point from the aircraft is read from point C.

i.e. $= \theta_3 + \theta_4 + C$

$\theta_3 = 90^0$

$= 90^0 + 50^0 + 80^0$

$= 220^0$

(iii) $A + B + C = 180^0$ *[sum of Ls in a Δ]*

$\theta_5 + 75^0 + 80^0 = 180^0$

$\theta_5 = 180^0 - 155^0$

$\theta_5 = 25^0$

The bearing of the aircraft from the starting point is $= 90^0 - (\theta_5 + \theta_1)$

$= 90^0 - (25^0 + 25^0)$

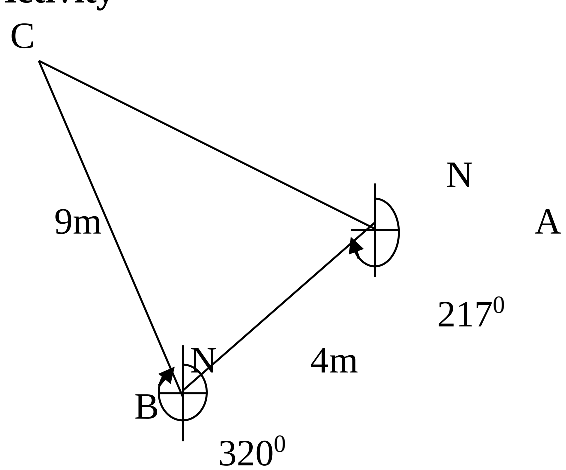
$= 90^0 - 50^0$

$= 040^0$

(read from the point A)

Class Activity

(1)



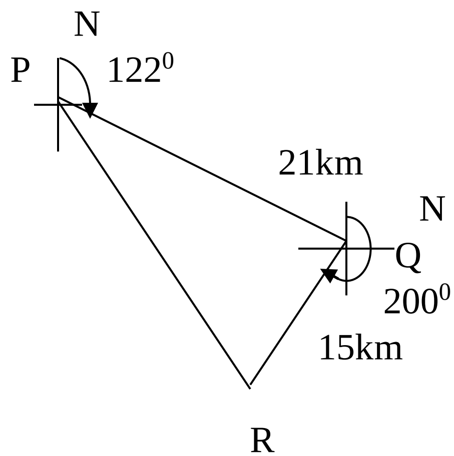
From the diagram above, find the following

(i) ABC

(ii) /AC/

(iii)The bearing of A from C.

(2)



From the diagram above, find the following

(i) PQR

(ii) /PR/

(iii) The bearing of P from R.

(3) A town B is 12km from another town A on a bearing of 047^0 and another town C is 8km from town B on a bearing of 124^0.

(i) How far is town A from town C?

(ii) What is the bearing of town A from C?

(4) A ship sailing in an open sea moves from a point A on a bearing of 055^0 at a speed of 50km/h for 1½ hour to another point B. It then moves on a bearing of 143^0 at a speed of 40km/h for 2 hours to another point C.

(i) How far is the ship from the starting point?

(ii) What is the bearing of the ship from the starting point?

TWO DIRECTIONS WITH DISTANCE GIVEN

(Under this case, we shall be considering the bearing of TWO OBJECTS at different locations read from the same point or TWO OBJECT moving from the same point in two different directions AND the DISTANCES covered by the two objects GIVEN)

Examples:

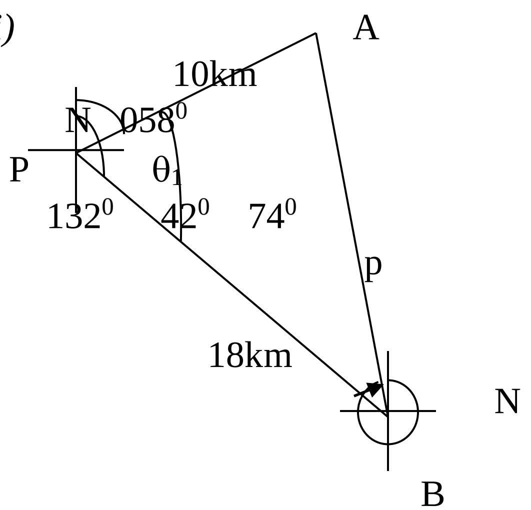
(1) Two missiles A and B shot from the same point, Missile A was shot on a bearing of 058^0 and at a distance of 10km and missile B was shot on a bearing of 132^0 at a distance of 18km.

(i) How far apart are the missiles?

(ii) What is the bearing of missile A from missile B? **(WAEC)**

<u>Solution:</u>

(i)



To get angle P,

$\theta_1 + 58^0 = 90^0$ *[complementary angles]*

$\theta_1 = 90^0 - 58^0$

$\theta_1 = 32^0$

$P = \theta_1 + 42^0$

$= 32^0 + 42^0$

$= 74^0$

OR $P = 132^0 - 58^0$

$= 74^0$

$18^2 + 10^2 - 2 \times 18 \times 10 \cos 74$

$= 324 + 100 - 360 \times 0.2756$

$= 424 - 99.216$

$= 324.784$

$P = \sqrt{324.784}$

$P = 18.0$km

The two missiles are 18km apart.

(ii) ***Using sine rule***

To find angle B,

$$\frac{b}{\sin B} = \frac{p}{\sin P}$$

$$\frac{10}{\sin B} = \frac{18}{\sin 74^0}$$

$$10 \sin 74 = 18 \sin B$$

$$\sin B = \frac{10 \sin 74^0}{18}$$

$$\sin B = 0.5340$$

$$B = \sin^{-1} 0.5340$$

$$B = 32.3^0$$

To get the bearing of A from B

$= 270^0 + \theta_2 + B$ *[θ_2 = 42 (alternate <s)]*

$= 270^0 + 42^0 + 32.3^0$

$= 344.3^0$

$\approx 344^0$

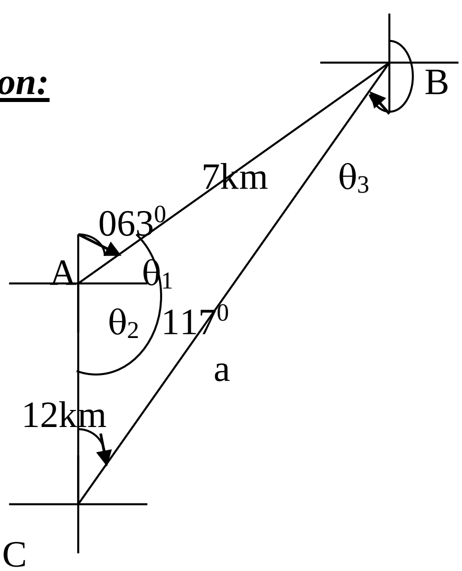
(2) Two points B and C are observed from a watch tower at point A. If B is 7km on a bearing of 063^0 and the other point C is 12km due south of A.

(i) How far apart are the two points?

(ii) What is the bearing of B from C?

(iv) What is the bearing of C from B?

Solution:



$\theta_1 + 63^0 = 90^0$ *[complementary angles]*

$\theta_1 = 90^0 - 63^0$

$\therefore \theta_1 = 27^0$

$\theta_2 = 90^0$

$A = \theta_1 + \theta_2$

$= 27^0 + 90^0$

(i) **Using cosine rule**

$a^2 = b^2 + c^2 - 2bc \cos A$

$= 12^2 + 7^2 - 2x12x7 \cos 117$

$= 144 + 49 - 168 [-\cos 180 - 117]$

$= 193 - 168 [-\cos 63]$

$= 193 + 168 \text{ x } 0.4540$

$= 193 + 76.27$

$a^2 = 269.27$

$a = \sqrt{269.27}$

$a = 16.4\text{km}$

The two points are 16.4km apart.

(ii) ***Using sine rule***

to find angle C

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{16.4}{\sin 117} = \frac{7}{\sin C}$$

$16.4 \sin C = 7 \sin 117$

$\sin C = \frac{7 \sin 63}{16.4}$

$\sin C = 0.3803$

$C = \sin^{-1} 0.3803$

$\therefore C = 22.4^0$

$\approx 22^0$

$\therefore$ The bearing B from C is 022^0

(iii) The bearing of C from B is

$=180 + \theta$

$\theta_3 = C = 22^0$

$= 180 + 22^0$
$= 202^0$

Class Activity

(1) Two men P and Q set off from a base camp R prospecting for oil. P move 20km on a bearing 205^0 and Q moves 15km on a bearing of 060^0.Calculate the
(a) Distance of Q from P
(b) Bearing of Q from P
(Give answers in each case correct to the nearest whole number).
SSCE, June 1996, No 12 (WAEC).

(2) Two boats A and B left a port C at the same time along different routes. B traveled a distance of 9km on a bearing of 135^0 and A traveled a distance of 5km on a bearing of 062^0.
(a) How far apart are the two ships?
(b) What is the bearing of ship B from A?

PRACTICE EXERCISE

(1) Two flying boats A and B left port P at the same time, A sailed on a bearing of 115^0 at an average speed of 8km/h and B sailed on a bearing of 241^0 at an average speed of 6km/h.
(a) How far apart are the flying boats after 1½ hour?
(b) What is the bearing of boat A from boat B?

(2) A man observed two boats P and Q at a sea sailing towards him at the point R. He observes P at a bearing of N43^0W moving at an average speed of 20km/h and Q is on a bearing of S52^0W moving at an average speed of 30km/h. If P took 2 hours to get to R and Q took 2½ hours to get to R.
(a) How far apart were the two boats when the man first noticed them?
(b) What was the bearing of P from Q?

(3)An aeroplane flew from city G to city H on a bearing of 150^0. The distance between G and H is 300km. It then flew a distance of 450km to city J on a bearing of 060^0. Calculate and correct to a reasonable degree of accuracy.
(a) The distance from G to J,
(b) How far north of H is J,
(c) How far west of H is G.

SSCE, Nov 1994, No 4 (WAEC).

(4) *A girl moves from a point P on a bearing of 060^0 to a point Q, 40m away. She then moves from the point Q, on a bearing of 120^0 to a point R. The bearing of P from R is 255^0. Calculate, correct to three significant figures the distance between P and R.*

SSCE, Nov 1993, No 2b (WAEC).

ASSIGNMENT

(1) A man travels from a village X on a bearing of 060^0 to a village Y which is 20km away. From Y, he travels to a village Z, on a bearing of 195^0. if Z is directly east of X, calculate, correct to three significant figures, the distance of (i) Y from Z (ii) Z from X.

SSCE, June 1995, No 10a (WAEC).

(2) A surveyor standing at a point X sights a pole Y due east of him and a tower Z of a building on a bearing of 046^0. After walking to a point W, a distance of 180m in the south-east direction, he observes the bearing of Z and Y to be 337^0 and 050^0 respectively.

(a) Calculate, correct to the nearest metre.

(i) /XY/

(ii) /ZW/

(b) If N is on XY such that XZ = ZN, find the bearing of Z from N.

SSCE, June 1998, No 10 (WAEC).

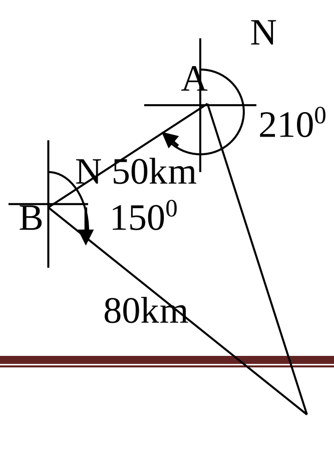
(3) An aeroplane flies from a town X on a bearing of $N45^0E$ to another town Y, a distance of 200km. It then changes course and flies to another town Z on a bearing of $S60^0E$. If Z is directly east of X, calculate correct to 3 significant figures.

(a) The distance from X to Z.

(b) The distance from Y to XZ.

(WAEC).

(3)



C

(a) In the diagram, A, B and C represent three locations. The bearing of B from A is 210^0 and the bearing of C from B is 150^0. Given that /BA/ = 50km and /BC/ = 80km, calculate:

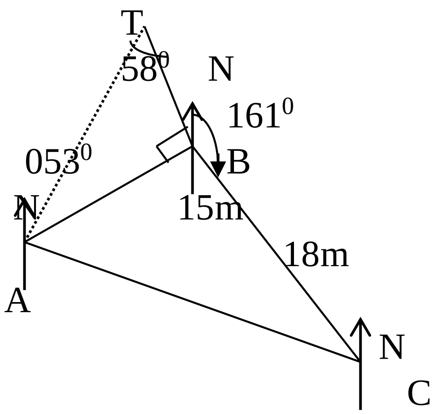
(i) The distance between A and C correct to the nearest kilometer

(ii) The bearing of A from C to the nearest degree.

(b) How far east of B is C?

WASSCE, Nov 1999. No 9 (WAEC).

(5)



In the diagram, three points A, B and C is on the same horizontal ground. B is 15m from A, on a bearing of 053^0. C is 18m from B on a bearing of 161^0. A vertical pole with top T is erected at B such that angle ATB = 58^0. Calculate, correct to three significant figures,

(a) The length of AC;

(b) The bearing of C from A;

(c) The height of the pole BT.

WASSCE, June 2001, N0 12. (WAEC)

(3) Two planes left Lagos international airport at the same time. The first traveled on a bearing of 048^0 at an average speed of 500km/h for $1^2/_5$ hour before landing.

The second traveled on a bearing of 332^0 at an average speed of 400km/h for ¾ hour before landing at its destination.

(a) How far apart are their destinations?

(b) What is the bearing of the first from the second?

KEYWORDS: BEARING,TRIGONOMETRY, SINE,COSINE,TANGENT,SPEED,VELOCITY,DISTANCE ETC