FIRST TERM: E-LEARNING NOTES

SS 2 MATHEMATICS

SCHEME FIRST TERM

WEEK	TOPIC	CONTENT
1	LOGARITHM 1	(a) Revision of logarithm of numbers greater than 1. (b) Comparison of characteristics of logarithms and standard form of numbers.
2	LOGARITHM 2	(a) Logarithm of numbers less than one, involving: Multiplication, Division, Powers and roots. (b) Solution of simple logarithmic equations.
3	SEQUENCE AND SERIES 1	(a) Meaning and types of sequence. (b) Example of an A. P. (c) Calculation of: (i) first term (ii) common difference (iii) nth term (iv) Arithmetic mean (v) sum of an A. P. (d) Practical problems involving real life situations.
4	SEQUENCE AND SERIES 2	(a) Examples of geometric progression. (b) Calculation of; (i) First term (ii) Common ratio (iii) nth term, (iv) Geometric menu (v) sum of terms of geometric progression. (vi) Sum to infinity. (c) Practical problems involving real life situation.
5	QUADRATIC EQUATION	(a) Revision of factorization of perfect squares. (b) Making quadratic expression perfect squares by adding a constant K. (c) Solution of quadratic equation by the method of completing the square. (d) Deducing the quadratic formula from completing the square. (e) Construction of quadratic equation from sum and product of roots. (f) Word problems leading to quadratic equations.
6	SIMULTANEOUS LINEAR AND QUADRATIC EQUATIONS	(a) Simultaneous linear equations (Revision). (b) Solution to linear and quadratic equations. (c) Graphical solution of linear and quadratic equations. (d) Word problems leading to simultaneous equations (capital market). (e) Gradient of curve.
7	MID-TERM BREAK	
8	*COORDINATES GEOMETRY OF STRAIGHT LINES	(a) Distance between two points. (b) Midpoint of line joining two points. (c) Gradients and intercept of a straight line. (d) Determination of equation of a straight line. (e) Angle between two intersecting straight lines. (f) Application of linear graphs to real life situation.
9	APPROXIMATIONS	(a) Revision of approximation. (b) Accuracy of results using logarithm table and calculators. (c) Percentage error. (d) Application of approximation to everyday life.
10	REVISION	
11	EXAMINATION	

WEEK 1:

DATE……………………….

Subject: Mathematics

Class: SS 2
TOPIC: logarithm 1
Content:

- Comparison of characteristics of logarithms and standard form of numbers.
- Revision of logarithm numbers greater than 1.

Comparison of characteristics of logarithms and standard form of numbers

There is a relationship between the standard form and the logarithm of a number.

For instance, (a) 189.7 = 1.897×10^2

This shows that the logarithm of a number is the power to which the base 10 is raised. Hence, Logarithm of 189.7 = 2.2781, where 189.7 = $10^{2.2781}$

(b) 850.9 = 8.509$\times 10^2$ (standard form)

Log850.9 = 2.9299

The integer (characteristics) is the same with the power 10

CLASS ACTIVITY

Show how the characteristics following Logarithms are related to standard form

1. 82000
2. 68.9
3. 6895
4. 605.8

Revision of logarithm numbers greater than 1

Logarithm of numbers is the power to which 10 is raised to give that number. Logarithms used in calculations are normally expressed in base 10.

- Rules for the use of Logarithms
  i. Multiplication: find the logarithms of the numbers and add them together

ii. Division: find the logarithm of each number. Then subtract the logarithm of the denominator from that of the numerator

iii. Powers: find the logarithm of the number and then multiply it by the power or the index

iv. Roots: find the logarithm of the number and then divide it by the root

Example1: Evaluate using logarithm tables

$$19.28 \times 2.987 \times 195.8$$

Numbers	Log
19.28	1.2851
2.987	0.4752
195.8	2.2918
11270	**4.0521**

$$Antilog\ of\ 4.0521\ = 11300\ to\ 3s.f$$

Example 2: Evaluate using logarithm tables

$$\sqrt{\frac{173.8\times(14.7)^2}{(2.61)^3}}$$

Numbers	Log			
173.8	2.2400	2.2400		
$(14.7)^2$	1.1673× 2	2.3346		
	Numerator	4.5746	4.5746	
$(2.61)^3$	0.4166× 3	1.2498	1.2498	
$\frac{173.8\times(14.7)^2}{(2.61)^3}$			3.3248	
$\sqrt{\frac{173.8\times(14.7)^2}{(2.61)^3}}$				3.3248÷ 2
45.96				**1.6624**

$\therefore$ $antilog\ of\ 1.6624 = 45.96\ (4\ s.f)$

CLASS ACTIVITY

Use logarithm tables to evaluate correct to 4 s.f.

1. $786.1 \times 89.5 \times 63.7$

2. $\frac{107.8 \times 38.97}{81.65}$

PRACTICE EXERCISE

Use log tables to find the value of

1. $\frac{\sqrt{17.45} \times (35.2)^2}{(3.15)^4 \times 8.15}$
2. $\frac{(27.1)^2 \times 327}{\sqrt{27500000}}$
3. $\frac{95.3 \times \sqrt[3]{18.4}}{(1.29)^5 \times 2.03}$

(SSCE 1991)

4. $\frac{298.6 \times 10.52}{2.56 \times 32.8}$
5. $\sqrt[3]{\frac{321000 \times 40}{175 \times 6000}}$

ASSIGNMENT

Use log tables to find the value of

1. $\frac{6.705 \times 3.68^3}{\sqrt[4]{35.81}}$
2. $\frac{23.67 \times 73.59}{(2.5134)^5 \times 2.03}$
3. $(\mathbf{987.3})^{\frac{1}{3}}$
4. $\sqrt[3]{\frac{\mathbf{875.4}}{\mathbf{4.234^3}}}$
5. $\sqrt{\mathbf{23.56 \times 66.45}}$

KEYWORDS: base, logarithm, integer, antilogarithm, mantissa, characteristics etc.

WEEK 2:
DATE……………………….
Subject: Mathematics
Class: SS 2
TOPIC: logarithm 2
Content:

- Logarithm of numbers less than one, involving: Multiplication, Division, Powers and roots.
- Solution of simple logarithmic equations.

Logarithm of numbers less than one

Simple logarithm operations

To find the logarithms of numbers less than 1, (i.e. numbers between 0 and 1), we use negative powers of 10.

For example, 0.08356 = 8.356 $\times 10^{-2}$ (standard form)

$$0.08356 = 10^{0.9220} \times 10^{-2} \quad \text{(from log tables)}$$

$$= 10^{-2+0.9220}$$

So Log0.08356 = -2+0.9220

Characteristics (i.e. power of 10) = -2

Mantissa = 0.9220

$\therefore -2 + 0.9220 = \bar{2}.9220$

Note: -2 is called bar 2 i.e. $\bar{2}$

Example 1: Work out the following giving the answers in bar notation

(a) $\bar{4}.3 \times 5$

(b) $\bar{1}.6043 \times 4$

SOLUTION

(a) $\bar{4}.3 \times 5 = (\bar{4} + 0.3)5$

$$= \overline{20} + 1.5$$
$$= \overline{19}.5$$

(b) $\bar{1}.6043 \times 4 = \bar{1} + 0.6043$

$$\underline{\times \qquad 4}$$
$$\underline{\bar{4} + 2.4172}$$
$$\Rightarrow \quad \bar{2}.4172$$

Example 2: Work out the following giving the answers in bar notation

$$\bar{5}.806 \div 4$$

SOLUTION

$$\bar{5}.806 \div 4 = \frac{\bar{5}+0.806}{4}$$
$$= \frac{\bar{8}+3.806}{4}$$
$$= \bar{2} + 0.9515$$
$$= \bar{2}.9515$$

CLASS ACTIVITY

Work out the following in bar notation form

(a) i. $\bar{3}.7$ + $\underline{\bar{5}.8}$ ii. $\bar{2}.9$ + $\underline{5.6}$ iii. $\bar{5}.7$ - $\underline{\bar{2}.3}$ iv. $\bar{2}.8$ - $\underline{\bar{6}.1}$ v. $\bar{5}.3$ - $\underline{\bar{2}.7}$

(b)

(i) $\bar{3}.4 \times 5$
(ii) $\bar{2}.823 \times 4$
(iii) $\bar{3}.7538 \div 5$
(iv) $\bar{6}.509 \div 5$

Logarithm of numbers less than one, involving: Multiplication, Division, Powers and roots.

Example 1: a. Evaluate, using logarithm tables 0.9807 x 0.007692

Solution: 0.9807 x 0.007692

Number	Log
0.9807	$\bar{1}.9915$
0.007692	$\bar{3}.8860$
0.007543	$\bar{3}.8775$

∴ antilog of $\bar{3}.8775$= 0.00754 to 3s.f

Evaluate the following using logarithm tables

b. 0.00889 ÷ 204.6

Numbers	Log

0.00889	$\bar{3}.9489$
204.6	2.3109
0.00004345	$\bar{5}.6380$

∴ antilog of $\bar{5}.6380$ = 0.0000435 to 3s.f

Note: In Logarithm, powers take multiplication while roots take division.

Example 2: a. Evaluate $(0.05872)^4$

Numbers	Log
0.05872	$\bar{2}.7687$
0.05872^4	$\bar{2}.7687 \times 4$
0.00001188	$\bar{5}.0748$

$$\therefore \mathbf{(0.05872)^4 = 0.000012}\ to\ 2s.f$$

b. $\sqrt[7]{0.0004786}$

Solution:

Numbers	**Log**
0.0004786	$\bar{4}.6799$
$\sqrt[7]{0.0004786}$	$\bar{4}.6799 \div 7$
	$\bar{7} + 3.6799 \div 7$
	$\bar{1} + 0.5257$

0.3355	$\bar{1}.5257$

$$\therefore \sqrt[7]{0.0004786} = \mathbf{0.3355}$$

$c.\ \frac{(0.0099)^2}{\sqrt[6]{0.000907}}$

Numbers	Log	
0.0099	$\bar{3}.9956$	
$(0.0099)^2$	$\bar{3}.9956 \times 2$	
	$\bar{6} + 1.9912$	
Numerator	$\bar{5}.9912$	$\bar{5}.9912$
0.000907	$\bar{4}.9576$	
$\sqrt[6]{0.000907}$	$\bar{4}.9576 \div 6$	
	$\bar{6} + 2.9576 \div 6$	
Denominator	$\bar{1} + 0.4929$	$\bar{1}.4929$
0.000315		$\bar{4}.4983$

$$\therefore \frac{(0.0099)^2}{\sqrt[6]{0.000907}} = 0.000315$$

CLASS ACTIVITY

Evaluate the following using Logarithm tables

(1)i. $(0.896 \times 0.791)^3$

ii. $(0.898)^6$

(2) i. $\sqrt{0.7164 \times 0.082}$

ii. $\frac{(0.7451)^4}{\sqrt[5]{0.00874}}$

SOLUTION OF SIMPLE LOGARITHMIC EQUATIONS

In this lesson, the equations have to be solved first, then tables used to evaluate

If $\log_a y = x$

Then $y = a^x$

RULES OF LOGARITHM

- note that any logarithm to the same base is 1, that is, $\log_a a = 1$.

  e.g $\log_3 3 = 1, \quad \log_{\frac{1}{4}} \left(\frac{1}{4}\right) = 1, \quad \log_{0.5} \left(\frac{1}{2}\right) = 1 \quad e.t.c$

- $note\ also\ that\ \ \log_a y^x = x \log_a y$

  e.g $\log_2 5^4 = 4 \log_2 5, \quad \log_6 (9)^{\frac{1}{2}} = \frac{1}{2} \log_6 9$, e.t.c

Example 1: Solve the logarithmic equations

$\log_x 81 = 4$

Solution:

i. $\log_x 81 = 4$

from the formula above, we have that

$x^4 = 81$

$x^4 = 3^4$

this implies that $x = 3$ because the powers will cancel each other

$\therefore x = 3$

EXAMPLE 2: Solve the logarithmic equations

$\log_3 9 = x$

ii. $\log_3 9 = x$

$3^x = 9$

$3^x = 3^2$

$\therefore \quad x = 2$

CLASS ACTIVITY

1. Solve for x, $(i) \log_x 7 = \frac{1}{2}$

   $(ii) \log_4 x = 3.5$

2. Evaluate the following

   i. $\log_3 243 + \log_{0.5} \left(\frac{1}{16}\right)$

   ii. $\frac{\log_3 27}{\log_3 \left(\frac{1}{9}\right)}$

PRACTICE EXERCISE

1. Use logarithm tables to evaluate

$$\frac{(3.68)^2 \times 6.705}{\sqrt{0.3581}}$$

2. Evaluate using logarithm tables, correct to 3 significant figures

$$\frac{\sqrt[3]{1.376}}{\sqrt[4]{0.007}}$$

3. Evaluate using tables

$$\sqrt{\frac{2.067}{0.0348 \times 0.538}}$$ (SSCE 1993)

4. Evaluate using tables, leaving your answer in standard form

$\sqrt{\frac{P}{Q}}$ Where P $=3.6 \times 10^{-3}$ and Q $= 2.25 \times 10^{6}$

5. Evaluate using logarithm table, correct to 1 decimal place

$$\sqrt{\frac{0.81 \times 10^{-5}}{2.25 \times 10^{7}}}$$

ASSIGNMENT

1. Evaluate using logarithm table, leaving your answer in standard form

$$\sqrt{\frac{8.1 \times 10^{-3}}{1.44 \times 10^{4}}}$$

2. Use logarithm tables to evaluate

$$\frac{(3.68)^2 \times 6.705}{\sqrt{0.3581}}$$ (SSCE 1991)

3. Use logarithm table to Evaluate, correct to 3 significant figures;

$$\sqrt{\frac{0.897 \times 3.536}{0.00249}}$$ (SSCE 1994)

4. Use logarithm tables to Evaluate, correct to 3 significant figures;

$$\frac{15.05 \times \sqrt{0.00695}}{6.95 \times 10^{2}}$$ (SSCE 1997)

5. Evaluate using logarithm tables $\sqrt[3]{\frac{0.1532}{0.01371}}$

6. Evaluate using logarithm tables $\frac{6.421 \times 0.00592}{0.04129}$

7. Given that $\frac{1}{3}\log_{10}p = 1$, find the value of p.

   A. $1/10$ B. 3 C. 10 D. 100 E.1000

8. Evaluate $\log_5 0.04$

   A. 0.008 B. – 1.4 C. – 2 D. 1

KEYWORDS: base, logarithm, integer, antilogarithm, mantissa, characteristics etc.

WEEK 3:
DATE……………………….
Subject: Mathematics
Class: SS 2
TOPIC: Sequence and Series 1
Content:

- Meaning and types of sequence.
- Example of an A. P.
- Calculation of: (i) first term (ii) common difference (iii) nth term (iv) Arithmetic mean (v) sum of an A. P.
- Practical problems involving real life situations.

Meaning and types of sequence

SEQUENCES:

A sequence is an ordered list of numbers whose subsequent values are formed based on a definite rule. The numbers in the sequence are called terms and these terms are normally separated from each other by commas.

Examples:

2, 4, 6, 8, 10,……

Rule: Addition of 2 for subsequent terms.

70, 66, 62, 58, 54,……

Rule: Subtraction of 4 for subsequent terms.

3, -6, 12, -24,……

Rule: *Multiply each term by –2.*

There are many types of sequences. We shall be considering the Arithmetic Progression and Geometric progression.

Finite and Infinite Sequences

A finite sequence is a sequence whose terms can be counted. i.e. it has an end. These types of sequences are usually terminated with a full stop. e.g. (i) 3,5,7,9,11,13. (ii) -7,-10,-13,-16,-19,-21.

If however, the terms in the sequence have no end, the sequence is said to be infinite. These types of sequences are usually ended with three dots, showing that it is continuous. e.g. (i) 5,8,11,14,17,20… (ii) -35,-33,-31,-29,-27,…

ARITHMETIC PROGRESSION (AP) {*LINEAR SEQUENCE*}

If in a sequence of terms $T_1, T_2, T_3, ...T_{n-1}, T_n$ the difference between any term and the one preceding it is constant, then the sequence is said to be in arithmetic progression (A.P) and the difference is known as the common difference, denoted by d.

$\therefore d = T_n - T_{n-1}$, where n = 1, 2, 3, 4, …

i.e $d = T_2 - T_1 = T_3 - T_2 = T_4 - T_3$ and so on.

Examples of A.P

(i) 1, 3, 5,7, 9, …

$T_n - T_{n-1} \Rightarrow 5 - 3 = 2$

$7 - 5 = 2$

$9 - 7 = 2$

$\therefore \quad d = 2$

The difference is common, hence it is an A.P.

(ii) 2, 4, 8, 16, 32, …

$T_n - T_{n-1} \Rightarrow 4 - 2 = 2$

$8 - 4 = 4$

$16 - 8 = 8$

32 - 16 = 16

*The difference is **NOT** common; therefore it is not an A.P.*

(iii) 70, 66, 62, 58, 54, …

$T_n - T_{n-1} \Rightarrow$ 66 - 70 = -4

62 - 66 = -4

58 - 62 = -4

$\therefore$ d = -4

The difference is common; hence it is an A.P.

(iv) –2, -5, -8, -11, …

$T_n - T_{n-1} \Rightarrow$ (-5) - (-2) = -5 + 2 = -3

(-8) - (-5) = -8 + 5 = -3

(-11) - (-8) = -11 + 8 = -3.

The difference is common; hence it is an A.P.

CLASS ACTIVITY

Which of the following are arithmetic progressing sequence?

(a) 4,6,8,10,…
(b) 3,7,9,11,..
(c) 1,6,11,16,21,26…
(d) 100,96,92,88,84,…
(e) 20,17,15,11,…
(f) 45,42,39,36,…

<u>THE nth TERM OF AN A.P</u>

If the first term of an A.P is 3 and the common difference is 2. The terms of the sequence are formed as follows.

1st term = 3

2nd term = 3+2 = 3 + (1)2

3rd term = 3+2+2 = 3 + (2)2

4th term = 3+2+2+2 = 3 + (3)2

5th term = 3+2+2+2+2 = 3 + (4)2

nth term = 3+2+2+2+ … = 3 + (n - 1)2

Hence, the nth term (T_n) of an A.P whose first term is "a" and the common difference is "d" is given as

$$T_n = a + (n - 1)d$$

Example 1:

a.Find the 21^{st} term of the A.P 3, 5, 7, 9, …

Solution

a = 3

d = 2

n = 21

$T_n = a + (n - 1)d$

$T_{21} = 3 + (21 - 1)2$

$= 3 + 20 \times 2$

$= 3 + 40$

$= 43.$

b.Find the 27th *term of the A.P*

100, 96, 92, 88, …

Solution

a = 100

d = -4

n = 27

$T_n = a + (n - 1)d$

$T_{27} = 100 + (27 - 1)(-4)$

$= 100 + 26 \times -4$

$= 100 - 104$

$= -4.$

Example 2:

a.Find the value of n given that 77 is the nth term of an A.P 3½, 7, 10½, …

Solution:

a = 3½

d = 7 - 3½

$\therefore$ d = 3½

$T_n = 77$

$T_n = a + (n - 1)d$

$77 = 3\frac{1}{2} + (n - 1)3\frac{1}{2}$

$77 = 3\frac{1}{2} + 3\frac{1}{2}n - 3\frac{1}{2}$

$77 = 3\frac{1}{2}n$

$77 = {}^{7}/_{2}n$

$7n = 77 \times 2$

$n = \frac{77 \times 2}{7}$

$n = 11 \times 2$

$\therefore n = 22.$

b. *What is the first term of an A.P whose 21st term is* 43 and *the common difference is* 2 *?*

Solution:

$T_{21} = 43$

$n = 21$

$d = 2$

$T_n = a + (n - 1)d$

$43 = a + 20 \times 2$

$43 = a + 40$

$a = 43 - 40$

$\therefore a = 3$

C. *Find the common difference of an A.P given that* 43 *is the* 21st *term of the sequence and the first term is* 3.

Solution:

$a = 3$

$T_{21} = 43$

$n = 21$

$T_n = a + (n - 1)d$

$43 = 3 + (21 - 1)\, d$

$43 = 3 + 20d$

$43 - 3 = 20d$

$20d = 40$

$d = \frac{40}{20}$

$\therefore\ d = 2.$

CLASS ACTIVITY

(1) Find the 31^{st} term of the sequence –7, -10, -13, -16, …
(2) What is the 26th term of the A.P 5, 10, 15, 20, …?

FURTHER Example 1:

The first three terms of an A.P are
x, 3x + 1*, and* (7x - 4)*. Find the*

(i) Value of x

(ii) 10th *term*

Solution:

(i) Recall that given an A.P T_1, T_2, T_3

$$T_2 - T_1 = T_3 - T_2$$

Hence for, x, (3x + 1), (7x - 4)

$$(3x + 1) - x = (7x - 4) - (3x + 1)$$
$$3x+1 - x = 7x - 4 - 3x - 1$$
$$2x + 1 = 4x - 5$$
$$1+ 5 = 4x - 2x$$
$$2x = 6$$
$$x = \frac{6}{2}$$
$$\therefore\ x = 3.$$

The sequence x, (3x + 1), (7x - 4) is

= 3, (3x3 + 1), (7x3 - 4)

= 3, 10, 17.

(ii) a = 3

n = 10

d = 7

$$T_n = a + (n - 1)d$$
$$T_{10} = 3 + (10 - 1)7$$
$$= 3 + 9x7$$
$$= 3 + 63$$
$$= 66.$$

Example 2:

The 6th term of an A.P is –10 *and the 9th term is* –28*.*
Find the (i) Common difference

(ii) First term

(iii) 26th *term of the sequence.*

Solution:

(i) $T_6 = -10$, $n = 6$ $\Big\}$ $T_n = a + (n - 1)d$

$-10 = a + (6 - 1)d$

$-10 = a + 5d$ ----------- (1)

$T_9 = -28$, $n = 9$ $\Big\}$ $-28 = a + (9 - 1)d$

$-28 = a + 8d$ ---------- (2)

Solve equation (1) and (2) simultaneously.

Eqn. (1): $-10 = a + 5d$

Eqn. (2): $-28 = a + 8d$

$18 = -3d$

$d = \frac{18}{-3}$

$\therefore\ d = -6.$

(ii) Put $d = -6$ in equation (1)

$-10 = a + 5(-6)$

$-10 = a - 30$

$-10+30 = a$

$\therefore\ a = 20.$

(iii) To find the 26th term of the sequence.

$a = 20$

$d = -6$

$n = 26$

$T_n = a + (n - 1)d$

$T_{26} = 20 + (26 - 1)(-6)$

$= 20 + 25(-6)$

$= 20 - 150$

$= -130.$

CLASS ACTIVITY

(1) The 6th term of an AP is –10 and the 9th term is 18 less than the 6th term. Find the
(a) common difference (b) first term
(c) 26th term of the sequence.

(2) The 7th term of an AP is 17 and the 13th term is 12 more than the 7th term. Find the (i) common difference (ii) first term (iii) 21st term of the AP.

Arithmetic Series:

These are series formed from an arithmetic progression. e.g.

1 + 4 + 7 + 10 + …

In general, if S_n is the sum of n terms of an arithmetic series then

$S_n = a + (a + d) + (a + 2d) + \dots + (l - d) +$

l -- (1)

Where l is the nth term, a is the first term and d is the common difference.

Rewriting the series above starting with the nth term, we have.

$S_n = l + (l - d) + (l - 2d) + \dots + (a + d) + a$ ----- (2)

Adding equation (1) and (2) we have

$2S_n = (a + l) + (a + l) + \dots + (a + l) + (a + l)$ in n places

$2S_n = n(a + l)$

$\therefore S_n = {}^{n}/_{2}(a + l)$

But l is the nth term i.e a + (n - 1) d

$S_n = {}^{n}/_{2}\{a + a + (n - 1)d\}$

$\therefore S_n = {}^{n}/_{2}\{2a + (n - 1)d\}$

Example 1 :

Find the sum of the first 20 *terms of the series* 3+5+7+9+ …

Solution:

a = 3

d = 2

n = 20

$S_n = {}^{n}/_{2}\{2a + (n - 1)d\}$

$S_{20} = {}^{20}/_{2}\{2\text{x}3 + (20 - 1)2\}$

$S_{20} = 10\{6 + 19 \text{ x } 2\}$

$= 10\{6 + 38\}$

$= 10\{44\}$

$\therefore S_{20} = 440$

Example 2:

Find the sum of the first 28 *terms of the series* –17 + (-14) + (-11) + (-8) + …

Solution:

a = -17

d = 3

n = 28

$S_n = {}^n/_2\{2a + (n - 1)d\}$

$S_{28} = {}^{28}/_2 \{2 (-17) + (28 - 1) 3\}$

$= 14 \{-34 + 27 \times 3\}$

$= 14 \{ -34 + 81\}$

$= 14 \{47\}$

$\therefore S_{28} = 658$

CLASS ACTIVITY

(1) Find the sum of the numbers from 1 to 100.

(2) Find the sum of the first 26 terms of the A.P –18, -15, -12, -9, …

<u>FURTHER Example :</u>

The sum of the first 9 *terms of an A.P is* 117 *and the sum of the next* 4 *terms is* 104.

Find the(i) Common difference

(ii) First term

*(iii)*25th *term of the A.P*

(WAEC)

<u>Solution:</u>

$T_1, T_2, T_3, T_4 \ldots T_8, T_9,$ $\longrightarrow$ 117 $\longleftarrow$ $\longrightarrow$ $T_{10}, T_{11}, T_{12}, T_{13}$ 104 $\longleftarrow$

$S_9 = 117$ } ------------------------(*)
n = 9

since a sequence is normally summed from the first term

$S_{9+4} = 117 + 104$
$\therefore S_{13} = 221$ } --------------(**)
n = 13

$S_n = {}^n/_2 \{2a + (n - 1) d\}$

From (*) above;

$117 = {}^9/_2 \{2a + (9 - 1) d\}$

$117 = {}^9/_2 \times 2a + {}^9/_2 \times 8d$

$117 = 9a + 36d$

Divide through by 9 to have;

$13 = a + 4d$ ----------------------(1)

From (**) above;

$221 = {}^{13}/_2 \{2a + (13 - 1) d\}$

$221 = {}^{13}/_2 \times 2a + {}^{13}/_2 \times 12d$

$221 = 13a + 78d$

Divide through by 13 to have;

$$17 = a + 6d \text{ ----------------------(2)}$$

From equation (1) and (2)

Eqn. (1): $13 = a + 4d$

Eqn. (2): $\underline{17 = a + 6d}$

$$-4 = -2d$$

$$d = \frac{-4}{-2}$$

$$\therefore d = 2.$$

(ii) From equation (1) we have

$$13 = a + 4 \times 2$$

$$13 = a + 8$$

$$a = 13 - 8$$

$$\therefore a = 5$$

(iii) $a = 5$

$d = 2$

$n = 25$

$$T_n = a + (n - 1) d$$

$$T_{25} = 5 + (25 - 1) 2$$

$$= 5 + 24 \times 2$$

$$= 5 + 48$$

$$T_{25} = 53$$

CLASS ACTIVITY

(1) The sum of the first 9 terms of an A.P is 171 and the sum of the next 5 terms is 235.Find the (a) Common difference

(b) First term

(c) Sequence

(2) The sum of the first 8 terms of an A.P is 172 and the sum of the next three terms is 15. Find the

(a) Common difference

(b) First term

(c) 21^{st} term of the A.P

PRACTICAL PROBLEMS INVOLVING REAL LIFE SITUATION

Example 1:

A clerk employed by a private establishment on an initial salary of ₦5000 *per annum. If his annual increment in salary is* ₦300. *Find the total salary earned by the clerk in* 20*years.*

Solution:

a = 5000

d = 300

n = 20

$S_n = {}^{n}/_{2}\ \{2a + (n - 1)\ d\}$

$S_{20} = {}^{20}/_{2}\ \{2 \times 5000 + (20 - 1)\ 300\}$

$= 10\ \{10000 + 19 \times 300\}$

$= 10\ \{10000 + 5700\}$

$= 10\ \{15700\}$

$\therefore\ S_{20} = 157000$

$\therefore$ The total amount earned in 20years is ₦157000.00

EXAMPLE 2

A sum of money is shared among nine people so that the first gets N75, the next N150, the next N225, and so on.

a. How much money does the ninth person get?
b. How much money is shared altogether?

Solution:

a=75 d=150-75=75 n=9

T_n=a + (n-1)d

T_9=75 + (9-1)75

=75+600

=N675

$S_n = \frac{n}{2}(a + l)$

$S_9 = \frac{9}{2}(75 + 675)$

=4.5 × 750

=N3375

CLASS ACTIVITY

(1) The value of a machine depreciates each year by 5% of its value at the beginning of that year. If its value when new on 1st January 1980 was N10,250.00, what was its value in January 1989 when it was 9years old? Give your answer correct to three significant figures.

(WAEC) 1989

(2) The houses on one side of a particular street are assigned odd numbers, starting from 11. If the sum of the numbers is 551, how many houses are there? (SSCE 1999)

PRACTICE EXERCISE

(1) The 6th term of an A.P is 26 and the 11th term is 46. Find the

(i) Common difference

(ii) First term

(iii) 25th term of the A.P

(2) The 5th term of an A.P is 11 and the 9th term is 19. Find the

(i) common difference

(ii) First term

(iii) 21^{st} term of the A.P

(3) The fourth term of an A.P is 37 and the 6th term is 12 more than the fourth term. Find the first and seventh terms.

SSCE, June 1994, No 11a (WAEC)

(4) The first three terms of an A.P are (x+2) ,(2x-5) and (4x+1). Find the

(i) Value of x

(ii) 7th term.

(5) The first three terms of an A.P are x, (2x-5) and (x+6). Find the

(i) Value of x

(ii) 21^{st} term.

ASSIGNMENT

1. If the first three terms of an A.P are (4x+1), (2x-5) and (x+3). Find the
   (i) Value of x
   (ii) Sequence
   (iii) 11th term of the sequence
2. Given that 9, x, y, 24 are in A.P, find the values of x and y.
   (10) If –5, a, b, 16 are in A.P, find the values of a and b.
3. The 8th term of an arithmetic progression (A.P) is 5 times the third term while the 7th term is 9 times greater than the 4th term. Write the first five terms of the A.P. (SSCE 2009)

4. If 3, x, y, 18 are the arithmetic progression (A.P). Find the values of x and y.

(SSCE 2008)

5. If $\frac{1}{2}, \frac{1}{x}, \frac{1}{3}$ are successive terms of an arithmetic progression (A.P), show that $\frac{2-x}{x-3} = \frac{2}{3}$

(SSCE 2007)

6. The 3rd and 8th terms of an arithmetic progression (A.P)are -9 and 26 respectively. Find the:
   i. Common difference
   ii. First term

(SSCE 2007)

7. The 2nd, 3rd and 4th terms of an A.P. are $x-2, 5 \text{ and} x+2$ respectively. Calculate the value of x.

(SSCE 2006)

8. An arithmetic progression (A.P) has 3 as its first term and 4 as the common difference.
   i. Write an expression, in its simplest form for the nth term.
   ii. Find the least term of the A.P that is greater than 100

(SSCE 2003)

9. The first term of an arithmetic progression (A.P)is 3 and the common difference is 4. Find the sum of the first 28 terms.

(SSCE 2002)

10. The first term of an arithmetic progression (A.P) is -8. The ratio of the 7th term to the 9th term is 5:8. Calculate the common difference of the progression. (SSCE 2000)

11. The 6th term of an A.P. is 35 and the 13th term is 77. Find the 20th term. (SSCE 1997)

KEYWORDS: sequence, series, first term, common difference, last term, arithmetic progression, finite sequence etc.

WEEK 4:
DATE……………………….
Subject: Mathematics
Class: SS 2
TOPIC: Sequence and Series II
Content:

- Examples of geometric progression

- ➢ Calculation of; (i) First term (ii) Common ratio (iii) nth term, (iv) Geometric progression (v) sum of terms of geometric progression. (vi) Sum to infinity
- ➢ Practical problems involving real life situation.

GEOMETRIC PROGRESSION (G.P) OR *EXPONENTIAL SEQUENCE*

Given any sequence of terms T_1, T_2, T_3, T_4, … T_{n-1}, T_n. If the ratio between any term and the one preceding it is constant then the sequence is said to be in geometric progression (G.P). The ratio is called the common ratio denoted by r. i.e.

$$r = \frac{T_n}{T_{n-1}}$$

where n = 1, 2, 3, 4, …

Example1: find out which of the sequence is a GP

(i) 1, 2, 4, 8, 16, …

$\frac{T_n}{T_{n-1}} \Rightarrow \frac{2}{1} = 2$

1

$\frac{4}{2} = 2$

$\frac{8}{4} = 2$

The ratio is common, hence the sequence is a G.P $\therefore r = 2$

(ii) 16, 8, 4, 2, …

$\frac{T_n}{T_{n-1}} \Rightarrow \frac{8}{16} = \frac{1}{2}$

$\frac{4}{8} = \frac{1}{2}$

$\frac{2}{4} = \frac{1}{2}$

The ratio is common, hence the sequence is a G.P $\therefore r = \frac{1}{2}$

Example 2: find out which of the sequence is a GP

(i) 3, 7, 9, 12, …

$\frac{T_n}{T_{n-1}} \Rightarrow \frac{7}{3} = 2\frac{1}{3}$

3

$$\frac{9}{7} = 1\frac{2}{7}$$

$$\frac{12}{9} = 1\frac{3}{9} = 1\frac{1}{3} =$$

The ratio is <u>NOT</u> common, hence not a G.P.

(ii) 2, -10, +50, -250, …

$\frac{T_n}{T_{n-1}} \Rightarrow \frac{-10}{2} = -5$

$$\frac{+50}{-10} = -5$$

$$\frac{-250}{+50} = -5$$

The ratio is common, hence the sequence is a G.P $\therefore r = -5$

CLASS ACTIVITY

Which of the following sequences is G .P.?

(a) 3,6,12,24…
(b) 2,4,6,8,…
(c) 5,15,45,…
(d) 1,4,16,…

THE NTH TERM OF A G.P

Let 5 be the first term of a G.P whose common ratio is 2. Then

The 2nd term is 5x2 $= 5(2)^1$

The 3rd term is 5x2x2 $= 5(2)^2$

The 4th term is 5x2x2x2 $= 5(2)^3$

The 5th term is 5x2x2x2x2 $= 5(2)^4$

The nth term is 5x2x2 … 2 $= 5(2)^{n-1}$

In general, the nth term of a G.P denoted by T_n, whose first term is "a" and whose common ratio is "r" is ar^{n-1}. i.e.

$T_n = ar^{n-1}$ where n = 1, 2, 3, 4, …

NOTE

The four examples below show how the formula can be used to find the nth term, n, r and a.

Example 1:

I. *Find the 8th term of the G.P*

3, 6, 12, 24, …

Solution

a = 3

r = 2

n = 8

$T_n = ar^{n-1}$

$T_{21} = 3(2)^{8-1}$

$T_{21} = 3 \times 2^7$

$T_{21} = 3 \times 128$

$\therefore T_{21} = 384$

II. *Find the value of n given that the nth term of a G.P is* 2916 *and the first term and common ratio are* 4 *and* 3 *respectively.*

Solution:

$T_n = 2916$

r = 3

a = 4

$T_n = ar^{n-1}$

$2916 = 4(3)^{n-1}$

$\frac{2916}{4} = 3^{n-1}$

$729 = 3^{n-1}$

$3^6 = 3^{n-1}$

$n-1 = 6$

$n = 6+1$

$\therefore n = 7.$

EXAMPLE 2:

1. *Find the common ratio of an exponential sequence whose* 10th *term is* –512 *and the first term is* 1.

Solution

$a = 1$

$T_{10} = -512$

$n = 10$

$T_n = ar^{n-1}$

$-512 = 1(r)^{10-1}$

$-512 = r^9$

$(-512)^{1/9} = r$

$r = (-2^9)^{1/9}$

$\therefore r = -2$

2. *Find the first term of an exponential sequence whose* 7th *term is* 4096 *and the common ratio is* 4.

Solution

$T_7 = 4096$

$n = 7$

$r = 4$

$T_n = ar^{n-1}$

$4096 = a(4)^{7-1}$

$4096 = a4^6$

$4096 = 4096a$

$a = \frac{4096}{4096}$

$\therefore a = 1$

CLASS ACTIVITY

(1) The 3rd and 6th term of a geometric progression (G.P) are 48 and $14\,^2/_9$ respectively. Write down the first four terms of the G.P.

SSCE, June. 1993, No9 (WAEC)

(2) The first and third terms of a G.P are 5 and 80 respectively. What is the 4th term?

SSCE, Nov. 1993, No 11b (WAEC)

GEOMETRIC SERIES

The general expression for a geometric series is given as

$S_n = a + ar + ar^2 + ar^3 + \ldots + ar^{n-1}$ ----- (1)

Where S_n represents the sum of n terms of the series

Multiply both sides of equation (1) by r to have

$rS_n = ar + ar^2 + ar^3 + ar^4 + \text{---} + ar^n$ ---- (2)

Subtract (2) from (1) to have

$S_n - rS_n = a - ar^n$

$S_n(1 - r) = a(1 - r^n)$

$S_n = \frac{a(1-r^n)}{1-r}$ ------------------- (3)

If the numerator and denominator of equation (3) is multiplied by –1, we have

$S_n = \frac{a(r^n-1)}{r-1}$ -------------------- (4)

If r < 1, formula (3) is more convenient

If r >1, formula (4) is more convenient

Example 1:

Find the sum of the first 8 terms of the G.P 3, 6, 12, 24, …

Solution:

a = 3

n = 8

r = 2

$S_n = \frac{a(r^n - 1)}{r - 1} \qquad r > 1$

$S_8 = 3\frac{(2^8 - 1)}{2 - 1}$

$= 3\underline{(256 - 1)}$

$$= \frac{3(256-1)}{1}$$

$= 3 (255)$

$S_8 = 765.$

Example 2:

Find the sum of the first 10 *terms of the G.P* 2, -6, 18, -54, …

Solution:

$a = 2$

$n = 10$

$r = -3$

$$S_n=\frac{a(1-r^n)}{1-r} \quad r < 1$$

$$S_{10} = \frac{2(1-(-3)^{10})}{1-(-3)}$$

$$= \frac{2(1-59049)}{1+3}$$

$$= \frac{2(-59048)}{4}$$

$$= \frac{-59048}{2}$$

$$S_{10} = -29524$$

CLASS ACTIVITY

(1) Find the sum of the first 9 terms of the sequence 84, 42, 21, 10½, …

(2) Find the sum of the first 10 terms of the G.P 4, 8, 16, 32, …

SUM TO INFINITY

In general, if the common ratio, r, is a fraction such that $-1<x<1$, the value of r^n approaches zero as n increases towards infinity. It then follows that the sum $S=\frac{a(1-r^n)}{1-r}$ becomes

$$\boldsymbol{S_\infty = \frac{a}{1-r}}$$

This formula gives the **sum to infinity** of a geometric progression.

EXAMPLE 1:

Find the sum to infinity of the series S_n=36+24+6+4+….

Solution

$$S_\infty = \frac{a}{1-r}$$

a=36 r= $\frac{24}{36} = \frac{2}{3}$

$$S_\infty = \frac{36}{1-\frac{2}{3}}$$

$$S_\infty = \frac{36}{\frac{1}{3}}$$

$$S_\infty = 36 \times 3$$

$$S_\infty = 108$$

EXAMPLE 2:

The sum to infinity of a GP series is 15/7 and its second term is -6/5. Find the common ratio.

Solution

$$S_\infty = \frac{a}{1-r}$$

$$\frac{15}{7} = \frac{a}{1-r}$$

$$15(1-r) = 7a$$

Now, ar=-6/5

a=-6/5r

15 (1-r)=7 (-6/5r)

15- 15r =-42/5r

5r(15-15r)=-42

$75r-75r^2$= -42

$25r – 25r^2$ = -14

$25r^2-25r-14=0$

$25r^2-35r+10r-14=0$

$5r(5r-7)+2(5r-7)=0$

$(5r+2)(5r-7)=0$

$5r+2=0$ or $5r-7=0$

$r=-2/5$ or $7/5$

Note that $r=7/5$ is not a true value since $(7/5)^n$ will not approach zero(0) as n increases towards infinity. Hence the valid common ratio of the series is -2/5.

CLASS ACTIVITY

1. Find the sum to infinity of $S_n=200+120+72+43\frac{1}{5}+........$

2. the first term of a GP series is 4/9 and the sum to infinity is $1\frac{1}{3}$. find the common ratio.

Practical problems involving real life situation

Example 1:

i. A ball was dropped from a height 80m above a concrete floor. It rebounded to the height of ½ of its previous height at each rebound. After how many bounces is the ball 2.5m high? ***NECO 2001***

Solution

The rebounds forms a sequence of the order

80, 40, 20, …,2.5.

$a = 80$

$r = ½$

$T_n = 2.5$ $\quad T_n = ar^{n-1}$

$2.5 = 80(½)^{n-1}$

$\frac{2.5}{80} = (½)^{n-1}$

$\frac{\frac{5}{2}}{80} = (½)^{n-1}$

$\frac{5}{2x80} = (½)^{n-1}$

$^1/_{32} = (½)^{n-1}$

$(½)^5 = (½)^{n-1}$

$n – 1 = 5$

$n = 5 + 1$

$\therefore\ n = 6$

ii. The 3rd *term of a G.P is* 54 *and the* 5th *term is* 486. *Find the*

(a) *Common ratio*

(b) *First term*

(c) 7th *term of the G.P*

Solution

(a) $\left.\begin{matrix} T_3 = 54 \\ n = 3 \end{matrix}\right\}$ $T_n = ar^{n-1}$

$54 = ar^{3-1}$

$54 = ar^2$ ---------------- (1)

$\left.\begin{matrix} T_5 = 486 \\ n = 5 \end{matrix}\right\}$ $486 = ar^{5-1}$

$486 = ar^4$ -------------- (2)

Equation (2) ÷ equation (1)

$$\frac{486}{54} = \frac{ar^4}{ar^2}$$

$9 = r^2$

$r = \pm\sqrt{9}$

$r = \pm 3$

(b) Substitute in equation (1)

$54 = a(\pm 3)^2$

$54 = 9a$

$a = {}^{54}/_{9}$

$\therefore\ a = 6$

(c) $a = 6$

$r = \pm 3$

$n = 7$

$T_n = ar^{n-1}$

$T_7 = 6(\pm 3)^{7-1}$

$= 6 \times 3^6$

$= 6 \times 729$

$= 4373$

Example 2:

i. *If* 2, x, y, 54 *are in G.P, find x and y.*

Solution:

$a = 2$

$n = 4$

$T_4 = 54$

$T_n = ar^{n-1}$

$54 = 2(r)^{4-1}$

$54 = 2r^3$

${}^{54}/_{2} = r^3$

$r^3 = 27$

$r^3 = 3^3$

$\therefore r = 3$

$a = 2$

$r = 3$

$T_2 = x$

$n = 2$

$T_n = ar^{n-1}$

$x = 2(3)^{2-1}$

$x = 2x3$

$\therefore x = 6$

$a = 2$

$r = 3$

$T_3 = y$

$n = 3$

$y = 2(3)^{3-1}$

$y = 2(3)^2$

$y = 2x9$

$\therefore y = 18$

ii.Given that x, (x-2), (2x-1) *are in G.P. Find the*

(a) Value(s) of x

(b) Sequences

(c) *Possible value(s) of the* 8th *term of the G.P*

Solution:

Since x, (x-2), (2x-1) are in G.P, the ratio between any of the terms and the one preceding must be common.

i.e $\underline{(x-2)} = \underline{(2x-1)}$

$x \qquad (x-2)$

Cross-multiplying

$(x - 2)(x - 2) = x(2x - 1)$

$x^2 - 2x - 2x + 4 = 2x^2 - x$

$0 = 2x^2 - x - x^2 + 2x + 2x - 4$

$0 = x^2 + 3x - 4$

i.e $x^2 + 3x - 4 = 0$ (Factorize the equation)

$-4x^2$

$x^2 + 4x - x - 4 = 0$

$x(x + 4) - 1(x + 4) = 0$

$(x + 4)(x - 1) = 0$

$x + 4 = 0$ or $x - 1 = 0$

$x = -4$ or $x = 1$

(b) Since x = -4 and 1, we shall have two sequences

For x = -4

$x, (x-2), (2x-1)$

$= -4, (-4 - 2), (2(-4)-1)$

$= -4, -6, -9$

For x = 1

$x, (x-2), (2x-1)$

$= 1, (1-2), (2x1-1)$

$= 1, -1, 1$

(c) For x = -4 the sequence = -4, -6, -9

$a = -4$

$n = 8$

$r = \frac{3}{2}$

$T_8 = -4(^3/_2)^{8-1}$

$= -4(^3/_2)^7$

$= -4 \times \frac{2187}{128}$

$\therefore T_8 = -\frac{2187}{32}$

For x =1, the sequence = 1, -1, 1

$a = 1$

$n = 8$

$r = -1$

$T_8 = -1(-1)^{8-1}$

$= 1(-1)^7$

$= -1$

Class activity

1. Three consecutive terms of a geometric progression are given as n-2,n and n+3. Find the common ratio.
2. The 3rd and 6th terms of a geometric progression are 48 and $14\frac{2}{9}$ respectively. Write down the first four terms of the GP.

PRACTICE EXERCISE

(1) Find the sum of the first 7 terms of the G.P 3, 9, 27, 81, …

(2) Find the sum of the first 11 terms of the G.P 5, 10, 20, 40, …

(3) If the 3rd and 7th terms of a G.P are 12 and 192 respectively, find the sum of the first 6 terms of the sequence.

(4) The third and fifth terms of a geometric progression are $^9/_2$ and $^{81}/_8$ respectively.

Find the (i) Common ratio

(ii) First term.

(5) If 2, x, y, -250, … is a geometric progression, find x and y.

(JAMB)

ASSIGNMENT

1. Given that 2, a, b, 686 are in G.P, find the value of a and b.

2. The first three terms of a G.P are x+1, (x+4) and 2x. Find the
   (i) Value (s) of x
   (ii) Sequence(s)
   (iii) 6th term of sequences
3. What is the 25th term of 5, 9, 13? The 3rd and 6th terms of a Geometric progression are 48 and $14\frac{2}{9}$.Write down the first four terms of the G.P. (SSCE 1993)

4. The third term of a Geometric Progression is 360 and the sixth term is 1215.Find the
   (a) common ratio (b) first term (c) sum of the first four terms. (SSCE 1998)

5. The 1st and 3rd terms of a geometric Progression are 2 and $\frac{2}{9}$ respectively. Find
   (i) the common difference (ii) the 5th term. (SSCE 1999)

6. Write down the 15th term of the sequence $\frac{2}{1 \times 3}, \frac{3}{2 \times 4}, \frac{4}{3 \times 5}, \frac{5}{4 \times 6}, \ldots,$ (SSCE 2003)

7. The sum of the second and third terms of a geometric progression is six times the fourth term. Find the two possible values of the common ratio.(ii) If the second term is 8 and the common ratio is positive, find the first six terms (SSCE 2008)

8. The third term of a Geometric Progression (G.P) is 24 and its seventh term is $4\frac{20}{27}$.
   Find its first term. (SSCE 2010)

9. Given the Geometric Progression 6, 12, 24, 48, ...
   Find (i) its common ratio (ii) the 24th and 40th term of the G.P.

10. The sum of the Geometric Progression 2 + 6 + 18 + 54 +…+ 1458 is 2186. Find the nth term of the G.P.

11. Find the sum of the G.P 3 + 9 + 27 + …. up to the 7th term.

12. Given the Geometric Progression (G.P) 4, 16, 64…

    Find the 6th and 7th term of the G.P respectively.
13. The sum to infinity of a geometric progression is -1/10 and the first term is -1/8. Find the common ratio of the progression. JAMB 2012
14. Find the sum to infinity of the series ½ ,1/6, 1/18,…. JAMB 2004

KEYWORDS:

sequence, series, first term, common ratio,last term, geometric progression, infinite series etc.

WEEK 5:
DATE……………………….
Subject: Mathematics
Class: SS 2
TOPIC: Quadratic Equations
Content:

- Revision of factorization of perfect squares
- Making quadratic expression perfect squares by adding a constant K.
- Solution of quadratic equation by the method of completing the square
- Deducing the quadratic formula from completing the square.
- Construction of quadratic equation from sum and product of roots.
- Word problems leading to quadratic equations.

Revision of factorization of perfect squares

A quadratic expression is a perfect square if it can be expressed as the product of two linear factors that are identical. For example, $x^2+4x+4=(x+2)(x+2)$, $x^2+6x+9=(x+3)(x+3)$ are perfect squares.

Example 1: Factorize : $x^2 - 22x + 121$

The expression is a perfect square if the first term and the constant terms are both perfect squares. The sign of the middle term of the quadratic expression can be put between the terms of the linear factors.

For $x^2 - 22x + 121$, the first term is the square of x and the constant term is 121 which is the square of 11.

$$x^2 - 22x + 121 = (x - 11)(x - 11)$$
$$= (x - 11)^2$$

Notice that the middle term is twice the product of the terms of the linear factor. In this case, $-22x = 2(x)(-11)$

Example 2: Factorize $400 + 120t + 9t^2$

The leading term (term containing the highest power of the variable) is a perfect square. Also the constant term, 400 is a perfect square.

$$\sqrt{400} = 20$$

$$\sqrt{9t^2} = 3t$$

So, $400 + 120t + 9t^2 = (20 + 3t)(20 + 3t)$

$$= (20 + 3t)^2$$

Example 3: Factorize $w^2 + 2wxy + x^2y^2$

Solution; $w^2 + 2wxy + x^2y^2 = (w + xy)(w + xy)$

$$= (w + xy)^2$$

Notice that the middle term of the quadratic expression is twice the product of the terms of the linear factors.

CLASS ACTIVITY

Factorize the following quadratic expressions

1. $e^2 - 8e + 16$

2. $100x^2 + 40x + 4$

3. $36p^2 - 12p + 1$

MAKING QUADRATIC EXPRESSION PERFECT SQUARES BY ADDING A CONSTANT K

In this section, we shall consider the constant K to be added to a quadratic expression to make it a perfect square.

Example 1: What must be added to $m^2 + 8m$ to make the expression a perfect square?

Solution:

Let the constant term be k, $m^2 + 8m + k$ is a perfect square if

$$m^2 + 8m + k = (m + a)^2$$

i.e. $m^2 + 8m + k = m^2 + 2am + a^2$

equating the coefficients of m, we have

$$2a = 8$$

$$a = 4$$

Equating the constant terms,

$$k = a^2$$

$$k = 4^2 = 16$$

Therefore 16 must be added to the expression to make it a perfect square.

In general, the quantity to be added is the square of half of the coefficient of m (or whatever letter is involved)

Example 2: What must be added to $w^2 - \frac{3}{4}w$ to make it a perfect square?

Solution:

What is to be added is the square of half of the coefficient of w

The coefficient of w is $\frac{-3}{4}$

Half of the coefficient is $\frac{1}{2} \times \frac{-3}{4} = \frac{-3}{8}$

The square of half of the coefficient is $\frac{9}{64}$, so $\frac{9}{64}$ must be added to $w^2 - \frac{3}{4}w$ to make it a perfect square. i.e $w^2 - \frac{3}{4}w + \frac{9}{64} = \left(w - \frac{3}{8}\right)^2$

CLASS ACTIVITY

What must be added to …

1. X^2+4x
2. M^2-3m

SOLUTION OF QUADRATIC EQUATION BY THE METHOD OF COMPLETING THE SQUARE

In this section, we shall solve quadratic equations using method of completing the square

Example 1:

Solve the quadratic equation using completing the square method:

$$x^2 - 8x + 4 = 0$$

Solution

the L.H.S of the equation does not factorize. So, one can rearrange the equation to make the L.H.S a perfect square

$$x^2 - 8x = -4$$

We then find half of the coefficient of x, square it and add it to both sides of the equation. The reason for that is to make the L.H.S a perfect square.

$$\text{i.e} \quad x^2 - 8x + \left(\frac{1}{2} \times -8\right)^2 = -4 + \left(\frac{1}{2} \times -8\right)^2$$

$$x^2 - 8x + 16 = -4 + 16$$

$$(x - 4)^2 = 12$$

Take square root of both sides,

$$(x - 4) = \pm\sqrt{12}$$

$$x = 4 \pm \sqrt{12}$$

EXAMPLE 2:

Solve $3x^2 + 7x + 3 = 0$ using completing the square method.

Solution

L.H.S of $3x^2 + 7x + 3 = 0$ cannot be factorized,

Make the coefficient of x^2 one by dividing through by 3 to get

$$x^2 + \frac{7}{3}x + 1 = 0$$
$$x^2 + \frac{7}{3}x = -1$$
$$x^2 + \frac{7}{3}x + \left(\frac{1}{2} \times \frac{7}{3}\right)^2 = -1 + \left(\frac{1}{2} \times \frac{7}{3}\right)^2$$
$$x^2 + \frac{7}{3}x + \frac{49}{36} = -1 + \frac{49}{36}$$
$$\left(x + \frac{7}{6}\right)^2 = \frac{13}{36}$$
$$\left(x + \frac{7}{6}\right) = \pm\sqrt{\frac{13}{36}}$$
$$x = -\frac{7}{6} \pm \sqrt{\frac{13}{36}}$$
$$x = \frac{-7 \pm \sqrt{13}}{6}$$

CLASS ACTIVITY

Solve the following using completing the square method

1. $x^2 - 6x + 4 = 0$
2. $p^2 + 3p + 1 = 0$

DEDUCING THE QUADRATIC FORMULA FROM COMPLETING THE SQUARE

The general form of a quadratic equation is $ax^2 + bx + c = 0,\ a \neq 0.$

We shall now derive the formula for solving the equation by method of completing the square

Given that, $ax^2 + bx + c = 0$

Since $a \neq 0$ we can divide through by a to get

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

We shall now make the left hand side a perfect square by adding $\left(\frac{b}{2a}\right)^2$ to both sides i.e half of the coefficient of x all squared.

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$
$$\left(x + \frac{b}{a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Take square root of both sides;

$$x + \frac{b}{a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is known as **quadratic formula**

Example 1: Use quadratic formula to solve, $2x^2 + x = 1$

We shall write the equation in the form $ax^2 + bx + c = 0$

So, $2x^2 + x - 1 = 0$

$\therefore a = 2, b = 1 \; and \; c = -1$

Use the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{9}}{4}$$

$$x = \frac{-1+3}{4} \; or \; x = \frac{-1-3}{4}$$

$$x = \frac{1}{2} \; or \; x = -1$$

EXAMPLE 2: i. Solve using the quadratic formula, $3p^2 - 8p + 2 = 0$

Solution

Considering the coefficients of $3p^2 - 8p + 2 = 0$

$a = 3, b = -8, c = 2$

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4ac}}{2(3)}$$

$$= \frac{8 \pm \sqrt{64 - 24}}{6}$$

$$= \frac{8 \pm 6.3246}{6}$$

$$\therefore p \simeq 2.39 \quad or \quad p \simeq 0.28$$

ii. Use quadratic formula to solve;

$\frac{x}{4} - \frac{1}{x} = \frac{5}{2}$

Solution; first is to clear the fractions by multiplying each term of both sides of the equation by the LCM of the denominators which $4x$,

$$4x\left(\frac{x}{4}\right) - 4x\left(\frac{1}{x}\right) = 4x\left(\frac{5}{2}\right)$$

$$x^2 - 4 = 10x$$

On rearranging, we have;

$$x^2 - 10x - 4 = 0$$

$$so\ that, a = 1, \qquad b = -10, \quad c = -4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(-4)}}{2(1)}$$

$$= \frac{10 \pm \sqrt{116}}{2}$$

$$= \frac{10 \pm 2\sqrt{29}}{2}$$

$$= 5 \pm \sqrt{29}$$

$$= 5 + 5.3852 \qquad or \quad 5 - 5.3852$$

$$\therefore x \simeq 10.39 \qquad or\ \ x \simeq -0.39$$

CLASS ACTIVITY

Use quadratic formula to solve the following equations:

1. i. $2x^2 + 3x = 3$

   ii. $1 = 6x^2 - x$

2 i. $16 + 8x + x^2 = 0$

   ii. $\frac{2x}{2x+3} = \frac{4x-2}{15}$

CONSTRUCTION OF QUADRATIC EQUATION FROM SUM AND PRODUCT OF ROOTS

We shall now consider how to construct quadratic equations from sum and product of roots. The root on an equation is the value that satisfies the equation.

Let $p\ and\ q$ be roots of a quadratic equation then $(x-p)(x-q) = 0$

$$x^2 - xq - xp + pq$$

$$x^2 - (p+q)x + pq = 0 \qquad \ldots\ldots\ldots(i)$$

The general form of a quadratic equation is $ax^2 + bx + c = 0 \quad ; \quad a \neq 0$

Divide through by a

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \qquad \ldots\ldots\ldots(ii)$$

Comparing equations (i) and (ii); equating coefficients of x, we have,

$$-(p+q) = \frac{b}{a}$$

$$(p+q) = -\frac{b}{a} \qquad i.e\ sum\ of\ roots$$

Equating their constant terms; we have $pq = \frac{c}{a}$ $\quad i.e\ product\ of\ roots$

Consequently, a quadratic equation can be expressed or written as;

$$x^2 - (sum\ of\ roots)x + (product\ of\ roots) = 0$$

Example 1: Find the quadratic equation whose roots are $\frac{1}{2}$ and $\frac{-2}{3}$

Solution

The equation is $\quad x^2 - \left(\frac{-2}{3} + \frac{1}{2}\right)x + \left(\frac{1}{2}\right)\left(\frac{-2}{3}\right) = 0$

$$x^2 - \left(\frac{-1}{6}\right)x - \frac{1}{3} = 0$$

Clear the fraction by multiplying through by the LCM of the denominator i.e. 6

$\Rightarrow \quad 6x^2 + x - 2 = 0$

EXAMPLE 2:

Construct the quadratic equation whose sum and product of roots are respectively $2 - \sqrt{3}$ and $2 + \sqrt{3}$.

Solution

$$x^2 - (2 - \sqrt{3} + 2 + \sqrt{3})x + \left((2 + \sqrt{3})(2 - \sqrt{3})\right) = 0$$

$$x^2 - (2 + 2)x + (4 - 3) = 0$$

$\Rightarrow$

$$x^2 - 4x + 1 = 0$$

CLASS ACTIVITY

1. Find the sum and product of the roots of the following quadratic equations:
   i. $x^2 - 2x = 10$
   ii. $x + \frac{1}{x} - 3 = 0$
2. Find the quadratic equation whose roots are;
   i. $-3\ and\ 2$
   ii. $1\frac{1}{2}\ and\ 1$
   iii. $-0.5\ and\ -0.25$
   iv. $1 - \sqrt{2}\ \ and\ \ 1 + \sqrt{2}$

WORD PROBLEMS LEADING TO QUADRATIC EQUATIONS

In this section, we shall consider some word problems leading to quadratic equation.

Example 1: If 63 is subtracted from twice the square of a number, the result is the same as five times of the number. Find the number

Solution

Let $x\ be$ the number,

Twice the square of the number is $2x^2$

The equation required is $2x^2 - 63 = 5x$

On rearranging, we have $2x^2 - 5x - 63 = 0$

This is then solved using any of the known methods (completing the square method or quadratic formula or factorization method)

$$\therefore the\ numbers\ are\ x = 7\ and\ x = -4\frac{1}{2}$$

Example 2: Find two consecutive odd numbers whose product is 399

Solution:

Let the smaller of the odd number be x, the other one will be $x + 2$ (note that this approach also goes for consecutive even numbers)

$$x(x + 2) = 399$$

$$x^2 + 2x - 399 = 0$$

Solving this equation, we obtain, $x = 19\ or\ x = -21$

If $x = 19$, then the other will be $x + 2 = 19 + 2 = 21$

If $x = -21$, then the other will be $-21 + 2 = -19$

Therefore, the numbers are -19 & -21 or 19 & 21.

CLASS ACTIVITY

1. A mother is 36 years old and her son is 6 years. When will the product of their ages be 451
2. The hypotenuse of a right angled triangle is one unit more than twice the shortest side. The third side is one unit less than twice the shortest side. Find the; (a) shortest side (b) hypotenus
3. A man is 3 times as old as his son, 8 years ago, the product of their ages was 112. Find their present ages.

PRACTICE EXERCISE

1. What value of k makes the given expression a perfect square? $\mathrm{m}^2 - 8\mathrm{m} + \mathrm{k}$.

   A. 2 B. 4 C. 8 D. 16 E. 64 (SSCE 1988)

2. Factorize: $5y^2 + 2ay - 3a^2$
   A. (5y-a)(y+3a) B. (5y+a)(y-3) C. $(5y^2+a)(2y-3a)$
   D. (y-a)(5y+3a) E. (y+a)(5y-3a) (SSCE 1988)

3. Factorize the following expression: $2x^2+x-15$.
   A. (2x+5)(x-3) B.(2x-5)(x-3) C. (2x-5)(x-3) D.(2x-3)(x+5)
   E. (2x+5)(x+3) (SSCE 1989)

4. If $4x^2-12x+c$ is a perfect square, find the value of c.
   A. 36 B. 9 C. 9/4 D. -9/4 E. 24 (SSCE 1990)

5. Factorize completely $4a^3-a$
   A. $a(4a^2-1)$ B. (2a-1)(2a+1) C. a(2a-1) D. $2a^2(a-1)$ E. (2a+1)(2a-1)
   (SSCE 1990)

6. Factorize $3a^2-11a+6$
   A. (3a-2)(a - 3) B. (2a-2)(a-3) C. (3a - 2)(a - 3) D. (3a + 2)(a -3)

   E. (2a-3)(a+2) (SSCE 1991)

7. Factorize $a^2 - 3a - 10$
   A. (a+5)(a+2) B. (a - 5)(a - 2) C. (a + 5)(a - 2) D.(a - 5)(a + 2)

8. Factorize the expressions: $2y^2+xy-3x^2$
   A. $2y(y+x)-3x^2$ B. $(2y-x)(2y+x)$ C. $(3x-2y)(x-y)$
   D. $(2y+3y)(y-x)$ E. $(x-y)(2y+3x^2)$

9. What must be added to the expression x^2-18 to make it a perfect square?
   A. 3 B. 9 C.36 D.72 E. 81
   (SSCE 1992)

10. Factorize the expression $2s^2-3st-2t^2$
    A. (2s-t)(s+2t) B. (2s+t)(s-2t) C. (s+t)(2s-t) D. (2s+t)(s-t)
    E. (2s+t)(s+2t) (SSCE 1993)

11. Factorize $2x^2-2x+45$
    A. $(2x-9)(x-5)$ B. $(2x-15)(x-3)$ C. $(2x+15)(x-3)$
    D. $(2x-15)(x-3)$ E. $(2x-9)(x+5)$ (SSCE 1993)

12. Factorize $6x^2+7xy-5y^2$

A. $(6x+5)(x-y)$ B. $(2x+5y)(3x-y)$ C. $(3x+y)(2x-y)$

D. $(3x+5y)(2x-y)$ E. $(2x+y)(3x-5y)$ (SSCE 1994)

ASSIGNMENT

1. Factorize $x^2+4x-192$. (SSCE 1990)

2. Factorize $2e^2-3e+1$ (SSCE 1990)

3. Find the value of m which makes x^2+8x+m a perfect square. (SSCE 1990)

4. Given that $(2x+7)$ is a factor of $2x^2+3x-14$, find the other factor (SSCE 2000)
5. Derive an equation whose coefficients are integers and which has roots of $^1/_2$ and -7.
6. Three years ago a father was four times as old as his daughter is now. The product of their present ages is 430. Calculate present ages of the daughter and the father.(SSCE 1989)
7. Solve the equation correct to two decimal places: $2x^2+7x-11=0$. (SSCE 1991)
8. Using the method of completing the square, find the roots of the equation
$$x^2-6x+7=0$$
Correct to 1 decimal place. (SSCE 1995)
9. The product of two consecutive positive odd numbers is 195. By constructing a quadratic equation and solving it, find the two numbers. (SSCE 1995)
10. The area of a rectangular floor is $135m^2$. One side is $1.5m$ longer than the other. Calculate
    (a) The dimensions of the floor
    (b) If it costs $N250.00$ per square metre to carpet the floor and only $N2000.00$ is
    available, what area of the floor can be covered with the carpet?
    (SSCE 1998)
11. A rectangular lawn of length $(0+5)$ metres is $(x-2)$ metres wide. If the diagonal is
    $(x+6)$ metres, find;
    (i) the value of x

(ii) the area of lawn.

12. The sum of the ages of a woman and her daughter is 46 years. In 4 years' time, the ratio of their ages will be 7:2. Find their present ages. (SSCE 2001)

13. The sides of a rectangular floor are $xm\ and\ (x+7)m$. The diagonal is $(x+8)m$.
Calculate in metres;
(a) the value of x,
(b) the area of the floor. (SSCE 2001)

14. Solve correct to two decimal places the equation $4x^2 = 11x + 21$. (SSCE2008)

15. The lengths, in cm of the sides of a right angled-triangle are x, $(x+2)\ and\ (x+1)$ where $x > 0$. Find, in cm, the length of its hypotenuse.

KEYWORDS:
Quadratic equation, Factorize, perfect square, coefficient, roots, quadratic formula, etc.

WEEK 6:
DATE……………………….
Subject: Mathematics
Class: SS 2
TOPIC: Simultaneous Linear and Quadratic Equation
Content:

- Simultaneous linear equations (Revision).
- Solution to linear and quadratic equations.
- Graphical solution of linear and quadratic equations.
- Word problems leading to simultaneous equations.
- Gradient of curve.

SIMULTANEOUS LINEAR EQUATIONS (REVISION)

Simultaneous linear equations (revision)
Recall: Simultaneous means happening or done at the same time i.e. following each other. It can be solved either graphically or algebraically. Algebraically involves using either substitution or elimination methods
Example 1: Solve the following pairs of simultaneous equation

$$2x - y = 8$$

$$3x + y = 17$$

Solution:
ELIMINATION METHOD
(a) $2x - y = 8$, $3x + y = 17$

On adding both equations, we have

$5x = 25$

$x = 5$

Substituting for ? in equation (1)

$2x - y = 8$

$2(5) - y = 8$

$y = 2$

$\Rightarrow\ x = 5\ , y = 2$

Using **substitution** method;

(a) $2x - y = 8$ ………….(1)

,$3x + y = 17$ …………(2)

In (2), $y = 17 - 3x$ ……………….(3)

Substituting y in equation (1), we have

$$2x - (17 - 3x) = 8$$
$$2x - 17 + 3x = 8$$
$$5x = 25$$
$$x = 5$$

In (3), $y = 17 - 3(5)$

$= 17 - 15$

$y = 2$

EXAMPLE 2: Solve the pairs of equations;

$\frac{27^x}{81^{x+2y}} = 9$ and $x + 4y = 0$

Solution;

$$\frac{3^{3x}}{3^{4(x+2y)}} = 3^2$$

Recall; $\frac{2}{a^2} = \frac{2}{1} \times \frac{1}{a^2} = 2a^{-2}$

$$\Rightarrow \ 3^{3x} \times 3^{-4(x+2y)} = 3^2$$

$$3^{3x-4(x+2y)} = 3^2$$

Comparing the powers of 3, we have,

$$3x - 4(x + 2y) = 2$$

$$3x - 4x - 8y = 2$$

$$-x - 8y = 2 \quad \dots\dots(i)$$

$$x + 4y = 0 \quad \dots\dots\dots(ii)$$

Solving equations (i) and (ii) we obtain values for $x\ and\ y$

i.e $x = 2\ and\ y = \frac{-1}{2}$

CLASS ACTIVITY

1. Solve the following pairs of simultaneous equations
   (a) $3y - 2x = 21$
   ,$5x + 4y = 5$
   (b) $\frac{125^x}{25} = 5^{2x-y}$, $3^{2x-5y} = 81$

2. Solve the questions below; (WAEC)
   (a) $2^{x+2y} = 1$, $3^{2x+y} = 27$
   (b) $2x + 3y - 1 = 3x + y + 7 = x + 2y$
   (c) $2.32a + 1.44b = 15.6$
   ,$4.8a - 1.92b = 2.88$

SOLUTION OF LINEAR AND QUADRATIC EQUATIONS

When solving a simultaneous equation involving one linear and one quadratic such as x^2, y^2 (or xy) graphical or substitution not elimination method is frequently used.

EXAMPLE 1: Solve the pair of equation; $3x^2 + 5xy - y^2 = 3$ ………………. (i)

$x - y = 4$ …………………(ii)

Solution

from equation (ii),

$$x - y = 4$$

$$x = y + 4 \qquad \ldots\ldots\ldots\ldots\ldots (iii)$$

Substituting for x in (i),

$$3x^2 + 5xy - y^2 = 3$$

$$3(y+4)^2 + 5(y+4)y - y^2 = 3$$

$$3(y^2 + 8y + 16) + 5y^2 + 20y - y^2 = 3$$

$$3y^2 + 24y + 48 + 5y^2 + 20y - y^2 = 3$$

Collecting like terms and rearranging the equation; we have

$$7y^2 + 44y + 45 = 0$$

Using any of the methods learnt previously for solving quadratic equations, we have that

$$y = -5 \quad or\ y = \frac{-9}{7}$$

$$substituting\ for\ y\ in\ equation\ (iii)$$

When $y = -5$,

$$x = -5 + 4$$

$$= -1$$

When $y = \frac{-9}{7}$

$$x = \frac{-9}{7} + 4$$

$$= \frac{19}{7}$$

$$\therefore (-1, -5); \left(\frac{19}{7}, \frac{-9}{7}\right)$$

EXAMPLE 2: Solve completely; $m^2 - n^2 = 29$

$$m + n = 7$$

Solution:

Note: $m^2 - n^2 = (m+n)(m-n) \quad difference\ of\ two\ square$

$$(m+n)(m-n) = 29 \qquad \ldots\ldots (i)$$

$$(m+n) = 7 \qquad \text{..........(ii)}$$

Substituting for m+n in (ii),

$$7(m-n) = 29$$

$$7m - 7n = 29 \qquad \ldots\ldots (iii)$$

Multiply equation (ii) by 7

$$7m + 7n = 49 \quad \ldots\ldots..(iv)$$

Add (iii) & (iv)

$$14m = 78$$

$$m = \frac{78}{14} = \frac{39}{7}$$

Substitute for m in (ii),

$$\frac{39}{7} + n = 7$$

$$n = 7 - \frac{39}{7}$$

$$n = \frac{10}{7}$$

$$\therefore m = \frac{39}{7} \quad , \quad n = \frac{10}{7}$$

CLASS ACTIVITY

1. Solve: (a) $9y^2 + 8x = 12$
   $2x + 3y = 4$

   (b.) $y = 5x + 3$
   $x^2 - y^2 + 45 = 0$ $\qquad (WAEC)$

2. (a)A man is x years old while his son is y years old. The sum of their ages is equal to twice the difference of their ages. The product of their ages is 675. Write down the equations connecting their ages and solve the equations in order to find the ages of the man and his son. (WAEC)

   (b)Solve: $x^2 - 4y^2 = 5$
   $5 - 2y + x = 0$

GRAPHICAL SOLUTION OF LINEAR AND QUADRATIC EQUATIONS

Note: To use graphical method, plot the graphs of each equation on the same axes. Then read off the x & y coordinates of the points where both lines cross to obtain the required solution.

Example 1:

Solve the following simultaneous equations graphically:

$$y = 2x^2 + x - 5 \ , \ y = 3 - x$$

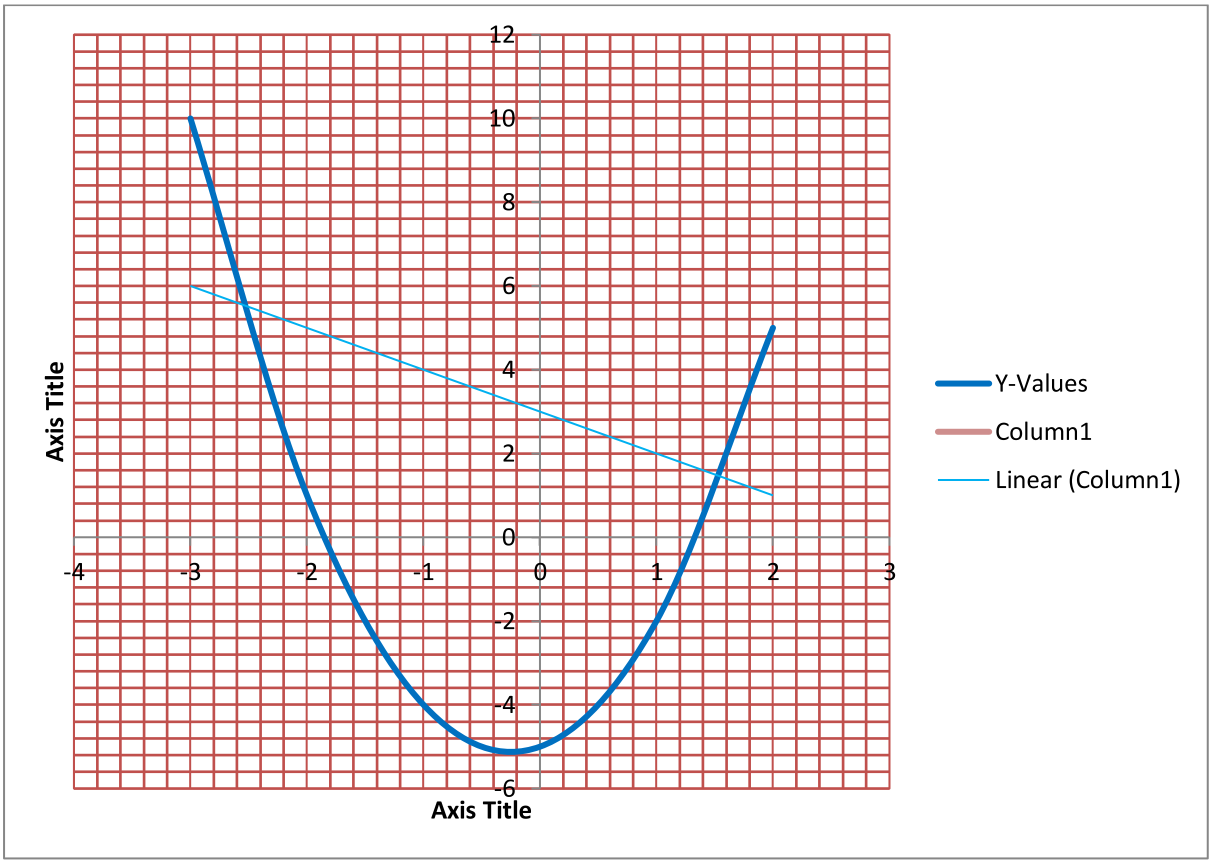
Solution

Tables of values ($y = 2x^2 + x - 5$)

$\boldsymbol{x}$	-3	-2	-1	0	1	2
$2x^2$	18	8	2	0	2	8
x	-3	-2	-1	0	1	2
-5	-5	-5	-5	-5	-5	-5
$\boldsymbol{y}$	10	1	-4	-5	-2	5

$y = 3 - x$

x	-2	0	2
y	5	3	1



From the graph, x≃1.6 and y≃1.5 or x≃ -1.6 and y=5.6

EXAMPLE 2:

Draw the graphs of $y = 3x^2 - 5x - 8$

(a) Use your graph to solve the following equations;

(i) $3x^2 - 5x - 8 = 0$

(ii) $3x^2 - 5x - 20 = 0$

(iii) $3x^2 - 8x - 6 = 0$

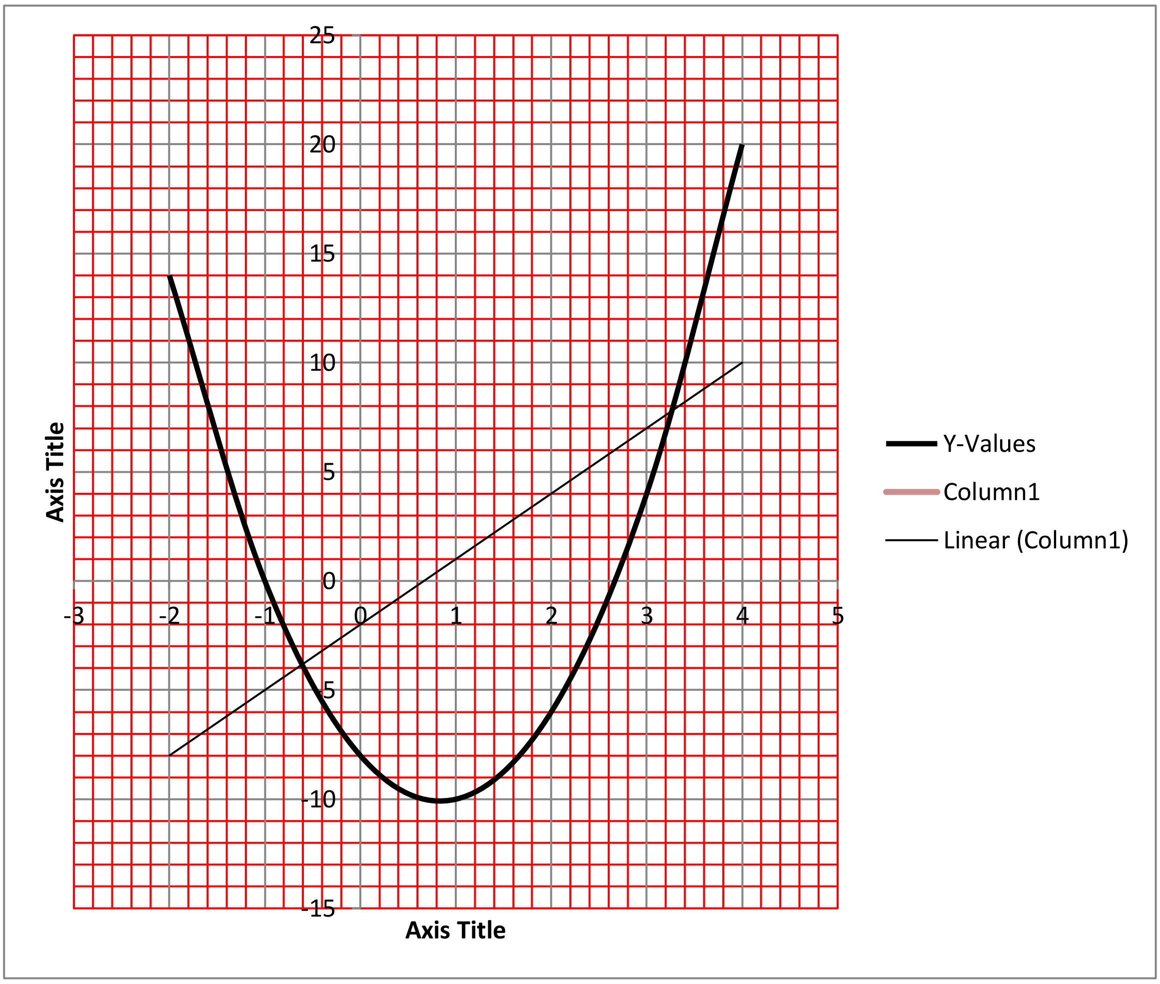
(b) Find the minimum value of $y = 3x^2 - 5x - 8$

Solution

Tables of values for $y = 3x^2 - 5x - 8$

x	-2	-1	0	1	2	3	4
$3x^2$	12	3	0	3	12	27	48

$-5x$	10	5	0	-5	-10	-15	-20
-8	-8	-8	-8	-8	-8	-8	-8
$\boldsymbol{y}$	14	0	-8	-10	-6	4	20



(i) From the graph, $x = -1 \; or \; 2.7$

(ii) For $3x^2 - 5x - 20 = 0$, we have to make it look like the first graphical equation given. So we add 12 to both sides;

$$3x^2 - 5x - 20 + 12 = 0 + 12$$

$$3x^2 - 5x - 8 = 12$$

The line $y = 12$ is drawn on the graph $\therefore x = -1.9 \; or \; 3.5$

(iii) For this, we add $3x$ to both sides ; $3x^2 - 8x + 3x - 6 = 0 + 3x$

$$3x^2 - 5x - 6 = 3x$$

Then subtract 2 from both sides as well,

$$3x^2 - 5x - 6 - 2 = 3x - 2$$
$$3x^2 - 5x - 8 = 3x - 2$$

The line $y = 3x - 2$ has tables of values as follows

x	-2	0	2
y	-8	-2	4

From the graph above, $x = -0.6$ *or* 3.3

(b) The minimum value of $3x^2 - 5x - 8$ is where the turning point of the curve is, i.e at $y = -10$ and it occurs at $x = 0.82$

CLASS ACTIVITY

1. Solve the following simultaneous equations graphically;
   (a) $y = x^2 - x - 8$, $y = 2x + 3$
   (b) $y = 3x^2 + 4x - 8$, $y + 5x + 4 = 0$

2. (a) Copy and complete the following table of values for the relation;

$$y = 2 + x - x^2$$

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5
y			0					1.25

.(b) Draw the graph of the relation using a scale of 2cm to 1 unit on each axis.

(c) Using the same axes, draw the graph of $y = 1 - x$

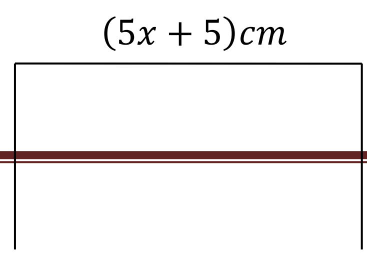
(d) From your graphs determine the roots of the equation $1 + 2x - x^2 = 0$ (WAEC)

WORD PROBLEMS LEADING TO SIMULTANEOUS EQUATIONS

Note: Read the question carefully so as to understand what you need to find. Choose letters for the unknowns. Solve the equations formed from the information.

Example 1:

The diagram below is a rectangle. Find the perimeter and the area of the rectangle



$(4x + 1)cm$ $(5x - y)cm$

$(x + 7y)cm$

solution

Opposite sides of a rectangle are equal, so we equate each of them and we obtain a simultaneous equation;

$$5x + 5 = x + 7y$$
$$5x - x - 7y = -5$$
$$4x - 7y = -5 \quad \dots\dots\dots(i)$$

$$4x + 1 = 5x - y$$
$$4x - 5x + y = -1$$
$$-x + y = -1 \quad \dots\dots\dots..(ii)$$

With the previous knowledge on simultaneous equation, solving equations (i) & (ii)

We obtain $x = 4\ , y = 3$

$$\therefore 5x + 5 = 5(4) + 5 = 25$$
$$4 + 7(3) = 4 + 21 = 25$$
$$4(4) + 1 = 16 + 1 = 17$$
$$5(4) - 3 = 20 - 3 = 17$$

Perimeter = 2L + 2B or 2(L+B)

= 2(25) + 2(17)

= 50 + 34

= 84cm

Area = LB

= 25cm x 17cm

= 425cm^2

EXAMPLE 2:

A number is made up of two digits. The sum of the digits is 11. If the digits are interchanged, the original number is increased by 9. Find the number. (WAEC)

Solution:

Let the digits be $p \ and \ q$, where p is the tens digit and q is the unit part, so the number is 10p+q. when the digits are interchanged, the tens digit becomes q and the unit digit becomes p. hence the number is 10q+p

But , $10q + p = 10p + q + 9$

$$10q + p - 10p - q = 9$$

$$9q - 9p = 9$$

Divide through by 9

$$q - p = 1 \ \dots\dots\dots (i)$$

We were told that the sum of the digits 11, i.e $q + p = 11 \quad \dots..(ii)$

Adding (i) & (ii), 2q = 12

q = 6

substituting for q in (ii)

p + 6 = 11

p = 5

∴ the number = $10p + q = 10(5) + 6 = 56$

CLASS ACTIVITY

1. The sum of two numbers is 110 and their difference is 20. Find the two numbers
2. The perimeter of a rectangular lawn is 24m. if the area of the lawn is $35cm^2$, how wide is the lawn? (JAMB)

LINEAR GRAPH (REVISION)

Recall that any equation whose highest power of the unknown is one is a linear equation. The expression y = ax + b where a & b are constants, represents a general linear function. Its graph is a straight line. To plot the graph of a linear function, two distinct points are sufficient to draw the straight line graph.

Example 1:

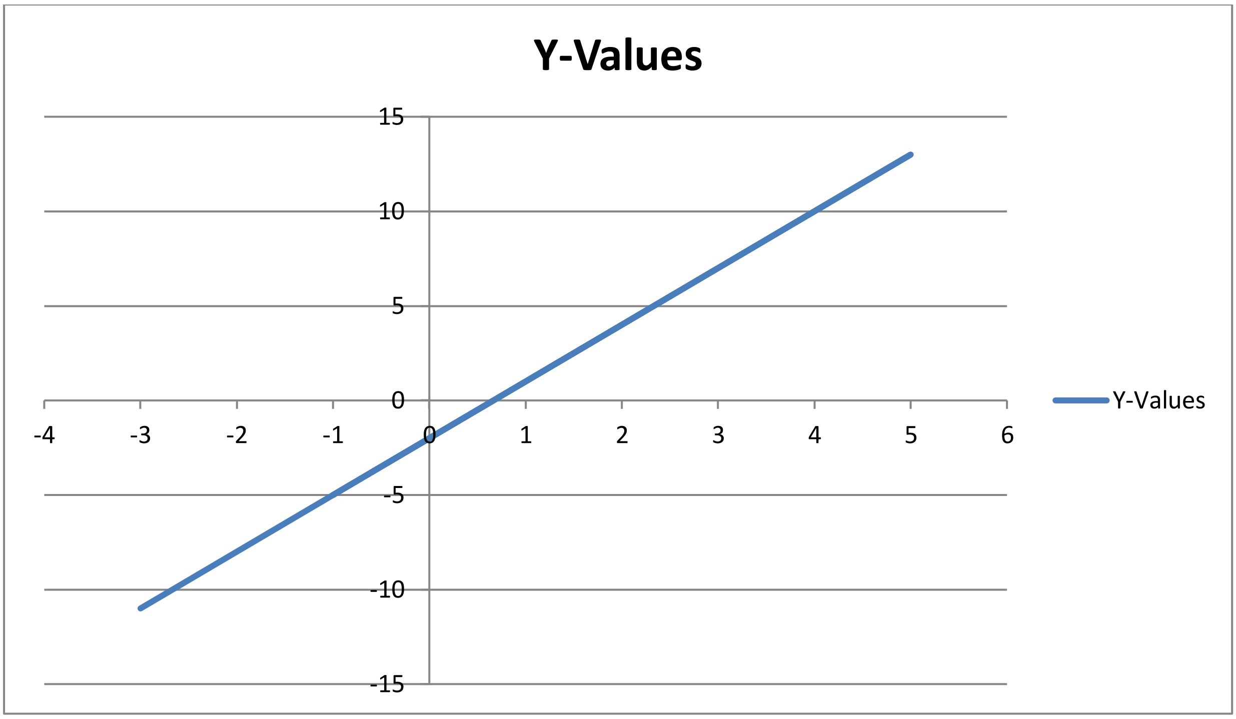
Draw the graph of y = 3x – 2

Solution: A table of three points is prepared below

x	-1	2	5
y	-5	4	13

Scale: 1cm to 1 unit on x-axis

1cm to 5 units on y-axis



GRADIENT OF A STRAIGHT LINE X & Y INTERCEPTS

The y-intercept of a line is the point $(0, b)$ where the line intersects the y-axis. To find b, substitute 0 for x in the equation of the line and solve for y.

The x-intercept of a line is the point (a,0) where the line intersects the x-axis. To find a, substitute 0 for y in the equation of the line and solve for x.

Example 1:

Solve the equation $4x - 2y = 8$, using the intercept method

Solution: To find the x-intercept, we let y be 0 and solve for x,

4x – 2(0) = 8

4x = 8

X = 2, the intercept is (2,0)

To find the y-intercept, we let x be 0 and solve for y

4(0) – 2y = 8

Y = -4, the intercept is (0,-4)

The graph of the given equation should then be plotted.

Gradient of a straight line: The gradient of a non-vertical line is a number that measures the line's steepness. The gradient of a straight line is the rate of change of y compared with x. For example, if the gradient is 3, then for any increase in x, y increases three times as much. We can calculate gradient by picking two points on the line and writing the ratio of vertical change (change in y) to the corresponding horizontal change (change in x).

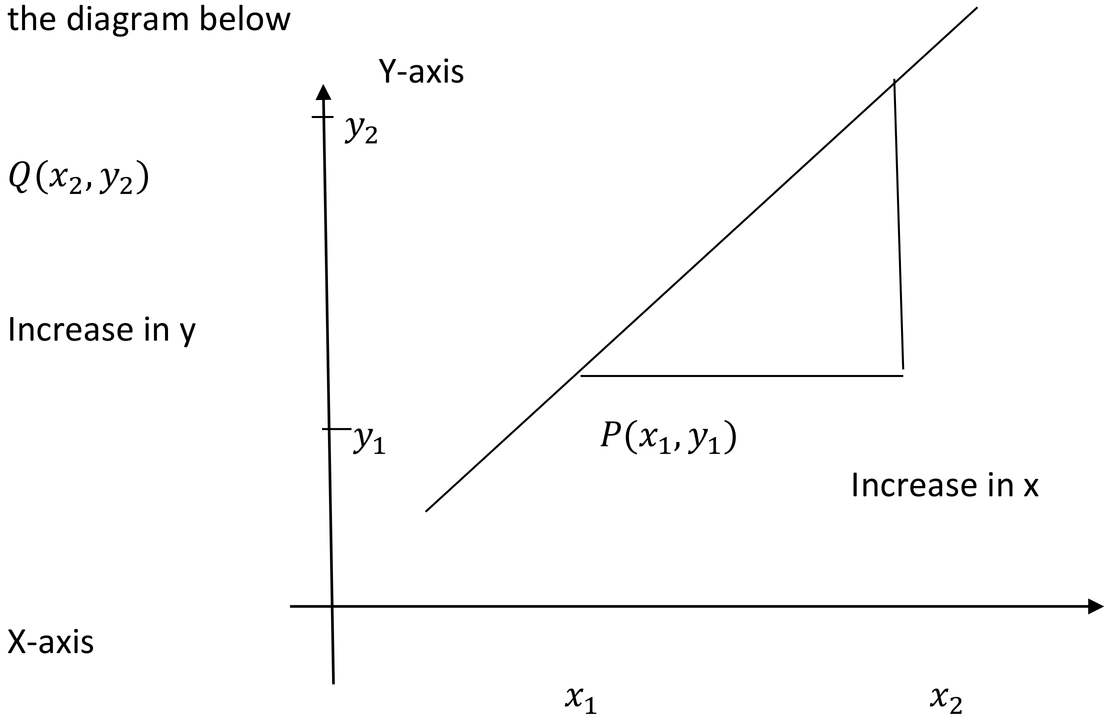
The gradient formula: The gradient of a straight line is generally represented by m.

∴ gradient of any line (m) = $\frac{vertical\ change}{horizontal\ change}$

$= \frac{change\ in\ y}{change\ in\ x}$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

Consider moving from point P to another point Q on a straight line PQ, as in the diagram below



The gradient of line PQ passing through points $P(x_1, y_1)$ *and* $Q(x_2, y_2)$ is;

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 2:

Find the gradient of the lines joining the following pairs of points(-3,2) , (4,4)

Solution: from the points $x_1 = -3, \quad y_1 = 2, \quad x_2 = 4, \quad y_2 = 4$

Gradient of AB = $\frac{increase\ in\ y}{increase\ in\ x}$

$= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{4-2}{4--3} = \frac{4-2}{4+3}$

$= \frac{2}{7}$

CLASS ACTIVITY

Find the gradients of the lines joining the following pairs of points

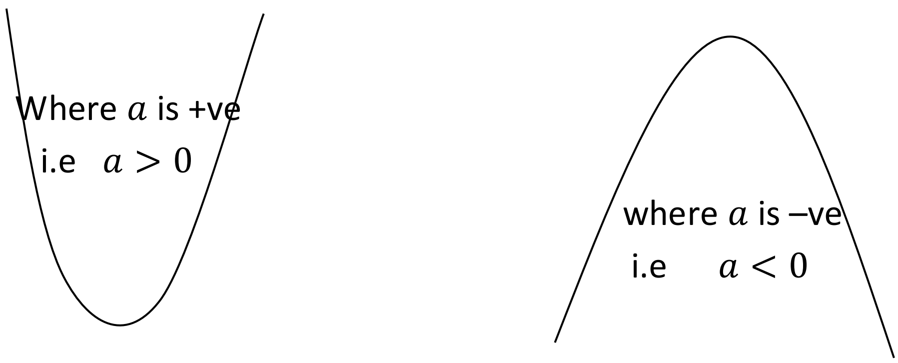
1. (9,7) , (2,5)
2. (-4,-4) , (-1,5)

3. (7,-2) , (-1,2)

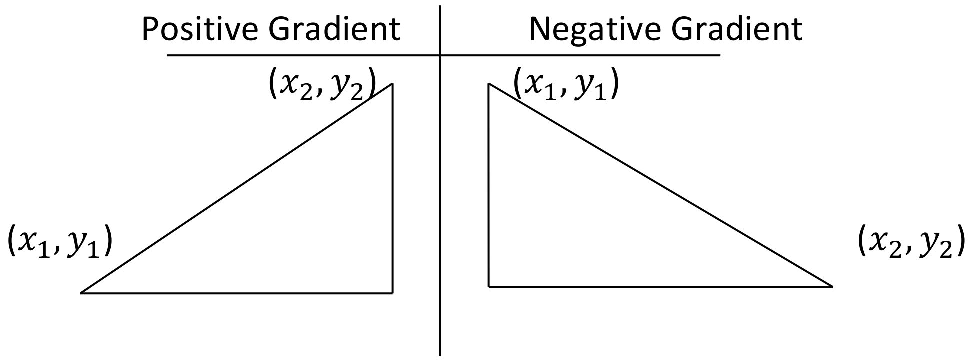
GRADIENT OF A CURVE

The gradient of any point on a curve is the gradient of the tangent to the curve at that point. The tangent must be produced at equidistant to the point.

Note: there are two major types of quadratic curves from the general form $ax^2 + bx + c$



There are two types of gradients namely: positive and negative gradients



Note: The gradient of a straight line is the same at any point on the line, but the gradient of a curve changes from point to point.

Example 1:

Copy and complete the following tables of values for $y = 2x^2 - 9x - 1$

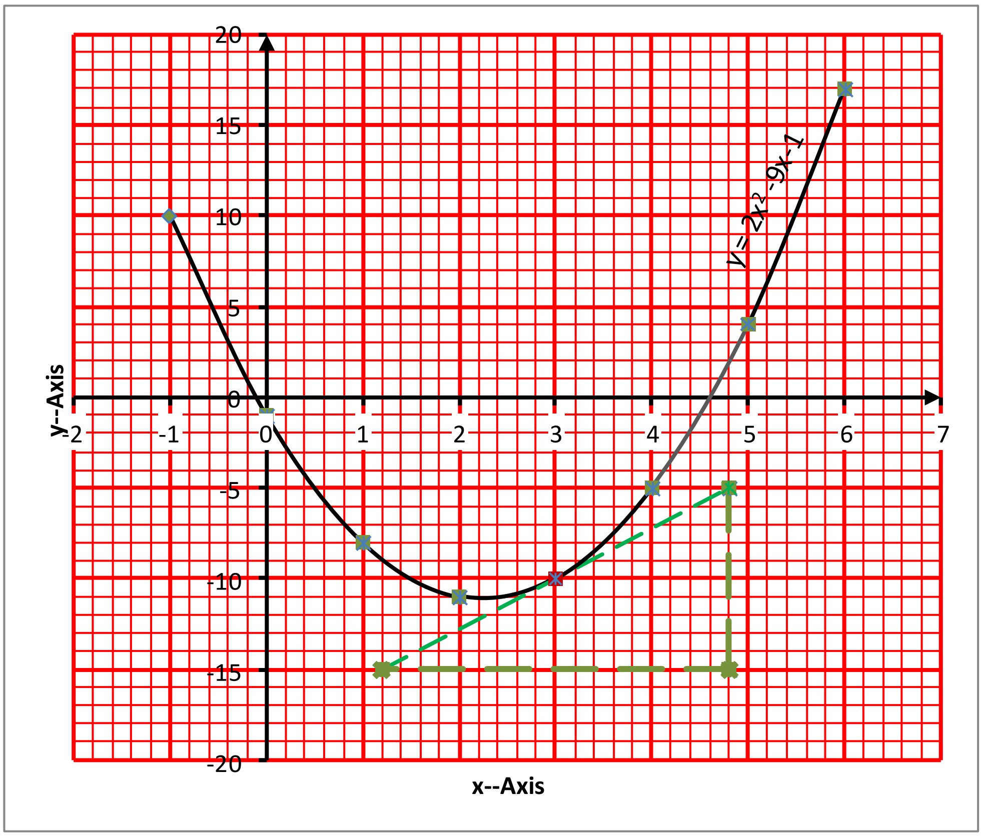
$\boldsymbol{x}$	-1	0	1	2	3	4	5	6
$\boldsymbol{y}$		-1	-8					17

Using a scale of 1cm to 5units on y-axis and 1cm to 1unit on x-axis, draw the graph of $y = 2x^2 - 9x - 1 \ for - 1 \leq x \leq 6$

Use the graph to find the gradient of the curve $y = 2x^2 - 9x - 1$ at $x = 3$ $\quad (WAEC)$

Solution

x	-1	0	1	2	3	4	5	6
$2x^2$	2	0	2	8	18	32	50	72
$-9x$	9	0	-9	-18	-27	-36	-45	-54
-1	-1	-1	-1	-1	-1	-1	-1	-1
y	10	-1	-8	-11	-10	-5	4	17



(b) At point x = 3 traced to the curve, we draw an equidistant line as shown, then take the slope.

Slope = $\frac{change\ in\ y}{change\ in\ x}$ = $\frac{-5-(-15)}{4.8-1.2}$ = 2.78

Example 2: The table below is for the curve $y = mx^2 - 2x + 5$

x	-4	-3	-2	-1	0	1	2	3
y	-3	2	5	6	5	2	-3	-10

(a) Find the value of the constant m

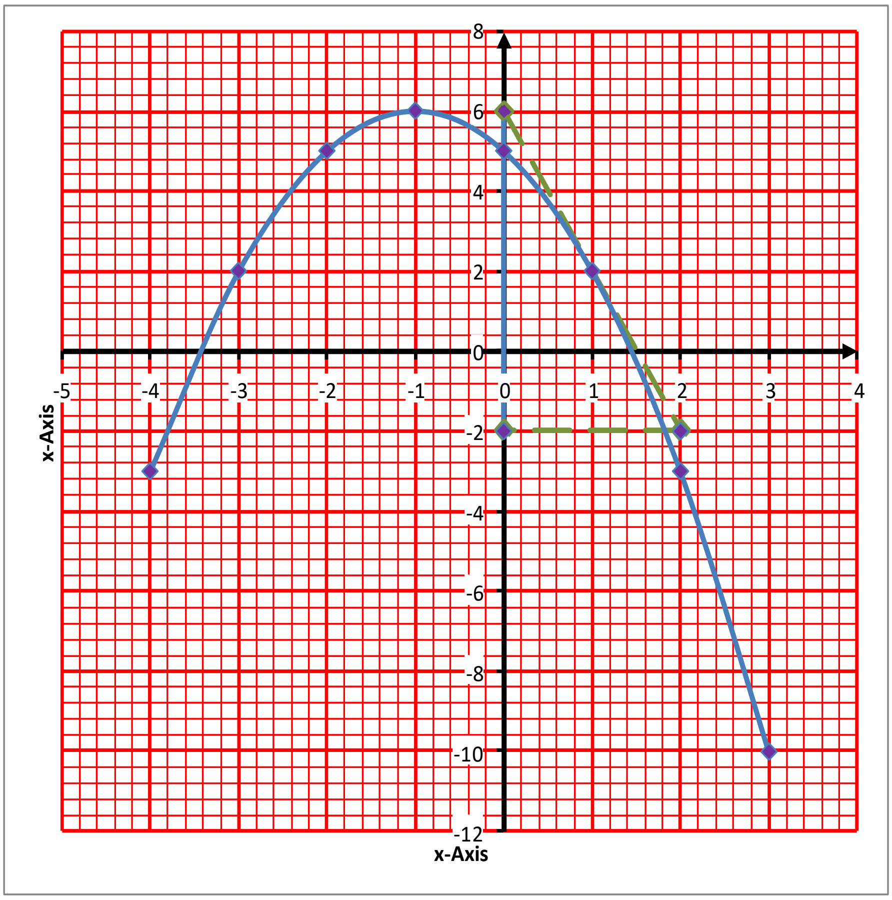
(b) Using a scale of 1cm to represent 1unit on the x-axis and 1cm to represent 2units on the y-axis, draw the graph of $y = mx^2 - 2x + 5$ for $-4 \leq x \leq 3$

(c) Use the graph to obtain the gradient of the curve at point $x = 1$ $(WAEC)$

Solution

(a) Using any two values of x and y we can solve for 'm', let us make use of (2, -3) i.e $x = 2$, $y = -3$

$$y = mx^2 - 2x + 5$$
$$-3 = m(2)^2 - 2(2) + 5$$
$$-3 = 4m - 4 + 5$$
$$\Rightarrow m = -1$$



(c.) Gradient = $\frac{change\ in\ y}{change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-6}{2-0} = \frac{-8}{2} = -4$

CLASS ACTIVITY

1. (a) Copy and complete the following table of values for the relation $y = 2x^2 - 7x - 3$.

x	-2	-1	0	1	2	3	4	5
y	19		-3		-9			

(b) Using 2cm to 1 unit on the x-axis and 2cm to 5 units on the y-axis, draw the graph of $y = 2x^2 - 7x - 3$ for -2$\leq x \leq 5$

2. (a)from your graph, find the:

i. minimum value of y

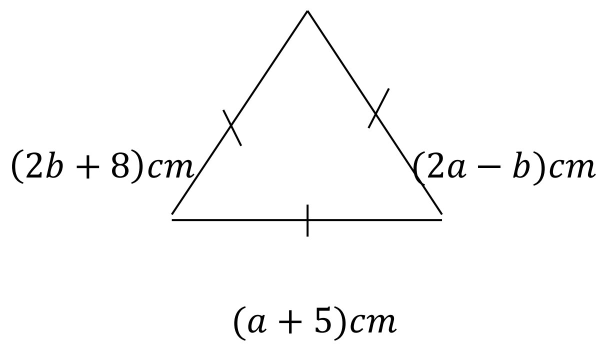
ii. gradient of the curve at x=1

(b) By drawing a suitable straight line, find the values of x for which

$2x^2 - 7x - 5 = x + 4$ (SSCE 2004)

PRACTICE EXERCISE

1. Find a two digit number such that three times the tens digit is 2 less than twice the units digit and twice the number is 20 greater than the number obtained by reversing the digits.(JAMB)
2. Find the perimeter and the area of the equilateral triangle in the diagram below.



3. Five years ago, a father was 3 times as old as his son, now their combined ages amount to 110 years, how old are they?
4. Solve for (x,y) in the equations;

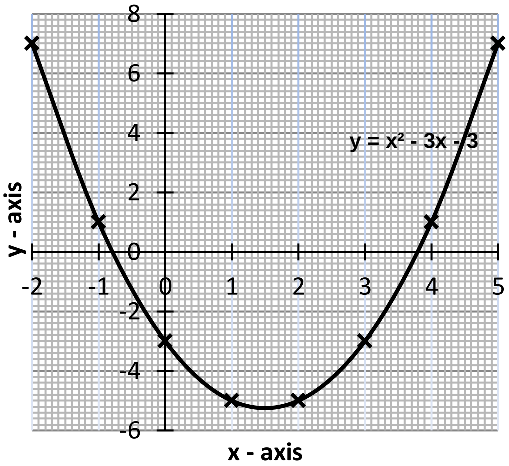
$2x + y = 4$

$x^2 + xy = -12$ $(jamb)$

5. Draw the graph of $y = 2x^2 - 3$ for values of x from -4 to 4. Using your graph, find
   (a) The roots of (i) $2x^2 - 3 = 0$ (ii) $2x^2 - 3 = 10$, giving your answer correct to one decimal place
   (b) The least value of y and the corresponding value of x (WAEC)

ASSIGNMENT

OBJECTIVE QUESTIONS



The graph above represents the relation $y = x^2 - 3x - 3$. Use it to answer questions 1 and 2.

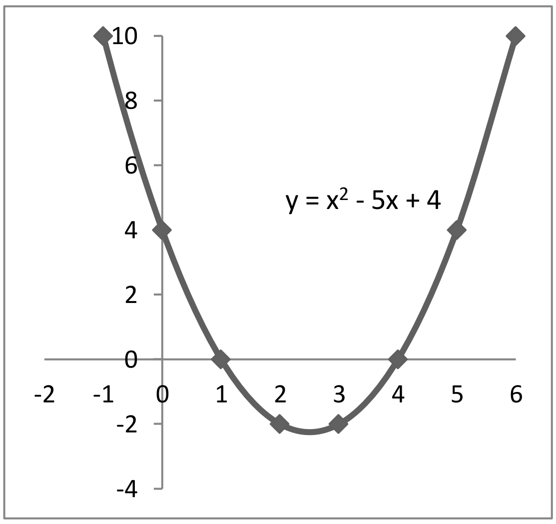
1. Find the values of x for which $x^2 - 3x = 7$

   A. -1.55, 4.55 B. 1.55, -4.55 C. -1.55, -4.55 D. 1.55, 4.55

   (SSCE 2011)

2. What is the equation of line of symmetry of the graph?

   A. $y = 0.5$ B. $x = 1.0$ C. $x = 1.5$ D. $y = 4.6$ (SSCE 2011)



The following is a graph of a quadratic function. Use it to answer question 3 and 4.

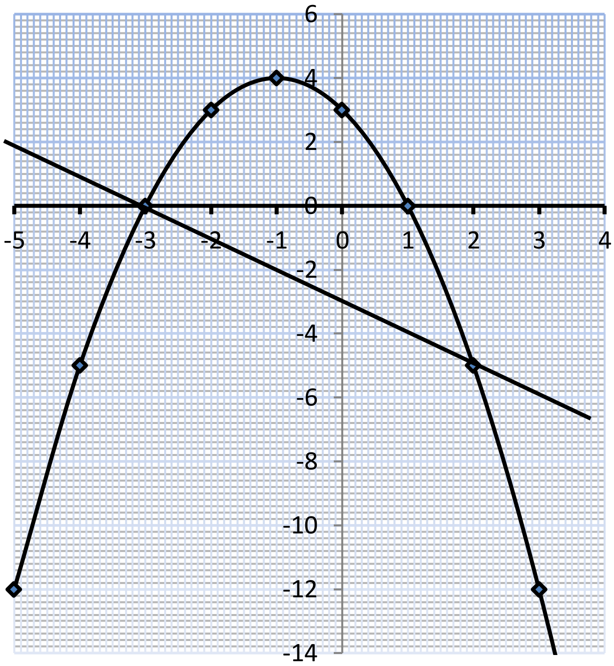
3. Find the co-ordinates of point **P.**

   A. (0, 4) B. (1, 4) C. (0, -4) D. (-4, 0) (SSCE2009)

4. Find the values of x when $y = 0$.

A. 1, 3 B. 1, 4 C. 2, 3 D.1, 6 (SSCE 2009)

Use the graph below to answer question below



5. What is the equation of the curve?

A. $y = 3 + 2x - x^2$ B. $y = 3 - 2x - x^2$ C. $y = x^2 + 2x - 3$ D. $y = x^2 + 2x + 3$ (SSCE 2006)

THEORY QUESTIONS

2. Draw the graph of $y = 4x^2$ +25 and $y = 3x + 2$for $-3 \leq x \leq 3$.

   Use the graph(s) to:

   i. find the roots of the equation $y = 4x^2$ +25 and $y = 3x + 2$

   ii. determine the line of symmetry of the curve $y = 4x^2$ +25.

3. (a) Copy and complete the table of values for the relation $y = -x^2 + x + 2$ for $-3 \leq x \leq 3$.

X	-3	-2	-1	0	1	2	3
Y		-4		2			-4

   (b)Using scales of 2cm to 1 unit on the x-axis and 2cm to 2 units on the y-axis.

   Draw a graph of the relation $y = -x^2 + x + 2$

(c) From the graph find the:

i. minimum value of y

ii. roots of the equation$x^2 - x - 2 = 0$

iii. gradient of the curve at $x = 0.5$ (SSCE 2010)

4. (a) Copy and complete the table

$y = x^2 - 2x - 2\ for - 4 \leq x4.$

X	-4	-3	-2	-1	0	1	2	3
Y	22				-2		1	6

(b) Using a scale of 2cm of 1 unit on the x-axis and 2cm to 5 units on the y-axis, draw the graph of $y = x^2 - 2x - 2$.

(c) Use your graph to find:

i. the roots of the equation $x^2 - 2x - 2 = 0$

ii. the values of x for which $x^2 - 2x - 4\frac{1}{2} = 0$

iii. the equation of the line of symmetry of the curve.

(SSCE 2005)

5. (a) Copy and complete the following table of values for the relation $y = 2x^2 - 7x - 3$.

x	-2	-1	0	1	2	3	4	5
y	19		-3		-9			

(b) Using 2cm to 1 unit on the x-axis and 2cm to 5 units on the y-axis, draw the graph of $y = 2x^2 - 7x - 3$ for -2$\leq x \leq 5$

(c)from your graph, find the:

i. minimum value of y

ii. gradient of the curve at x=1

(d) By drawing a suitable straight line, find the values of x for which $2x^2 - 7x - 5 = x + 4$ (SSCE 2004)

6. The table shows the values of the relation $y = 11 - 2x^2$ for -4$\leq x \leq 3$

x	-4	-3	-2	-1	0	1	2	3
y	-13				11			

(a) Copy and complete the table

(b) Using 2cm to 1 unit on the x-axis and 2cm to 5 units on the y-axis, draw the graph of $y = 11 - 2x - 2x^2$

(c) Use your graph to find:

i. the roots of the equation $11 - 2x - 2x^2 = 0$

ii. the value for which $3 - 2x - 2x^2 = 0$

iii. the gradient of the curve at x=1. (SSCE 2003)

6. a. Draw the table of values for the relation $y = 3 + 2x - x^2$for the interval $-3 \leq x \leq 4$

b. Using a scale of 2cm to 1 on the x-axis and 2cm to 2 units on the y-axis, draw the graphs of:

i.$y = 3 + 2x - x^2$

ii.$y = 2x + 3$ for $-3 \leq x \leq 4$.

c. Use your graph to find:

i.The roots of the equation $3 + 2x - x^2 = 2x + 3$

ii. The gradient of $y = 3 + 2x - x^2$ at $x = -2$

7. a. Draw the table of values for the relation y=$x^2$for the interval $-3 \leq x \leq 4$

b. Using a scale of 2cm to 1 on the x-axis and 2cm to 2 units on the y-axis, draw the graphs of:

i. $y = x^2$

ii. $y = 2x + 3$for$-3 \leq x \leq 4$.

c. Use your graph to find:

i. The roots of the equation $x^2 = 2x + 3$

ii. The gradient of $y = x^2$ at $x = -2$ (SSCE 2001)

8. a. Copy and complete and the following table of values for the relation $y = x^2 - 2x - 5$.

x	-3	-2	-1	0	1	2	3	4
y		-2		-6	-2	3	10	

b. Draw the graph of the relation $y = x^2 - 2x - 5$; using a scale of 2cm to 1 unit on the x-axis,

and 2cm to 2 units on the y-axis

c. Using the same axes, draw the graph of $y = 2x - 3$

d. Obtain in the form $ax^2 + bx + c = 0$ where a, b and c are integers, the equation which is

satisfied by the x-coordinate of the points of intersection of the two graphs.

e. from your graphs, determine the roots of the equation obtained in d. above.(SSCE 2000)

9. (a) Copy and complete the values for the relation $y = 5 - 7x - 6x^2$ for $-3 \leq x \leq 2$.

x	-3	-2	-1	0	0.5	0	1	2
y	-28		6			5		

(b) Using a scale of 2cm to 1 unit on the x-axis and 2cm to 5 units on the y-axis, draw the:

i. Graph of $y = 5 - 7x - 6x^2$

ii. line y=3 on the same axis

(c) Use your graph to find the

i. roots of the equation $2 - 7x - 6x^2 = 0$

ii. Maximum value of $y = 5 - 7x - 6x^2$ (SSCE 1998)

10. a. Copy and complete the following table of values for $y = 2x^2 - 9x - 1$

x	-1	0	1	2	3	4	5	6

y	-1	-8	-11					

b. Using a scale of 2cm to represent 1 unit on the x axis and 2cm to represent 5 units on

the y-axis, draw the graph of $y = 2x^2 - 9x - 1$.

c. Use your graph to find the:

i. Roots of the equation $2x^2 - 9x = 4$, correct to one decimal place

ii. Gradient of the curve $y = 2x^2 - 9x - 1$ at x=3 (SSCE 1994)

KEYWORDS: gradient, line of symmetry, linear graph, quadratic graph, coordinator, x-axis, y-axis, table of values etc.

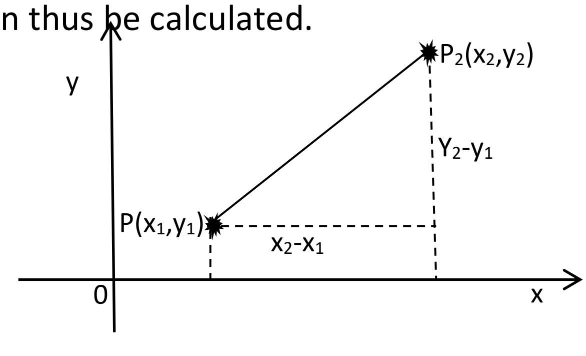
WEEK 7:
DATE……………………….
Subject: Mathematics
Class: SS 2
TOPIC:
MID TERM BREAK

WEEK 8:
DATE……………………….
Subject: Mathematics
Class: SS 2
TOPIC: Coordinates geometry of straight lines
Content:

- Distance between two points.
- Midpoint of line joining two points.
- Gradients and intercept of a straight line.
- Determination of equation of a straight line.
- Angle between two intersecting straight lines.
- Application of linear graphs to real life situation.

DISTANCE BETWEEN TWO POINTS

Let $P_1(x_1, y_1) and\ P_2(x_2, y_2)$ be two distinct points. The distance d between them can thus be calculated.



Applying Pythagoras theorem to right- angled triangle in the graph above,

$d^2 = (x_2 - x_1)2 + (y_2 - y_1)2$

$$\therefore \boldsymbol{d} = \sqrt{(\boldsymbol{x_2} - \boldsymbol{x_1})^2 + (\boldsymbol{y_2} - \boldsymbol{y_1})^2}$$

Example 1:

Calculated the distance between the points, (4,1) and (3,-2)

Solution:

The distance between the points (4,1) and (3, -2) is

$$\sqrt{[(4-3)^2 + (1-(-2))^2]} = \sqrt{10} = 3.16$$

EXAMPLE 2:

Find the length of the line segment with end points (2,8) and (6,5).

Solution

$X_1=2$

$Y_1=8$

$X_2=6$

$Y_2=5$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(6-2)^2 + (5-8)^2}$$

$$d = \sqrt{4^2 + (-3)^2}$$

$$d = \sqrt{25}$$

$$d = 5$$

CLASS ACTIVITY

Find the distances between the given points:

1. $(15,11),\ (3,6)$
2. $(\sqrt{2}, 1),\ \ (2\sqrt{2}, 3)$

MID-POINT OF A LINE SEGMENT

In the Cartesian plane above, let $R(x, y)$ be the mid-point of the line segment PQ, with the coordinate $P(x_1, y_1)$ and (x_2, y_2) .

As triangles $PRS\ and\ PQT$ are similar;

$\frac{PR}{RQ} = \frac{PS}{ST}$

Since R is the mid-point, $PR = RQ, PS = ST$

$\therefore x - x_1 = x_2 - x$

$2x = x_2 + \ x_1$

$x = \frac{x_2+x_1}{2}$

Similarly,

$y = \frac{y_2+y_1}{2}$

Hence, the co-ordinates of the mid-point of the line joining (x_1, y_1) and (x_2, y_2) are:

$\left[\frac{X_2+X_1}{2}, \frac{Y_2+Y_1}{2}\right]$

Example 1: Find the mid-point ‘R’ of the line segment AB where $A(1,5)$ and $B(-3, -1)$.

Solution:

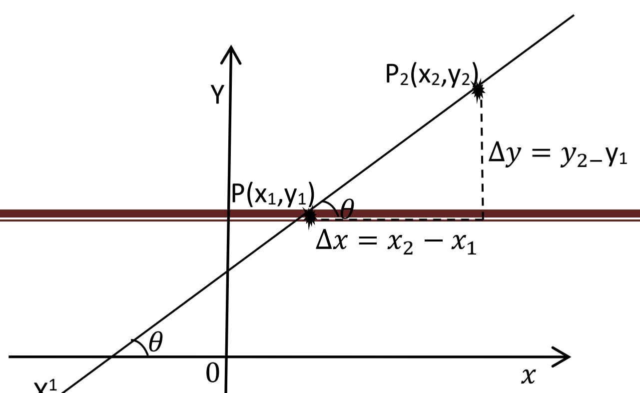
$x = \frac{x_2+x_1}{2} = \frac{1+(-3)}{2} = \frac{1-3}{2} = \frac{-2}{2} = -1$ and

$y = \frac{y_2+y_1}{2} = \frac{-1+5}{2} = \frac{4}{2} = 2$

$R(\ -1,2)$

GRADIENT OF A STRAIGHT LINE

The gradient of a line is defined as the ratio, increase in $y\ \div increase\ in\ x$, in going from one point to another on a line.



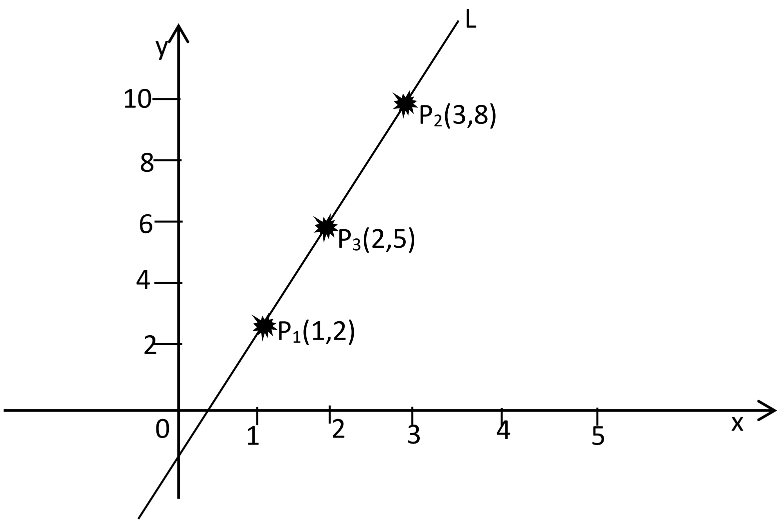
$\Delta x = x_2 - x_1$ Is the change in x as the variable x increases or decreases from x_1 to x_2 and $\Delta y = y_1 - y_2$ is the change in y with respect to y_1 and y_2 .

The slope (gradient) m of a straight line L is defined as

$$\boldsymbol{m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}}$$

If θ is the angle of inclination to the slope of L, then $= \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$;θ is called the angle of slope of the line.

Example 2: Find the slope m and the angle of inclination θ of the L through points $p_1(1,2), p_2(3,8)$ and $p_3(2,5)$



Solution:

The slope m of points P_1 and P_2 on L is

$$m = \frac{8-2}{3-2} = \frac{6}{2} = 3$$

The slope m_1 of the points P1 and P3 on L is

$m_2 = \frac{8-5}{3-2} = 3$

Therefore, $m_1 = m_2 = 3$ implies that the slope of the line L is 3.

Since $m = \tan\theta$, $i.e$ $\tan\theta = 3$ $\therefore \tan^{-1}(3) = 71.57°$

Therefore, the angle of inclination θ is 71.57°

It can therefore be concluded from the example above that any given line has one and only one slope.

CLASS ACTIVITY:

(1) Find the angle between lines L_1 , with slope -7 and L2 which passes through (2,-1) and (5,3)

(2) Find the gradients of the lines joining the following pairs of points :
(a) $(5,-4)$ and $(3,-2)$ (b) $(13,-4)$ and $(11,8)$

DETERMINATION OF EQUATION OF A STRAIGHT LINE

ONE POINT FORM OF A LINE

The equation of a line passing through any point(x_1,y_1) and with gradient m is given by $\boldsymbol{y - y_1 = m(x - x_1)}$

LINE THROUGH TWO POINTS

The equation of a line joining the two-points (x_1,y_1) and (x_2,y_2) is given by

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

EXAMPLE 1:

Find the equation of the straight line which passes through the following pairs of points: (3, -4) and (5,-3)

SOLUTION

X_1=3 y_1= -4 x_2=5 y_2= -3

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$

$$\frac{y-(-4)}{x-3}=\frac{-3-(-4)}{5-3}$$

$$\frac{y+4}{x-3}=\frac{1}{2}$$

$$2(y+4)\ =x-3$$

$$2y+8=x-3$$

$2y-x=-11$ or 2y-x+11=0

EXAMPLE 2:

Find the equation of the straight line with gradient 5 and passing through the points(3,-5)

SOLUTION

$X_1=3$ and $y_1=-5$ m=5

$$y-y_1=m(x-x_1)$$

$$y-(-5)=5(x-3)$$

$$Y+5=5x-15$$

$$y-5x=-20$$

PERPENDICULAR AND PARALLEL LINES

Two lines are said to be perpendicular to each other if the product of their gradient is equal to -1. If m_1 and m_2 are gradients of two perpendicular lines then **$m_1m_2=-1$.**

Two lines are said to be parallel to each other if their gradients are equal. If m_1 and m_2 are gradients of two parallel lines, then **$m_1=m_2$**

EXAMPLE 1:

Find the equation of the straight line which is perpendicular to the line 5x-2y=3 and passing through (3, -5).

SOLUTION

$5x - 2y = 3$

$5x - 3 = 2y$

$y = \frac{5}{2}x - \frac{3}{2}$

m_1= 5/2

$\mathbf{m_1} \times m_2$ = -1

$\frac{5}{2} \times m_2 = -1$

$m_2 = -\frac{2}{5}$

Using $\boldsymbol{y - y_1 = m(x - x_1)}$

$\boldsymbol{y - (-5) = -\frac{2}{5}(x - 3)}$

$\boldsymbol{y + 5 = -\frac{2x}{5} + \frac{6}{5}}$

$\boldsymbol{5y + 25 = -2x + 6}$

$\boldsymbol{5y + 2x = -19}$

EXAMPLE 2:

Find the equation of the straight line which is parallel to 4x-5=12-y and is passing through the point (4,7).

SOLUTION

4x-5=12-y

4x+y=17

Y=-4x+17

M_1=-4 and m_2= -4 since they are parallel

$using\ \mathrm{y - y_1 = m(x - x_1)}$

$\mathrm{y} - 7 = -4(\mathrm{x} - 4)$

$\mathrm{y} - 7 = -4x + 16$

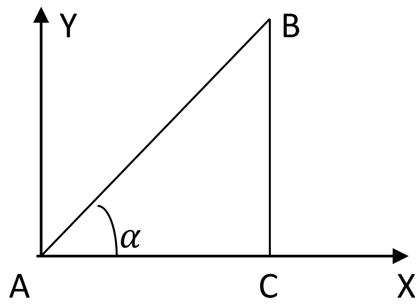
$y + 4x = 7 + 16$

$y + 4x = 23$

CLASS ACTIVITY

1. Find the equation of the line which is perpendicular to the line y=-4x, passing through the points(7,1)
2. Find the equation of the line parallel to 3x+5y=1 and passing through (4,-2)

ANGLE OF SLOPE



From the diagram above, line AB makes an angle α with positive x-axis. α is called the angle of slope of the line. The gradient of the line AB = $\frac{BC}{AC}$ = $\tan\alpha$, therefore the gradient of the line is equal to tangent of the angle the line makes with the positive x-axis.

Example 1: Find the gradient of the line joining (2,4) and (1,3); find also the angle of slope of the line.

Solution: Let $(x_1, y_1) = (2,4)$ and $(x_2, y_2) = (1,3)$

$$m = \frac{3-4}{1-2} = \frac{-1}{-1}$$

$$\therefore m = 1$$

But, $m = \tan\alpha$;

$$\tan\alpha = 1$$

$$\alpha = \tan^{-1}(1)$$

$$\Rightarrow \alpha = 45°$$

Example 2: Find the gradient of the line joining (6,-2) and (-3, 2), also the angle of slope.

Solution:

Let $(x_1, y_1) = (6, -2)$ and $(x_2, y_2) = (-3,2)$

$$m = \frac{2-(-2)}{-3-6} = \frac{4}{-9}$$

Also, $m = \tan\beta$ $\quad where\ \beta\ represent\ the\ angle\ of\ slope$

$\tan\beta = \frac{-4}{9}$

$\beta = \tan^{-1}\left(\frac{-4}{9}\right)$

$\beta = -23.75°$

$\therefore \beta = 336.2^0$ or 156.25^0

CLASS ACTIVITY

1. Find the gradient of the line joining $(-1,-12)$ and $(6,-12)$. Find the angle of slope of the line.
2. Find the gradient of the line joining $(-1,-12)$ and $(6,-12)$. Find the angle of slope of the line.

ANGLE BETWEEN TWO LINES

The angle between the lines

Y=xtanα+c_1 and

Y=xtanβ + c_2

is given by $\alpha{\sim}\beta$.

We can also use the formula

Tanθ=|$\frac{m_2-m_1}{1+m_1m_2}$|

where M_2 and m_1 are the gradients of the straight lines.

EXAMPLE 1:

Find the angle between the lines

Y=3x-4 and 2x-y+1=0

SOLUTION

y=3x-4

tan$\alpha = 3$

$\alpha = 71.6^0$

$also, 2x - y + 1 = 0$

Y=2x+1

tan$\beta = 2$

$\beta = 63.5^0$

$\therefore required\ angle = \ \alpha{\sim}\beta \qquad | \ \ \alpha > \beta$

=71.6^0-63.5^0

=8.1^0

2^{ND} METHOD

Using tanθ= $\left|\frac{m_2-m_1}{1+m_1m_2}\right|$

tanθ= $\frac{3-2}{1+2\times3}$

tanθ= $\frac{1}{7}$

tanθ= 0.1429

θ=$tan^{-1}0.1429$

θ=8.1^0

EXAMPLE 2:

Find the acute angle between the following pair of lines

2x-3y+5=0 and

4y-x+2=0

SOLUTION

Tanθ=$\left|\frac{m_2-m_1}{1+m_1m_2}\right|$

2x-3y+5=0

3y=2x+5

Y=$\frac{2}{3}$ x + $\frac{5}{3}$,m_1=2/3

4y-x+2=0

4y=x-2

Y= $\frac{x}{4}-\frac{1}{2}$,m_2= ¼

Tanθ=$\left|\frac{\frac{2}{3}-\frac{1}{4}}{1+\frac{2}{3}\times\frac{1}{4}}\right|$

Tanθ= $\frac{\frac{5}{12}}{\frac{7}{6}}$

Tanθ= $\frac{5}{14}$

Tanθ= 0.3571

$\theta = tan^{-1}(0.3571)$

Θ≃ 19.7^0

CLASS ACTIVITY

1. Find the acute angle between the pair of lines
   3x+4y=2 and
   2x+y=5
2. Find the acute angle between the following pairs of lines:
   Y=5x-1 and y-3x+2=0

APPLICATION OF LINEAR GRAPHS TO REAL LIFE SITUATION

EXAMPLE 1:

P(-6,1) and Q(6,6) are the two ends of the diameter of a given circle. Calculate the radius

SOLUTION

X_1=-6 Y_1=1 X_2=6 Y_2=6

$\mathrm{d} = \sqrt{(\mathrm{x}_2 - \mathrm{x}_1)^2 + (\mathrm{y}_2 - \mathrm{y}_1)^2}$

$\mathrm{d} = \sqrt{(6 - (-6))^2 + (6 - 1)^2}$

$\mathrm{d} = \sqrt{(12)^2 + (5)^2}$

$\mathrm{d} = \sqrt{144 + 25}$

$\mathrm{d} = \sqrt{169}$

$\mathrm{d} = 13$

length of diameter = 13units and radius = 6 ½ units

EXAMPLE 2:

$Find\ the\ value\ of\ \alpha^2+\beta^2\ \ if\ \alpha + \beta = 2$

$and\ the\ distance\ between\ the\ points\ (1, \alpha) and\ (\beta, 1) is\ 3\ units. JAMB$

SOLUTION

$X_1=1 \quad y_1=\alpha \quad x_2=\beta \quad y_2=1$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$3 = \sqrt{(\beta - 1)^2 + (1 - \alpha)^2}$

$9=\beta^2 - 2\beta + 1 + 1 - 2\alpha + \alpha^2$

$9 - 2 + 2\beta + 2\alpha = \alpha^2 + \beta^2$

$7+2(\boldsymbol{\alpha} + \boldsymbol{\beta})$ $= \alpha^2 + \beta^2$

$7+2 \times 2=\alpha^2 + \beta^2$

$\alpha^2 + \beta^2 = 11$

CLASS ACTIVITY

1. What is the value of r if the distance between the points (4,2) and (1,r) is 3 units.
2. Find the distance between the point Q(4,3) and the point common to the lines $2x - y = 4\ and\ x + y = 2$. JAMB

PRACTICE EXERCISE

1. Find the acute angle between the lines 3x+2y=1 and 7x+4y=5.
2. Find the coordinates of the mid-point of the x and y intercepts on the line 2y=4x-8
3. Find the equation of the line which is parallel to the line 5x+4y=18 and makes an intercept of 2 units on the x-axis. WAEC
4. Find the equation of the line which passes through the point P(4,-3) and is perpendicular to the line 2x+5y+1=0 WAEC
5. Find the coordinates of the mid-point of the x and y intercepts on the line 2y=4x-8 JAMB

   ASSIGNMENT

   1. What is the value of p if the gradient of the line joining (-1,P) and (p,4) is $\frac{2}{3}$? JAMB
   2. Find the value of p if the line that passes through (-1,-p) and (-2p,2) is parallel to the line 8x+2y-17=0 JAMB
   3. If the lines 3y=4x-1 and qy=x+3 are parallel to each other, find the value of q.

4. PQ and RS are two parallel lines. If the coordinates of P,Q,R and S are (1,q), (2,3),(3,4) and (5,2q) respectively, find the value of q. JAMB
5. Find the mid-points of the line segments with the following end points:
   a) (3,0) and (4,-4)
   b) (-4,-5) and (3,9)
   c) (-1/4 ,0) and (1,- ½)
6. Find the value of α if the lines 2y-αx+4=0 is perpendicular to the line y+ ¼ x-7=0 JAMB

7. Find the point of intersection of the two lines 3x-2y+5=0 and y-4x+3=0

KEYWORDS:

Coordinates, parallel, perpendicular, intercept, gradient, equation, mid-point, intersect etc.

WEEK 9:
DATE……………………….
Subject: Mathematics
Class: SS 2
TOPIC: Approximations
Content:

- Revision of approximation.
- Accuracy of results using logarithm table and calculators.
- Percentage error.

- Application of approximation to every day life.

REVISION OF APPROXIMATION

An approximation helps us to see and understand easily the size of a number. It is a number taken as close as possible to the actual value of the number.

In order to come close to the actual value, the number must be rounded off. This implies digits 1 to 4 are rounded down while those from 5 to 9 are rounded up. Other methods could be approximating to decimal places or significant figures.

Decimal places mean the number of places after the decimal point while significant figure is the first non-zero digit from the left.

Example 1: The distance between the earth and the sun is 148729440km. Round this number to the nearest (a) million (b) 2s.f (c) 3s.f

SOLUTION

a) 148729440km = 149000000km (i.e round 7 up)
b) 148729440km = 150000000km (i.e round 8 up)
c) 148729440km = 149000000km (i.e round 7 up)

EXAMPLE 2: (a) Write 78.45831kg to (i) 2d.p, (ii) 3d.p

SOLUTION

(i) 78.45831kg = 78.46kg (i.e round 8 up)

(ii) 78.45831kg = 78.458kg (round down 3)

(b) Round 0.0004996 to (i) 1s.f (ii) 2s.f

SOLUTION

(i) 0.0004996 = 0.0005 (round the first 9 up)

(ii) 0.0004996 = 0.00050 (round the second 9 up)

CLASS ACTIVITY

(1) Round off 586.5764 to (i) 1d.p (ii) 3d.p
(2) Approximate 964572183665 to (i) 3s.f (ii) 6s.f

ACCURACY OF RESULTS USING LOGARITHM TABLE AND CALCULATORS

All measurements are approximations, so they cannot be exact. Any stated measurement has been rounded off to some degree of accuracy.

Example 1:

A plot of land measuring 4532m by 431m.Calculate the area of the plot, using the calculator and the Logarithms table.

Solution: (i) 4532m x 431m = 1953292m^2

(ii) 4532m x 431m

Number	Log
4532	3.6563
431	2.6345
1954	**6.2908**

Antilog of 6.2908 = 1954000

EXAMPLE 2: Calculate the difference due to using two different methods of calculation by evaluating

(a) 2321 x 4122

(b) 12204 x 2123

SOLUTION

(a) i. 2321 x 4122 = 9567162

ii. 2321 x 4122

Number	Log
2321	3.3657
4122	3.6151
9568	**6.9808**

Antilog of 6.9808= 9568000

Difference = 9568000 – 9567000

= 838

.(b) (i) 12204 x 2123 = 25909092

(ii) 12204 x 2123

Numbers	Log
12204	4.0864
2123	3.3269
2590	7.4133

Antilog 0f 7.4133= 25900000

Difference = 25909092 - 25900000

= 9092

It can be seen that the result from the calculator is more accurate. The logarithm table results are higher because of premature approximation. The last example was different because the table deals with four-figures only while one is 5-figures

CLASS ACTIVITY

Solve this problems, using the calculator and the logarithm table. Calculate the difference from your answers and state why?

(a) 153 x 5234
(b) 12536 x 2413
(c) 213564 x 1215
(d) 1211015 x 123143

PERCENTAGE ERROR

Every measurement no matter how carefully carried out is an approximation, not exact. If the height of a wall is 25m, then the true or actual height is between 24.5m and 25.5m i.e ± 0.05 (range difference). The error involved cannot exceed ± 0.05, the maximum absolute error.

Note: The maximum absolute error is an allowance within which the actual measurement falls.

Relative error is the ratio of the maximum absolute error (precision) and the true measurement value.

In measurements,

$$\textbf{Percentage error} = \frac{error}{measurement} \times 100\%$$

If the true value is known,

$$\textbf{Percentage error} = \frac{error}{true\ value} \times 100\%$$

Example1:

A sales girl gave a balance of #1.15 to a customer instead of #1.25. Calculate her percentage error.

SOLUTION

Balance given = #1.15

Real or actual bal. = #1.25

Error = #1.25 - #1.15 = #0.10

Percentage error = $\pm \frac{0.10}{1.25} \times 100\%$

= 8%

EXAMPLE 2:

A student who was asked to correct 0.02539 to two significant figures gave its value to two decimal places. His percentage error is ………..

SOLUTION:

The correct value in significant figure is 0.025

His answer is 0.03

% error = $\frac{0.03-0.025}{0.025} \times 100\%$

= $\frac{0.5}{0.025} \times 100\% = \frac{500}{25}\% = 20\%$

CLASS ACTIVITY

(1) A Man made a table with a rectangular top of dimension 36cm by 44cm instead of 37cm by 41cm. what is the percentage error in the perimeter of the table correct to 1 decimal place?

(2) A boy measures the length and breadth of a rectangular lawn as 59.6m amd 40.3m respectively instead of 60m and 40m. what is the percentage error in his calculation of the perimeter of the lawn?

APPLICATION OF APPROXIMATION TO EVERYDAY LIFE

This aspect deals with real life situations.

Example 1:

A car travels a distance of 100km for 1h 39mins. Calculate the speed of the car. If the time is rounded up to the nearest 1hr.Find the difference between the actual speed and when the time was rounded off.

SOLUTION

Speed = $\frac{distance}{time}$ = $\frac{100}{1\frac{39}{60}}$

= $\frac{100}{1} \times \frac{60}{99}$

= 66.67km/hr

1hr 39mins to the nearest one hour is 2hrs, speed = $\frac{100}{2}$ = 50km/hr

Difference = 66.67km/hr – 50km/hr

= 16.67km/hr

EXAMPLE 2:

Find the sum of 34.25 (to 4s.f), 26 (to 3s.f) and 10 (to 2s.f) and leave your answer to a reasonable degree of accuracy.

Solution: the actual values

34.25 ± 0.005

26 ± 0.5

$\underline{10 \pm 0.5}$

Sum = 70.25 ± 1.105

∴ the actual sum lies between 69.145 and 71.355

Hence, the sum of the values to a reasonable degree of accuracy is 69 to 2s.f

Note: Maximum error can be derived thus;

Whole numbers = $\frac{1}{2} of\ 1$ = 0.5

1 d.p numbers = $\frac{1}{2} of \frac{1}{10} = 0.05$

2 d.p numbers = $\frac{1}{2} of \frac{1}{100} = 0.005$ etc

PRACTICE EXERCISE

(1) If it takes a proton to move $1.2 \times 10^{-1} m$ in 6.8×10^{-6}seconds, find the speed of the proton in metre per second correct to a suitable degree of accuracy

(2) Instead of writing $\frac{35}{6}$ as a decimal correct to 3s.f, a student wrote it correct to 3d.p, find his error in standard form.

(3) Find the value of each of the following and the degree of accuracy

(a) 717.3 + 200.5 + 670.3 + 504.4

(b) 41.56 – 9.062 – 4.147 – 10.20 – 5.108

(4) A string is 4.8m . a boy measured it to be 4.95m. Find the percentage error

(5) A sales boy gave a change of N68 instead of N72.calculate his percentage error.

ASSIGNMENT

(1) The length of a piece of stick is 1.75m. A girl measured it as 1.80m. Find the percentage error. NECO 2011

(2) A man estimated his transport fare a journey as N210.00 instead of N220.00. Find the percentage error in his estimate, correct to 3 s.f. NECO 2012

(3) A boy when rounding up a number wrote 98 instead of 980(to 2 significant figures).What is the percentage error? NECO 2007

(4) Approximate 0.0033780 to 3 significant figures

(5) Express 302.10495 correct to 5 s. f.

(6) Express the product of 0.007 and 0.057 to 2 s. f. NECO 2004

(7) Evaluate $(0.13)^3$ correct to 3 s. f. NECO 2006

(8) A rectangular room has sides 5m by 4m, measured to the nearest metre.

(a) Write down the limits of accuracy for each length

(b) Find the greatest area the room could have

(c) Find the smallest perimeter the room could have

(9) Evaluate: $\frac{6.42+2.13}{4.1-2.85}$, correct to 2 s. f. NECO 2008

KEYWORDS: error, percentage error, approximate, limits of accuracy, actual value,etc.

WEEK 10 REVISION

WEEK 11 EXAMINATION