SS1 THIRD TERM: E-LEARNING NOTES

SCHEME THIRD TERM

WEEK	TOPIC	CONTENT
1	**LOGICAL REASONING**	(a) Simple statements. (b) Meaning of simple statement – (i) True or false (ii) negation of simple statements. (c) Compound statements (i) Meaning (ii) Conjunction (iii) Disjunction (iv) Implication (v) bi-implication. (d) Logical operators and symbols. (i) list of logical operators and symbols (ii) Truth value of logical operators.
2	**TRIGONOMETRY 1**	(a) Basic Trigonometric Ratios – (i) sine (ii) cosine (iii) tangent with respect to right-angled triangles. (b) Trigonometric ratio of; - (i) Angle 30^0 (ii) Angle 45^0 (iii) Angle 60^0.
3	**TRIGONOMETRY 2**	(a) Application of trigonometric ratios (angle of elevation and depression; bearing). (b) Trigonometric ratios related to the unit circle. (c) Graphs of sines and cosines.
4	**MENSURATION 1**	(a) Length of arcs of circle. (b) Perimeter of sectors and segments. (c) Areas of sectors of a circle. (d) Areas of segments of a circle.
5	**MENSURATION 2**	(a) Relation between the sector of a circle and the surface area of a cone. (b) Surface area and volume of solids; (i) Cube, cuboids (ii) Cylinder (iii) Cone (iv) Prisms (v) Pyramids.
6	**MENSURATION 3**	(a) Surface areas and volume of frustum of a cone and pyramid. (b) Surface area and volume of compound shapes.
7	**MID-TERM BREAK.**	
8	**DATA PRESENTATION**	(a) Revision on collection, tabulation and presentation of data. (b) Frequency distribution (grouped data).
9	**DATA PRESENTATION**	(i) Line graph, (ii) Bar graph. (iii) Histograms. (iv) Pie Chart. (v) Frequency polygon. (vi)Deductions and Interpretations.
10	**REVISION**	
11	**EXAMINATION**	

WEEK 1 **Date……………………………………**

SUBJECT: MATHEMATICS

CLASS: SS 1

TOPIC: LOGICAL REASONING

CONTENT:

- (a) Simple statements.
- (b) Meaning of simple statement – (i) True or false (ii) negation of simple statements.
- (c) Compound statements (i) Meaning (ii) Conjunction (iii) Disjunction (iv) Implication (v) bi-implication.
- (d) Logical operators and symbols. (i) List of logical operators and symbols (ii) Truth value of logical operators.

✓ **SIMPLE STATEMENTS**

Logic is the science of thinking about or explaining the reason for something. It is a particular method or system of reasoning which arrive at conclusions by way of valid evidence.

Mathematical logic can be defined as the study of the relationship between certain objects such as numbers, functions, geometric figures etc. .

Example: The following are logical statements;

1. Nigeria is in Africa
2. The river Niger is in Enugu
3. $2 + 5 = 3$
4. $3 \leq 7$

   (**N.B The educator should ask the students to give their examples)**

Example: The following are not logical statements because they are neither true nor false.

1. What is your name?
2. Oh what a lovely day
3. Take her away
4. Who is he?
5. Mathematics is a simple subject (note that this statements is true or false depending on each individual, so it is not logical).

   Educator to ask the students to give their own examples.

✓ **MEANING OF SIMPLE STATEMENT.**

Statements are verbal or written declarations or assertions. The fundamental (i.e logical) property of a statement is that it is either true or false but not both. So logical statements are statements that are either reasonably true or false but not both

True or False statements:
To determine the truth or falsity of a simple statement, one requires pre-knowledge and/or definition of the main concepts related to the statements. For example, the simple statement 'it is hot' is true if 'it' refers to a hot object or weather. Otherwise the statement is false. A true statement is said to have a truth value **T** while a false statement is said to have a truth value **F**.

Example: indicate **T** or **F** for the truth value of each statement.

1. 10_{two} is equal to 10
2. Green is one of the colours on the Nigerian flag
3. How far is Abuja from here?
4. $3 \in \{2, 4, 6, 8, \ldots\}$
5. The perimeter of a room 2.5m by 3.5m is 6m

Solution:

1. F
2. T
3. Not applicable
4. F
5. F
   (note: educator to explain closed statements and open statements as in question 3)

NEGATION OF SIMPLE STATEMENTS
The opposite of a statement is called the negation of the statement. Given any logical statement P, the negation (or the contradiction or the denial) of P is written symbolically as ~P

Examples: write the negation of each of the following statements.

1. I am a Mathematician
2. $2 > 4$
3. $\frac{1}{5} < \frac{1}{2}$

4. $x + 1 \geq 4$
5. The sky is the limit

Solution:

1. I am not a mathematician
2. $2 \ngtr 4$
3. $\frac{1}{5} \nless \frac{1}{2}$
4. $x + 1 \leq 4 \; or \; x + 1 \ngeq 4$
5. The sky is not the limit

Truth table for ~P:

P	~P
T	F
F	T

Class Activity:

Write the negation of the following statements:

1. All polygons are quadrilaterals
2. It is a sunny day
3. XYZ is an isosceles right angled triangle
4. The figure is a cube
5. X is not a prime number

Compound statements:

When two or more simple statements are combined, we have a compound statement. To do this, we use the words: ‘and’, ‘or’, ‘if … then’, ‘if and only if’, ‘but’. Such words are called connectives.

Conjunction (or ^) of logical statements:

Any two simple statements p,q can be combined by the word ‘and’ to form a compound (or composite) statement ‘p and q’ called the conjunction of p,q denoted symbolically as p^q.

Example: 1. Let p be “The weather is cold” and q be “it is raining”, then the conjunction of p,q written as p∧q is the statement “the weather is cold and it is raining”.

2. The symbol ‘∧’ can be used to define the intersection of two sets A and B as follows;

$$A \cap B = \{x : x \in A \wedge x \in B\}$$

The truth table for p∧q is given below;

P	**Q**	**P∧Q**
T	T	T
F	F	F
T	F	F
F	T	F

Class Activity:

Form compound statements using **‘and’**, and express the following compound statements in symbol form.

1. P: It is cold.
   Q: It is wet.
2. P: $x + 3$
   Q: x= -3
3. P: f(x) = 5x^2 + 2
   Q: f(1) = 7
4. P: $(x + 2)^2$ is a perfect square.
   Q: when x = 1, $(x+2)^2$ = 9

Disjunction (or ∨) of logical statements.
Any compound statement formed by using the word **‘or’** to combine simple statements is called a disjunction. The symbol ‘∨’ stands for ‘or’.

Examples

1. Let `p ‘ be “Bola studied Mathematics”, and `q’ be “Ngozi studied French”. Then the disjunction of p, q (p∨q) is the statement “Bola studied Mathematics or Ngozi studied French”.
2. p: You will read your notes .
   q: You will fail
   p∨q: You will read your notes or fail
3. P: $x + 1 = 2$
   Q: $x = 1$.
   P ∨ Q : $x + 1 = 2 \; or \; x = 1$.
4. P: The solution of $x^2 \; - 2x \; - 15 \; = 0 \; is \; 5$.
   Q: The solution of $x^2 - 2x - 15 = 0 \; is$ -3.
   P∨Q: The solution of $x^2 - 2x - 15 = 0 \; or - 3$

The truth table for p∨q is illustrated below

P	**Q**	**P∨Q**
T	T	T
F	F	F
T	F	T
F	T	T

Class Activity:
Express the compound statements in symbolic form.

1. P: $\sqrt{2}$ is a rational number , Q: $\sqrt{2}$ is an even number.
2. P: the trade union is stubborn, Q: the workers strike will soon be ended.
3. P: 5 is a prime number, Q: 7 is an even number.
4. P: a person who has taken physics can go to geophysics, Q: a person who has taken geology can go for geophysics.

Implications (conditional statements)
When the connective ‘if…then’ is used to combine simple statements, the result is called an implicative or conditional proposition. We denote implication symbolically by ⇒ i.e p⇒q means if p is true, then q is true. (or p implies q or p only if q, etc.)

Examples: Form compound statements using ‘if … then’

1. P: The triangle is an equilateral triangle
   Q: The angles are equal
   P ⇒ Q: if the triangle is equilateral then the angles are equal.
2. P: $-\infty < x < 10$
   Q: $100 < x^2 < \infty$
   P ⇒ Q: if $-\infty < x < 10$ then $100 < x^2 < \infty$
3. P: Isa is a Mathematician .
   Q: He is intelligent.
   P ⇒ Q: if Isa is a mathematician then he is intelligent.
   N.B : Educator to explain antecedent and consequent and Examples should be given.
   The truth or falsity of the implication P ⇒ Q: is shown below;

P	**Q**	**P ⇒ Q**
T	T	T
T	F	F
F	T	T
F	F	T

Class Activity:

Form compound statements using ‘if … then ’

1. P: y = 2
   Q: $y^2 = 4$
2. P: A student reads Mathematics
   Q: the student reads science
3. P: Damilola is a youth corper
   Q: she has a degree
4. Identify the antecedent and consequent in the statement below;
   If Mathematics teachers work very hard then they will be compensated.

Bi-implication or Bi-conditional statement (equivalence).
Another common statement in Mathematics is of the form “p if and only if q”. This statement is actually the combination of two conditional statements and so it is called bi-conditional or

equivalence and is denoted by $p \leftrightarrow q$ or sometimes p iff q (if and only if) i.e implies and is implied by.

Examples:

1. Let p be "he is a handsome man" and q be "10 > 6" then $p \leftrightarrow q$ is the statement "he is a handsome man if and only if 10 > 6", then

p $\leftrightarrow q$ *is the statement* He is a handsome man if and only if 10 >6".

2. P: A number is divisible by 3

Q: the sum of the digits of the number is divisible by 3

$P \leftrightarrow Q$: A number is divisible by 3 iff the sum of its digits is divisible by 3

The truth table for $p \leftrightarrow q$ is shown below;

P	**Q**	**P ↔ Q**
T	T	T
T	F	F
F	T	F
F	F	T

Logical operators and symbols

The word 'not' and the four connectives 'and', 'or', 'if … then', 'if and only if' are called logic operators. They are also referred to as logical constants. The symbols adopted for the logic operators are given below.

Logic Operators	Symbols
'not'	$-$ *or* $\sim$
'and'	$\wedge$
'or'	$\vee$
'if … then'	$\rightarrow$

‘if and only if’ $\leftrightarrow$

When the symbols above are applied to propositions p and q, we obtain the representations in the table below:

Logic operation	Representation
‘not p’	$\sim p$ or $\bar{p}$
‘P and q’	$p \wedge q$
‘p or q’	$p \vee q$
‘if p then q’	$p \rightarrow q$
‘p if and only if q’	$p \leftrightarrow q$

PRACTICE EXERCISE:

1. What are the Truth values of this compound statement? $\sim (\mathbf{P} \wedge \sim \mathbf{Q})$
2. Determine the truth value of the compound statement $(\boldsymbol{S} \Rightarrow \boldsymbol{R}) \wedge \sim \boldsymbol{R}$
3. Use a truth table to prove that; $\sim(\boldsymbol{p} \Rightarrow \boldsymbol{q}) \leftrightarrow (\boldsymbol{p} \wedge \sim \boldsymbol{q})$ is a Tautology.
4. Copy and Complete the table below;

P	**Q**	**$\sim$Q**	**P $\Rightarrow$ Q**	**P $\vee\sim$Q**	**(P$\Rightarrow$Q)$\leftrightarrow$(P$\vee\sim$Q)**
T	T				
T	F				
F	T				
F	F				

5. If P and Q are two logical statements, copy and complete the following truth table

P	**Q**	**P$\vee$Q**	**$\sim$(P$\vee$Q)**	**(P$\vee$Q) $\wedge$ $\sim$Q**	**$\sim$(P$\vee$Q) $\Rightarrow$ $\sim$P**

ASSIGNMENT:

1. If P and Q are two logical statements, copy and complete the following truth table

P	**Q**	$P \vee Q$	$\sim(P \vee Q)$	$(P \vee Q) \wedge \sim P$	$\sim(P \vee Q) \Rightarrow (\sim P \wedge \sim Q)$

2. Study the following;

i. Antecedent

ii. Consequent

iii. Converse, inverse and contrapositive statements

iv. Tautology and contradiction

WEEK 2

SUBJECT: MATHEMATICS

CLASS: SS 1

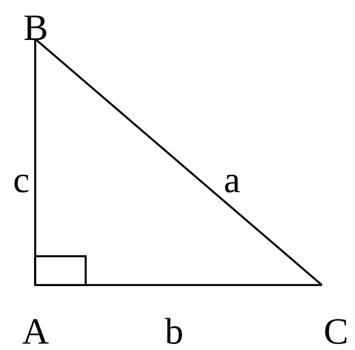
TOPIC: TRIGONOMETRY 1

CONTENT:

- (a) Basic Trigonometric Ratios – (i) sine (ii) cosine (iii) tangent with respect to right-angled triangles. (b) Trigonometric ratio of; - (i) Angle 30^0 (ii) Angle 45^0 (iii) Angle 60^0.

Basic trigonometric ratios (i) Sine (ii) Cosine (iii) Tangent with respect to right-angled triangles.

These trigonometric ratios are applicable to right – angled triangle. A right – angle triangle is 90^0. Thus the remaining two angles add up to 90^0 since every triangle contains two right angles.

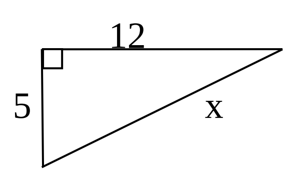


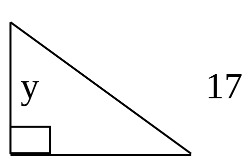
In ΔABC, B + C = 90^0.

Such angles whose sum is 90^0 are said to be complementary angles. While capital letter are used for angles, small (lower case) letters are used for sides. Notice that the side opposite A is labelled a, the one opposite B is labelled b etc.

The side opposite the right angle is called the hypotenuse. Every right – angled triangle obeys the Pythagoras theorem. This theorem states that the square of the hypotenuse of any right angled triangle is equal to the sum of the square of the other two sides.
Thus in the above triangle, $a^2 = b^2 + c^2$

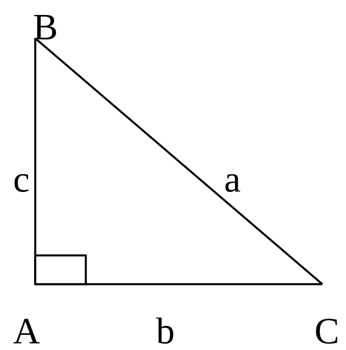
Apply Pythagoras theorem to the right – angled triangles below to find the lettered sides.





There are six basic trigonometric ratios viz: sine, cosine, tangent, cosecant, secant and cotangent. The first three are commonly used.

They are applicable only to right – angled triangles. Their short forms are: Sin, Cos, tan, cosec, sec, and cot respectively.



in the figure above, sin B = $\frac{opposite\ of\ B}{hypotenuse} = \frac{b}{a}$

sin C = $\frac{opposite\ of\ C}{hypotenuse} = \frac{c}{a}$

In short, sine = $\frac{opposite}{hypotenuse}$

Similarly, cos B = $\frac{Adjacent\ side\ to\ B}{hypotenuse} = \frac{c}{a}$

Cos C = $\frac{Adjacent\ side\ to\ C}{hypotenuse} = \frac{b}{a}$

In short, Cosine = $\frac{adjacent}{hypotenuse}$

Also Tan B = $\frac{opposite\ of\ B}{hypotenuse\ B} = \frac{b}{c}$

What is Tan c ?

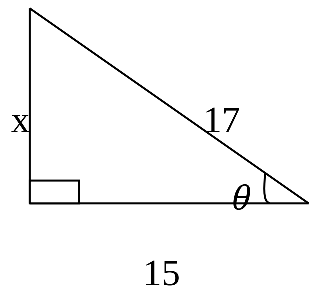
In short, tan = $\frac{opposite}{adjacent}$

Using the first letter of these three words of three formulae, we have SOH CAH TOA

SINE

The trigonometric ratio sine, is opposite divided by hypotenuse. Its reciprocal is cosecant.

Example 1:



Find sin θ in the above figure.

Solution

Sin θ = $\frac{opposite\ side\ of\ \theta}{hypotenuse}$

The opposite side of θ is not given. By using Pythagoras theorem, it can be found.

Let the opposite side to θ be x

By Pythagoras theorem

$x^2 + 15^2 = 17^2$

$x^2 + 225 = 289$

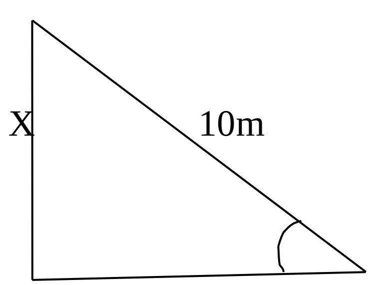
$x^2 = 289 - 225$

$x^2 = 64$

$x = 8$

Sin θ = $\frac{opposite}{hypotenuse} = \frac{8}{17}$

Example 2:



22^0

Find x.

Solution

Since the opposite of the given angle is known and the hypotenuse of the triangle, one can use sine ratio.

Sin $22^0 = \frac{x}{10m}$

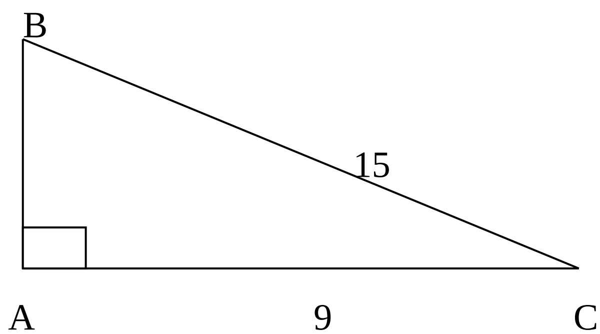
x = 10m x sin 22^0

= 10m x 0.3746

= 3. 746m

Class Activity:

1. Find SinB, SinC in the figure below.



(2) In right – angled triangle XYZ, with Z = 90^0. If l XY l = 5m and l YZ l = 3m. Find (i) Sin x (ii) y

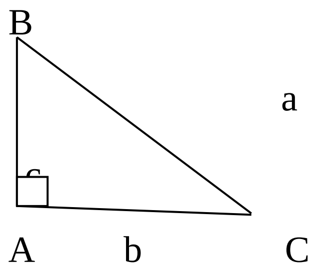
(3) If a ladder of length 2m leans against a wall and makes 30^0 with the floor, how high above the

floor does the ladder reach on the wall?

(4) New General Maths SS1 Ex. 11a; No. 1, 2a, b, d, 3a, b

COSINE

The trigonometric ratio cosine is adjacent divided by hypotenuse. Its reciprocal is secant.



In the triangle above, B and C are complementary Sin B = b/a = Cos C

Also Sin C = c/a = Cos B

For complementary angles, the sine of one is the cosine of the other i.e

Sin θ = Cos (90 - θ)

Example 1:

Solve sin2x = cos3x

Solution

Since Sin θ = cos (90 - θ)

So, sin2x = cos (90 – $2x$) (i)

But we are given that

Sin2x = cos3x (ii)

From the right hand sides of equation (i) and (ii)

We conclude that

$90^0 - 2x = 3x$

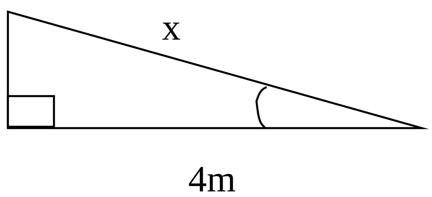
$90^0 = 5x$

So x = 90/5 = 18^0.

Example 2.

The angle of elevation of the top of a tree is 60^0. If the point of observation is 4m from the foot of the tree. How far is the point from the top of the tree?

Solution



Let the point of observation from the top of the tree be x metres.

$\text{Cos}60^\circ = 4/x$

$0.5 = 4/x$

$0.5x = 4$

$x = 4 \div 0.5$

$= 8\text{m}.$

Example 3

Given that Cos x = 0.7431, $0 < x < 90^0$, use tables to find the values of (i) 2sinx (ii) tan x/2

Solution

$\text{Cos } x = 0.7431$

$X = \cos^{-1} 0.7431$

$= 42^0.$

(i) $2\text{Sin}x = 2\text{Sin}42^0$

$= 2(0.6691)$

$= 1.3382$

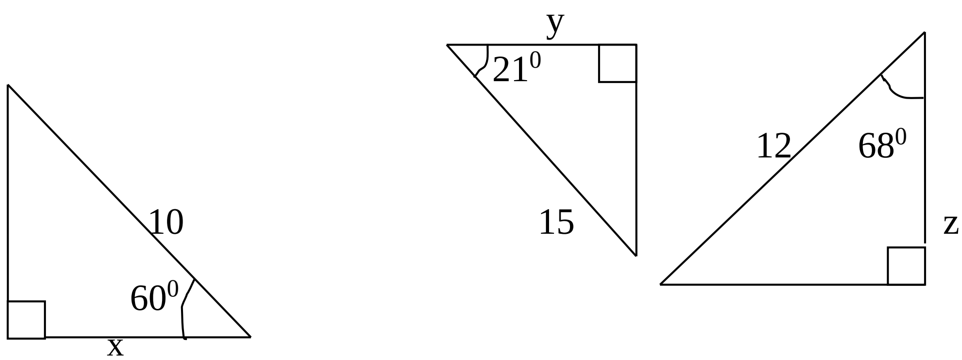
(ii) $\text{Tan } x/2 = \tan 42/2$

$= \tan 21^0$

$= 0.3839$

Class Activity:

1. Find the unknown sides of the following triangles.



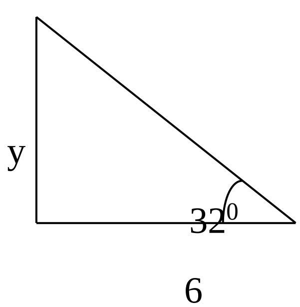
2. If sin θ = 0.3970 use the tables to find (i) cos 2θ (ii) tan 3θ

TANGENT

Tan = $\frac{opposite}{adjacent}$

Example 1:

Find y in the figure



Tan 32 = y/6

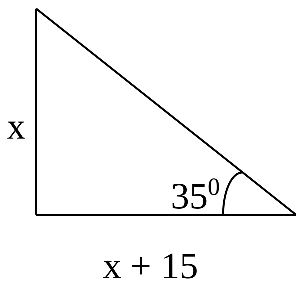
y = 6tan 32

= 6 x 0.6249

= 3.75

Example 2:

Find x in the figure below



Tan $35^0 = \frac{x}{x+15}$

$0.7002 = \frac{x}{x+15}$

$x = 0.7002\ (x+15)$

$x(1 – 0.7002) = 0.2998x$

$x0.2998 \qquad = 10.503$

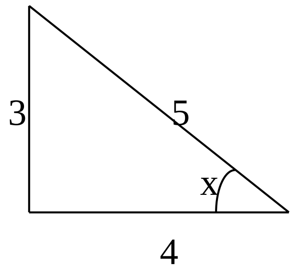
$x = \frac{10.503}{0.2998}$

$= 35.03$

Example 3.

If sin x = 3/5 and x is an acute angle, find cosx + tanx

Solution



Since sin x = 3/5

Opposite = 3, hypotenuse = 5,

Let the adjacent be a, by Pythagoras theorem,

$a^2 + 3^2 = 5^2$

$a^2 = 25 - 9$

$a^2 = 16$

Hence a = 4

so, cos x + tan x $= 4/5 + 3/4$

$= (16+15)/20$

$= 31/20$

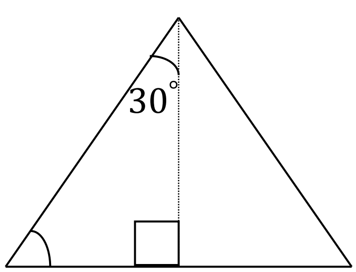
$= 1\frac{11}{20}.$

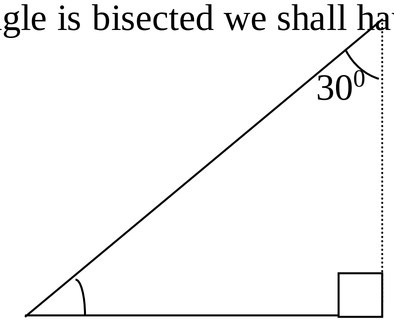
Class Activity:

1. If Cos P = 4/5 and P is an acute angle, what is the value of tan P?
2. Given that tan x = 8/15. What is the value of sin x + cos x?

Angles 30^0 and 60^0.

To consider the trigonometric ratios for the special angles 30^0 and 60^0, we shall consider an equilateral triangle of side 2units. If the triangle is bisected we shall have





2 2 2 h

60^0 60^0

1 1 1

If the value of the altitude h, can be obtained using Pythagoras theorem.

$h^2 + 1^2 = 2^2$

$h^2 = 4 - 1$

$h = \sqrt{3}$

consequently, $\text{Sin}30^0$ = opp/hyp = ½

$\text{Cos}30^0$ = adj/hyp

$= \sqrt{3}/2$

$\text{Tan}30^0$ = opp/adj

$= 1/\sqrt{3}$

$\text{Sin } 60^0 = \sqrt{3}/2$

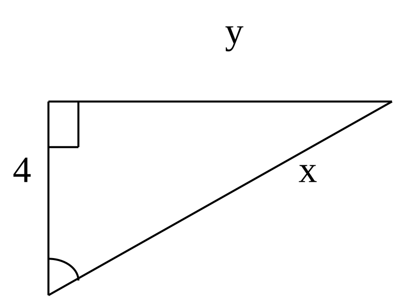
$\text{Cos } 60^0 = ½$

$\text{Tan } 60^0 = \sqrt{3}/1$

$= \sqrt{3}$

Example 1

Find x and y in the figure below



60^0

Look at the figure with respect to the given 60^0, the known side is adjacent. The unknown side x is the hypotenuse, the trigonometric ratio that connects the known (adjacent) and the unknown (hypotenuse) is cosine. Thus we write

Cos $60^0 = {}^4/_x$

But cos 60 = ½

So ½ = ${}^4/_x$

Hence x = 8.

To find y, y is the opposite side and 4 is the adjacent. Hence we use tangent

Tan $60^0 = {}^y/_4$

But tan $60^0 = \sqrt{3}$

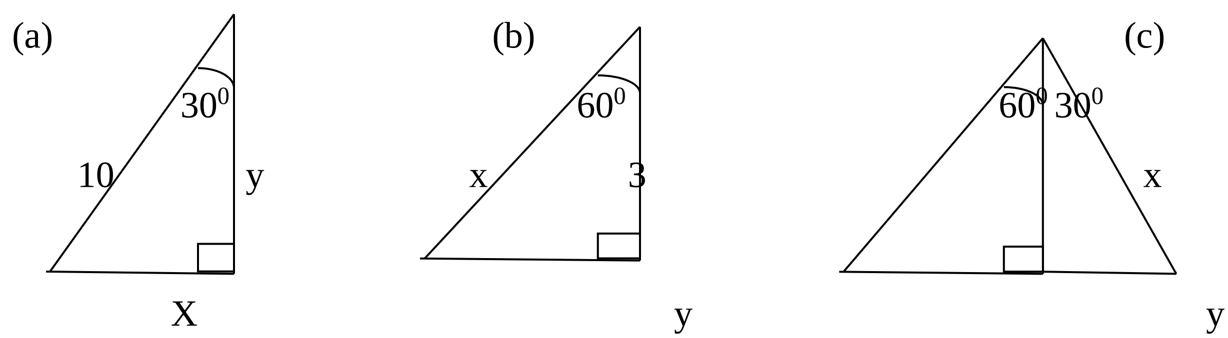
$\sqrt{3} = \frac{y}{4}$,

hence y = $4\sqrt{3}$

Class Activity:

Do not use tables but leave your answer in surds form

1. Find the values of the lettered sides;

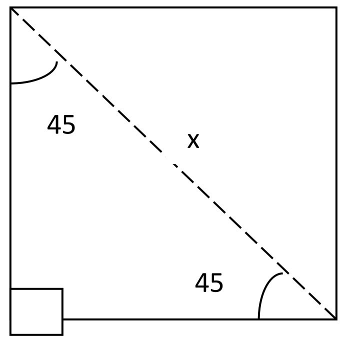


z

2.New General Mathematics SS 1, Ex 11d. Nos 3 – 8

ANGLE 45^0.

Consider a square one unit. 1



Suppose the length of its diagonal is x. 1

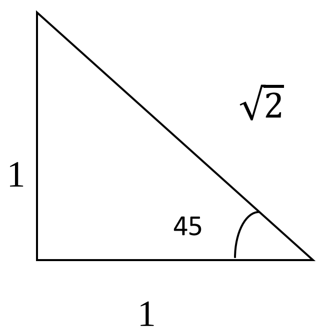
Using Pythagoras theorem,

$x^2 = 1^2 + 1^2$

$x^2 = 2$

$x = \sqrt{2}$

from this isosceles triangle obtained from above



Sin 45^0 = $^{opp}/_{hyp}$

$= 1/\sqrt{2}$

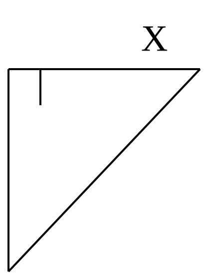
Cos 45^0 = adj/hyp

$= 1/\sqrt{2}$

tan 45^0 = opp/adj = = 1/1

Example 1:

Find x and y in the figure below:



— 45°

Y 6

To find x, use cosine

Cos 45^0 = adj/hyp

Cos 45^0 = x/6

$1/\sqrt{2} = x/6$

$x\sqrt{2} = 6$

$\therefore x = 6/\sqrt{2}$

Rationalize the denominator to get

$x = (6\sqrt{2})/(\sqrt{2}\sqrt{2})$

$= 6\sqrt{2}/2$

$= 3\sqrt{2}$

likewise Sin 45^0 = y/6

$1/\sqrt{2} = y/6$

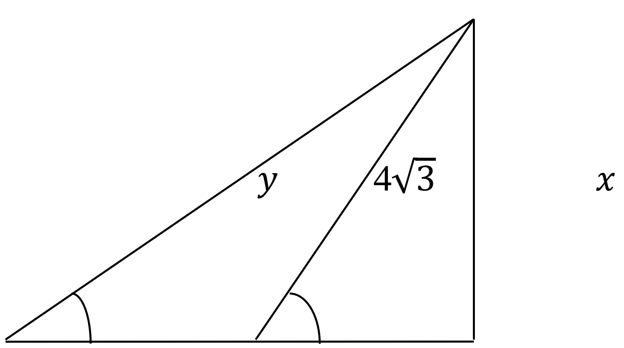
$y\sqrt{2} = 6$

$\therefore x = 6/\sqrt{2}$

$= 3\sqrt{2}$

Example 2.

Find x and y below



45°   60°

Solution

Sin $60^0 = x/4\sqrt{3}$

$\sqrt{3}/2 = x/4\sqrt{3}$

$2x = 4\sqrt{3} * \sqrt{3}$

$\therefore x = 4\sqrt{3} * \sqrt{3}/2$

$= 3(3)$

$= 6$

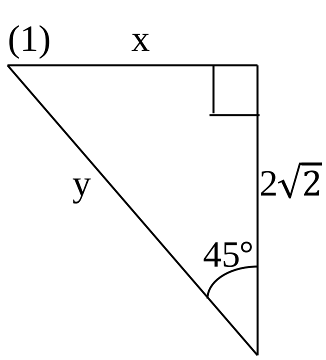
To get y, use Sine

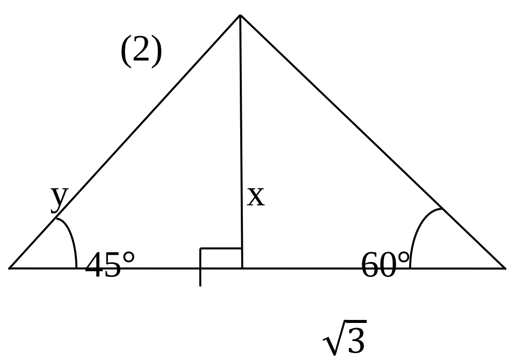
Sin $45^0 = x/y$

$1/\sqrt{2} = 6/y$

$y = 6\sqrt{2}$

Class Activity: Find the marked sides in the triangles below. Do not use tables.



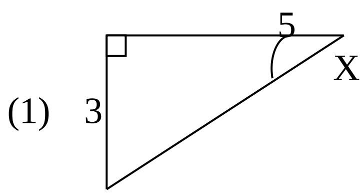


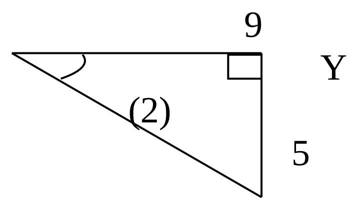
Below is the following summary of trigonometric ratio for 30^0 45^0 and 60^0.

$\boldsymbol{\theta}$	$\mathbf{30^0}$	$\mathbf{45^0}$	$\mathbf{60^0}$
Sin θ	½	$1/\sqrt{2}$	$\sqrt{3}/2$
Cos θ	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2
Tan	$1/\sqrt{3}$	1	$\sqrt{3}$

PRACTICE EXERCISE:

Find the marked angles below:





ASSIGNMENT

1. New General Mathematics for SS 1 Ex. 11a, Page 135, No. 6, 7, 8,9, 14
2. New General Mathematics for SS 1 Ex. 11b, Page 139, No. 5, 15, 19
3. New General Mathematics for SS 1 Ex. 11c, Page 142, No. 14, 15
4. New General Mathematics for SS 1, Ex 11d, Page 146, No. 11 – 20
5. New General Mathematics for SS 1, Ex 11e, Page 147, No. 1, 2, 7, 13

WEEK 3

SUBJECT: MATHEMATICS

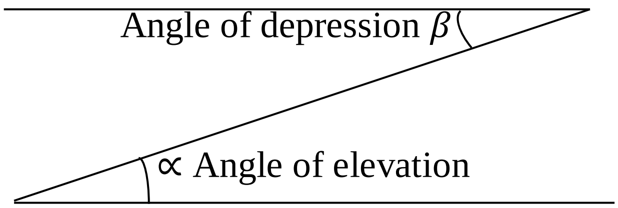
CLASS: SS 1

TOPIC: TRIGONOMETRY 2

CONTENTS:

- (c) Application of trigonometric ratios (angle of elevation and depression; bearing). (d) Trigonometric ratios related to the unit circle. (e) Graphs of sines and cosines.

Application of trigonometric ratios (angle of elevation and depression; bearing).

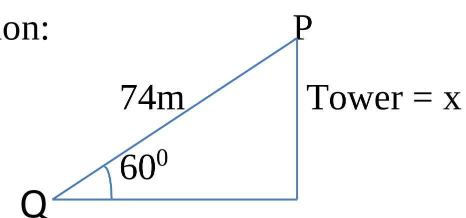


Angle of elevation:

Example 1:

The angle of elevation of a point P on a tower from a point Q on the horizontal ground is 60^0. If /PQ/=74m, how high is P above the ground?

Solution:



The relevant sides to 60^0 are Opp and Hyp (SOH)

$\sin 60^0 = \frac{x}{74}$

$\frac{\sqrt{3}}{2} = \frac{x}{74}$

$x = \frac{74\sqrt{3}}{2}$

$\therefore x = 37\sqrt{3}m$

Example 2:

A man 1.8m tall observes a bird on top of a tree. If the man is 21m away from the tree and his angle of sighting the bird is 30^0, calculate the height of the tree.

Solution:

Tan $30^0 = \frac{k}{21}$

$k = 21 \tan 30°$

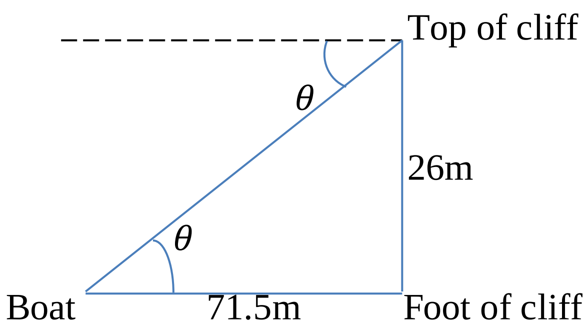
$k = 12.12m$

Thus height of the tree = k + 1.8m = 12.12m + 1.8m = 13.92m

Angle of depression:

Example 3:

A boat can be sighted at the sea 71.5m from the foot of a cliff which is 26m high. Calculate the angle of depression of the boat from the top of cliff, correct to 2 sig. figures.



Tan $\theta = \frac{26}{71.5} = 0.3636$

$\theta = \tan^{-1} 0.3636$

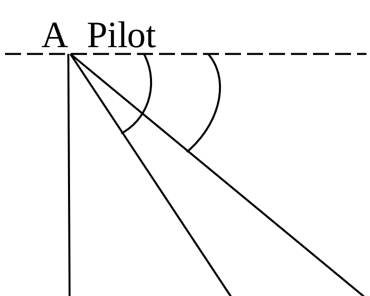
= 19.98

= 20^0 (2 s.f)

Example 4:

The pilot of an air craft 2,000m metres above the sea level observes at an instance that the angles of depression of two boats which are in direct straight line are 58^0 and 72^0. Find correct to the nearest metres, the distance between the two boats.

Solution:



72^0 58^0

2000m

B C x D

The distance between the two boats is $x = BD - BC$

In triangle ABC, tan 72^0 = $\frac{2000}{BC}$

$$BC = \frac{2000}{\tan 72^0}$$

$$= 649.84m$$

In triangle ABD, tan 58^0 = $\frac{2000}{BD}$

$$BD = \frac{2000}{\tan 58^0}$$

$$= 1249.74m$$

But, $x = BD - BC$

$= 1249.74m - 649.84m$

$= 599.9m$

$= 600m$ (to the nearest metres)

Bearing:

(a) A boy sets out to travel from A to C via B. From A he travels a distance of 4km on a bearing 030° to B. From B he travels a further 3km due east. Calculate how far is C

(i) North of A

(ii) East of A

(b) Hence, or otherwise, calculate the distance AC correct to 1 decimal place.

Class Activity:

OBJECTIVE QUESTIONS

1. A ladder 9m long leans against a vertical wall making an angle of 64^0 with the horizontal ground. Calculate correct to one decimal place. How far the foot of the ladder is from the wall?
   A. 4.0m B. 5.8m C. 7.1m D. 8.1m E. 18.5m

2. When an Airplane is 900m above the ground, its angle of elevation from a point P on the ground is 30^0. How far is the plane from P by time of right?
   A. 400m B. 800m C. 1500m D. 1600m E. 1700m

3. The angle of elevation of X from Y is 30^0. If /XY/ = 40m, how high is X above the level of Y?
   A. 10m B. 20m C. 30m D. 40m E. 50m

4. If the shadow of a pole 7m high is ½ its length, what is the angle of elevation of the sun, correct to the nearest degree?
   A. 90^0 B. 63^0 C. 60^0 D. 26^0 E. 0^0
5. From the top of a building 10 m high, the angle of depression of a stone lying on the horizontal ground is 69^0. Calculate, correct to 1 decimal place, the distance of the stone from the foot of the building
   A. 3.8m B. 6.0m C. 9.3m D. 26.1m

THEORY QUESTIONS

1. A ladder of length 4.5 m leans against a vertical wall making an angle of 50^0 with the horizontal, if the bottom of a window is 4m above ground what is the distance between the top of the ladder and the bottom of the window? [Answer correct to the nearest m].

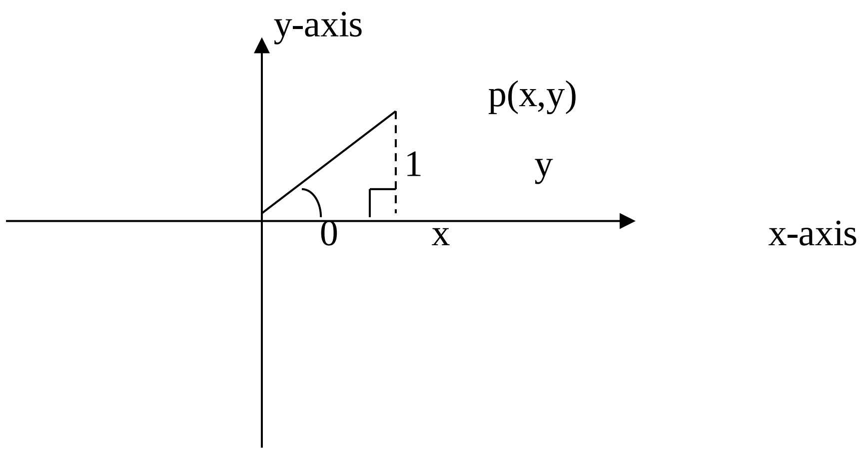
2. From a horizontal distance of 8.5 km, a pilot observes that the angle of depression of the top and base of a control tower are 30^0 and 33^0 respectively. Calculate, correct to 3 significant figures.

   (a) The shortest distance between the pilot and the base of the control tower;
   (b) The height of the control tower.

Trigonometric ratios related to the unit circle

1. Angles between 0^0 and 360^0.

Consider a circle of radius one unit and centre at O, the origin, in the XY plane.



The circle is called a unit circle. Point P has co-ordinate (x,y) and lOPl = 1 unit.

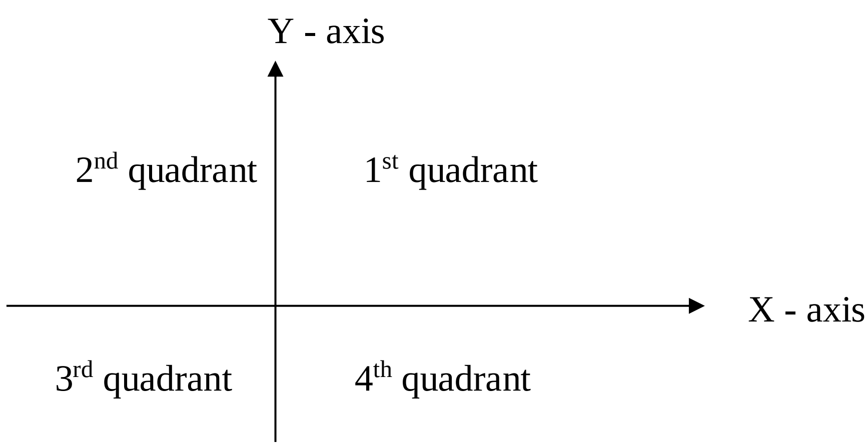
Line Op makes an angle of θ with Ox.

In the right – angled triangles,

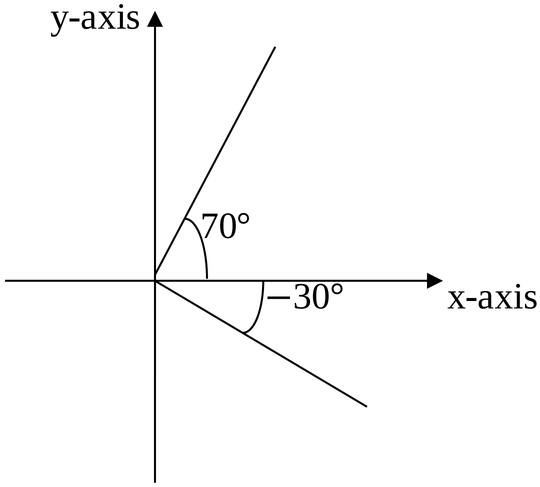
Sinθ = y/1 = y

Cosθ = x/1 = x

The Y – axis and x – axis divides the plane into 4 parts (or 4 quadrants)



Starting from the OX line, positive angles are measured in the anticlockwise direction while the negative angles are measured in the clockwise direction



Notice that $+70^0 = -290^0$ (same direction)

$-30^0 = +330^0$

In what quadrant is (i) $+170^0$ (ii) -170^0 (iii) -45^0 (iv) -260^0 (v)235^0.?

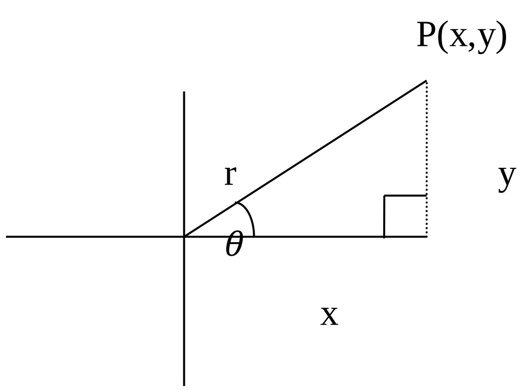
What positive angle is equivalent to

(i) -160^0 (ii) -180^0 (iii) -270^0?

What negative angle is equivalent to

(i) 167^0 (ii) 202^0 (iii) 285^0

For angles in the first quadrant, $0^0 < \theta < 90^0$.



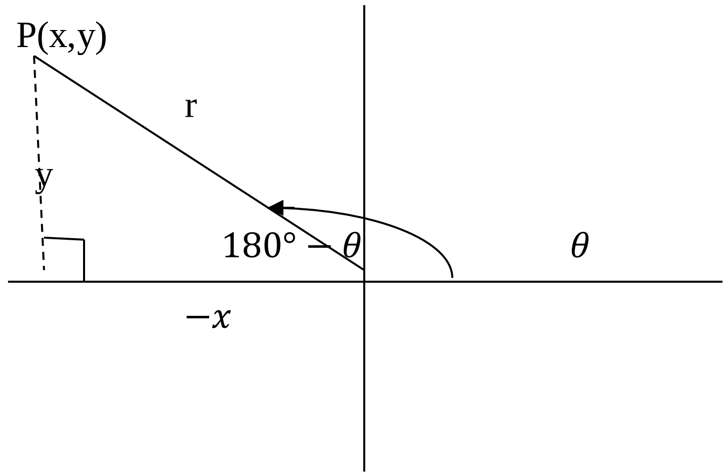
Sin θ = y/x

Cos θ = x/r

tan θ = y/r

All the trigonometric ratios are positive

For angles – the second quadrant, $90^0 < \theta < 180^0$.



In this quadrant, x is negative, y is positive.

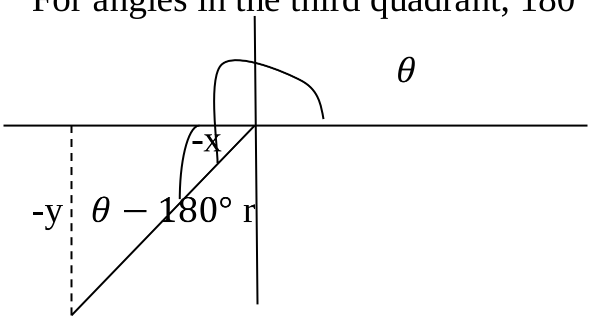
r is always taken to be positive r = $\sqrt{(x^2 + y^2)}$

Sin θ = y/r = Sin $(180 - \theta)$ which is positive

Cos θ = -x/r = -Cos $(180 - \theta)$ which is negative

Tan θ = y/-x = -tan $(180 - \theta)$ which is negative

For angles in the third quadrant, $180^0 < \theta < 270^0$.

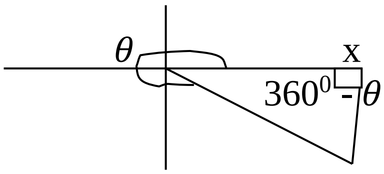


Sin θ = -y/r = -Sin $(\theta - 180^0)$. This is negative

Cos θ = -x/r = -Cos $(\theta - 180^0)$. This is negative

Tan θ = -y/-x = tan $(\theta - 180^0)$. This is positive

For angles negative, the fourth quadratic, $270^0 < 360^0$.



X is positive

Y is negative

Sin θ = -y/r = -Sin (360^0 - θ). This is negative

Cos θ = x/r = Cos(360^0 - θ). This is positive

Tan θ = -y/x = - tan(360^0 - θ). This is negative

Class Activity:

Use tables to find the values of the following.

(i) Cos 130^0
(ii) Tan (-130^0)
(iii) Sin 111^0
(iv) Sin (-320^0)

2. Fill the table below

θ	0^0	30^0	60^0	90^0	120^0	150^0	180^0	210^0	240^0	270^0	300^0	330^0	360^0
Sinθ													
Cosθ													
Tan θ													

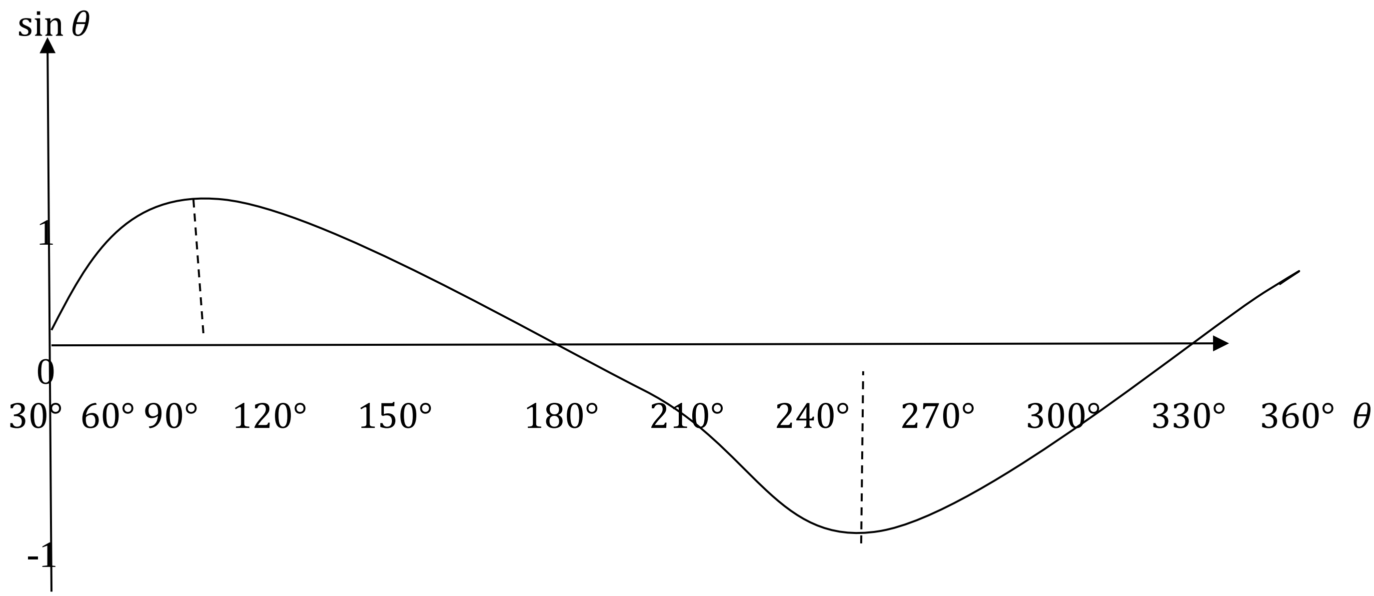
Graphs of Trigonometric function of sine and cosine

The graph of y = Sinθ can be drawn using the unit circle.

One can plot values of y against θ to get a wave-like curve.

For instance when $\theta = 360^0$, the corresponding y value is obtained by drawing dotted horizontal line from mark 30^0 on the unit circle to meet the y – axis.

When $\theta = 90^0$, the dotted horizontal line for 90^0 on the circle will meet the y – axis where y = 1



To draw the graph of Cos x, use the corresponding values of x and θ.

For instance on the circle, when θ = 0^0, that corresponds to x = 1, when θ = 60^0, dotted line from point 600 on the unit circle vertically downwards will meet the x-axis at x = ½

(i.e when θ = 60^0, x = ½).

Plotting values of x against corresponding values of θ gives the graph of Cos θ . it is another wave-shaped curve. As θ increases beyond 360^0, the curves of Sin θ and Cosθ repeat themselves.

Class Activity:

1. Draw the graph of $y = 2\cos\theta$ for $0° \leq \theta \leq 360°$
2. Draw the graph of $y = 3\sin\theta$ for $0° \leq \theta \leq 360°$

PRACTICE EXERCISE:

1. At a point 500m from the base of a water tank the angle of elevation of the top of the tank is 45^0, find the height of the tank.
A. 250m B. 353m C. 354m D. 433m E. 500m (SSCE 1993)

2. A ladder 6m long leans against a vertical wall so that it makes an angle of 60^0 with the wall. Calculate the distance of the foot of the ladder from the wall.
A. 3m B. 6m C. 2√3m D. 3√3m E. 6√3m (SSCE 1994)

3. The angle of elevation of the top X of a vertical pole from a point P on a level ground is 600, the distance from a point P to the foot of the pole is 55 m, without using tables, find the height of the pole.

A. 50/3m B. 50m C. 55√3m D. 60m E. 65cm (SSCE 1996)

4. A boat is on the same horizontal level as the foot of a cliff, and the angle of depression of the boat from the top of the cliff is 30^0. If the boat is 120m away from the foot of the cliff, find the height of the cliff correct to 3 significant figures. (SSCE 1992)

5. A simple measuring device is used at points X and Y on the same horizontal level to measure the angle of elevation of the peak P of a certain mountain. If X is known to be 5,200m above sea level,

/XY/ = 4000m and the measurement of the angles of elevation of P at X and Y are 15^0 and 35^0 respectively, find the height of the mountain. (Take tan 15^0 = 0.3 and tan 35^0 =0.7) (SSCE 1993)

6. Draw the graph of $y = 2sin\ x + 3cos\ x$ for $0° \leq x \leq 360°$ at intervals of 30^0. Using the graph find the solution set of the equation $4sin\ x + 6cos\ x = -3$

ASSIGNMENT:

1. The angle of elevation of a point T on a tower from a point U on the horizontal ground is 30^0, if

TU = 54 cm, how high is T above the horizontal ground?

A. 108m B. 72m C. 46.3m D. 31.2m E. 27m (SSCE 1997)

2. A ladder 5m long rest against a wall such that its foot makes an angle of 30^0 with the horizontal. How far is the foot of the ladder from the wall.

A. 5√2/3m B. 2½m C. 5√3/2m D. 10√3/3m E. 10√3m (SSCE 1998)

3. The angles of depression of the top and bottom of a building are 51^0 and 62^0 respectively from the top of a tower 72m high. The base of the building is on the same horizontal level as the foot of the tower. Calculate the height of the building correct to 2 significant figures. (SSCE 2004)

4. From two points on opposite sides of a pole33m high, the angles of elevation of the top of the pole are 53^0 and 67^0. If the two points and the base of the pole are on the same horizontal level, calculate, correct to three significant figures, the distance between the two points. (SSCE 2007)

5. (i) Copy and complete the following table for f:x → $9cosx + 6sinx$

x	0^0	30^0	60^0	90^0	120^0	150^0	180^0	210^0	240^0 360^0	270^0	300^0	330^0
f(x)							-9					

(ii) Draw the graph of f for $0° \leq x \leq 360°$

(iii) Use your graph to estimate; the maximum and minimum values of f correct to 1 decimal place, stating nearest degree for which they occur.

(iv) The truth set of the equation $3cosx + 2sinx = \frac{5}{3}$

WEEK 4

SUBJECT: MATHEMATICS

CLASS: SS 1

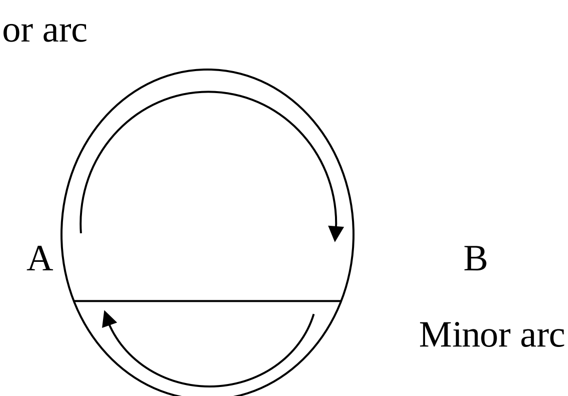
TOPIC: MENSURATION

CONTENTS:

(a) Length of arcs of circle.
(b) Perimeter of sectors and segments.
(c) Areas of sectors of a circle.
(d) Areas of segments of a circle.

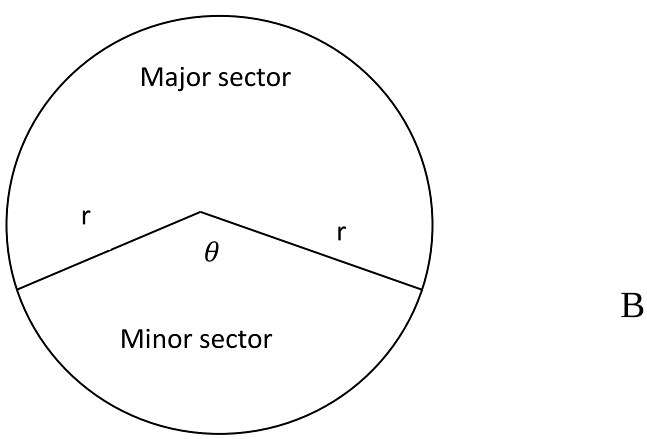
DEFINITION AND MEANING, LENGTH OF ARCS OF CIRCLE

(i) **AN ARC**: an arc of a circle is a part of the circumference of the circle.



Hence, an arc is a length or a distance along the circumference of a circle. It is never an area.

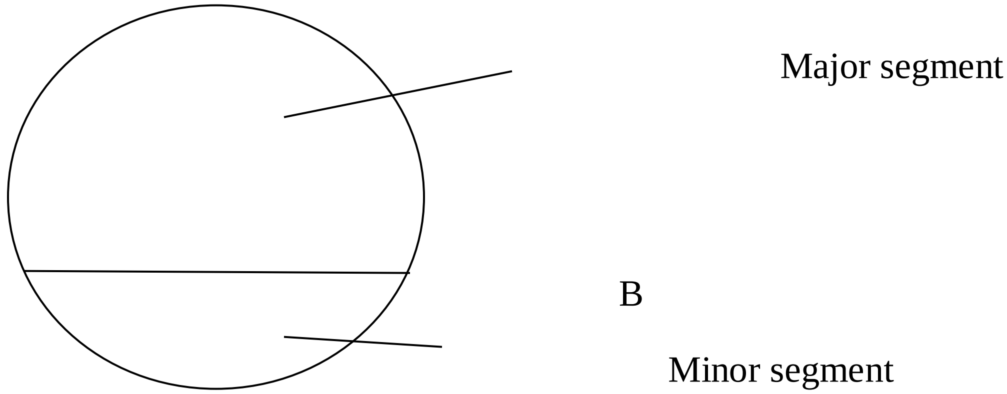
(ii) **A SECTOR**: a sector is a part or a fraction of a circle bounded by an arc and two radii.



Hence, an arc is a length whereas the sector covers an area of a circle.

(iii) A SEGMENT

The segment of a circle is the part cut off from the circle by a chord. A chord is the line segment AB.



LENGHT OF ARCS OF CIRCLES

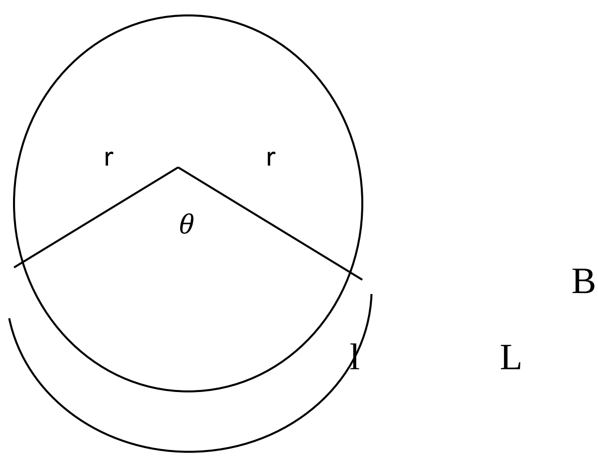
If 5 sectors are cut off from 5 different circles and the lengths of the arcs l, radii r and angles θ measured and compared.

Then

i/2π = θ/360 or L = θ/360 x 2 πr = 2 πr θ/360

hence, in a circle of radius r, the length l of an arc that subtends angle θ at the centre is given by

$$L = \frac{\theta}{360^\circ} \times 2\pi r$$



Example 1:

Find the length of an arc of a circle of radius 5.6cm which subtends an angle of 60^0 at the centre of the circle (Take π = 22/7)

Solution

Length of arc = $\theta/360 \times 2\pi r$

Given $\theta = 60^0$, r = 5.6cm, $\pi = 22/7$

Substituting into the formula,

Length of arc AB = $60/360 \times 2 \times 22/7 \times 5.6$

$= 1/6 \times 2 \times 22/7 \times 5.6$

$= 1/6 \times 2 \times 22/7 \times 0.8$

$= 17.6/3$

= 5.8667cm

= 5.87cm (2 decimal places)

Example 2;

What angle does an arc 6.6cm in length subtends at the centre of a circle of radius 14cm. Use $\pi = 22/7$)

Solution

Length of arc xy = $\theta/360 \times 2\pi r$

$6.6 = \theta/360 \times 2 \times 22/7 \times 14$

$\theta \times 2 \times 22 \times 14 = 6.6 \times 360 \times 7$

$\theta = \frac{6.6 \times 360 \times 7}{2 \times 22 \times 14}$

$= (33 \times 18)/11 \times 2$

$= 3 \times 9$

$= 27^0$

$\theta = 27^0$, the angles subtend by the arc

Example 3: An arc of length 12.57cm subtends an angle of 60^0 at the centre of a circle. Find

The radius of the circle

The diameter of the circle.

Solution

Arc = θ/360 x 2 πr

12.57 = 60/360 x 2 x 22/7 x r/1

12.57 x 360 x 7 = 60 x 2 x 22 x r

$r = \frac{12.57 \ x \ 360 \ x \ 7}{60 \ x \ 2 \ x \ 22}$

r = $\frac{12.57 \ x \ 6 \ x \ 7}{44}$

r = 527.94/44

r = 11.99

r = 12cm

Example 4:

An arc of a circle of diameter 28m subtends an angle of 108^0 at the centre of the circle. Find the length of the major arc.

Solution

Minor arc angle = 108^0

Major arc angle = $360^0 - 108^0$

= 252^0.

Arc = = θ/360 x 2 πr

= 252/360 x 2/1 x 22/7 x 14/1

= $\frac{252 \ x \ 44}{180}$

= $\frac{11088}{180}$

= 61.6m

The length of major arc = 61.6m

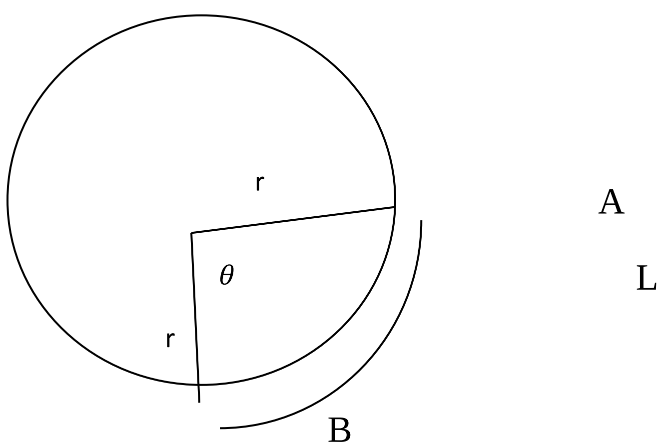
Class Activity:

1. Find the radius of a circle which subtends an angle of 120^0 at the centre of the circle and is of length 2.8cm ($\pi = 22/7$)
2. In terms of π, what is the length of an arc of a circle of radius $3^1/_2$m?

- **PERIMETER OF SECTORS AND SEGMENTS**

PERIMETER OF SECTORS

The word perimeter simply means the distance round an object. So the perimeter of a sector of a circle is the distance round the circle.



hence the perimeter of a sector AOB is the sum of two radii (2r) and length of arc l, where r is radius and l = length of arc.

Perimeter of sector AOB = $\frac{\theta}{360°} \times 2\pi r + 2r$

Example

Find the perimeter of the sector of radius 3.5cm which subtends an angle of (i) 45^0 (ii) 315^0.

Solution

Length of arc = $\frac{\theta}{360}$ x $2\pi r$

Here θ = 45^0, r = 3.5cm, π = $\frac{22}{7}$

Length of arc = $\frac{45}{360}$ x 2 x $\frac{22}{7}$ x 3.5

= $\frac{1}{8}$ x $\frac{2}{1}$x $\frac{22}{7}$ x $\frac{7}{2}$

= 22/8

= 2.75cm

The perimeter of the sector is 2r + l, here r = radius which is 3.5cm and l = 2.75cm.

Perimeter = 2r + length of arc

= 2r + 2.75cm

= (2 x 3.5) + 2.75cm

= 7.0 + 2.75cm

= 9.75cm

(ii) θ = 315^0, π = $\frac{22}{7}$, r = 3.5cm

Length of arc = $\frac{315}{360}$ x 2 x $\frac{22}{7}$ x 3.5cm

= $\frac{63}{72}$ x 2 x $\frac{22}{7}$ x $\frac{7}{2}$ cm

= $\frac{7}{8}$ x 2 x $\frac{22}{7}$ x $\frac{7}{2}$ cm

= 77/4 = 19.25cm

Perimeter = 2r + 19.25cm

= (2 x 3.5) + 19.25

= 7 + 19.25

= 26.25cm

Class Activity:

1. Calculate the perimeter of a sector of a circle of radius 14cm, where the sector angle is 60^0. Take $\pi = \frac{22}{7}$
2. The perimeter of a sector is 61.43cm. If the angle subtended by the sector at the centre is 120^0. Find the radius of the sector.

PERIMETER OF SEGMENT

The perimeter of a segment = length of arc + length of chord.

To find the length of chord AB, we bisect <AOB

Using trigonometric ratios

$\frac{AD}{r} = \text{Sin}\,\frac{\theta}{2}$

AD = r Sin $\frac{\theta}{2}$

But, AD = DB

Hence

AB chord AB = AD + DB

$= \text{r Sin}\,\frac{\theta}{2} + \text{r Sin}\,\frac{\theta}{2}$

$= \text{2r Sin}\,\frac{\theta}{2}$ units

Length of Chord = 2r Sin $\frac{\theta}{2}$ units

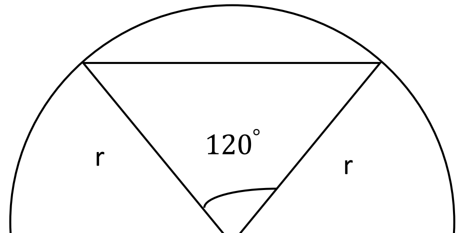
Thus,

Perimeter of segment = $\frac{\theta}{360}$ x $2\pi r$ + 2r Sin $\frac{\theta}{2}$

Example :

AB is a chord of a circle with centre o and radius 4cm, $\angle AOB = 120^0$. Calculate the perimeter of the minor segment ($\pi = \frac{22}{7}$).

Solution



A B B

Chord AB = rSin $\frac{\theta}{2}$, r = 4cm, θ = 120^0.

$\frac{\theta}{2} = \frac{120}{2} = 60^0$.

So chord AB = 2 x 4 Sin 60^0

= 8 x sin 60^0

= 8 x $\frac{\sqrt{3}}{2}$

= $4\sqrt{3}$cm

= 6.92cm

Length of arc AB = $\frac{120}{360}$ x $2 \; x \; 4$ x $\frac{22}{7}$

= $\frac{1}{3}$ x 8 x $\frac{22}{7}$

= $\frac{176}{21}$

= 8.38cm (2 d.p)

Perimeter of minor segment = length of arc + chord

= 8.38 + 6.92cm

= 15.30cm

AREA OF SECTORS OF A CIRCLE

The area of a sector = $\frac{\theta}{360}$ x $area\ of\ circle$

Area of a sector = $\frac{\theta}{360}$ x πr^2

where θ is the angle formed at the centre by the arc of the circle.

Example 1:

Find the area of the sector of a circle of radius 4.8cm which subtends an angle of 135^0 at the centre.(Take π = 3.142)

Solution

The area of a sector = $\frac{\theta}{360}$ x πr^2

Here θ = 135^0, r = 4.8cm, π = 3.142

The area of a sector = $\frac{135}{360}$ x 3.142 x $(4.8)^2$

= $\frac{135}{360}$ x 3.142 x 4.8 x 4.8

= 27 x 3.142 x 0.4 x 0.8

= $27.14688cm^2$

Area of sector = $27.15cm^2$ (decimal places)

Example 2:

AB is an arc of a circle of length 9.2cm with centre 0 and the radius is 4.6cm. Find the area of the sector AOB.

Solution

Length of arc AB = $\frac{\theta}{360}$ x $2\pi r$

r = 4.6cm, length of arc = 9.2cm

Length of arc AB = $\frac{\theta}{360}$ x 2 x π x 4.6

9.2 = $\frac{\theta}{360}$ x 2 x π x 4.6

9.2 x 360 = θ x 2 x π x 4.6

$\theta = \frac{9.2 \, x \, 360}{2 \, \pi \, x \, 4.6}$

$\theta = \frac{360}{\pi}$

Area of sector AOB

$= \frac{\theta}{360}$ x πr^2

$= \frac{\theta}{360}$ x πr^2

$= \frac{\theta}{360}$ x $\frac{1}{360}$ x π x $(4.6)^2$

$= (4.6)^2$

= (4.6) x (4.6)

$= 21.16cm^2$

Class Activity:

1. Calculate the area of a sector of a circle which subtends an angle 45^0 at the centre of the circle, radius 14cm.
2. A sector of 80^0 is removed from a circle of radius 12cm. What area of the circle is left? Use $(\pi = \frac{22}{7})$.
3. New General Mathematics for senior secondary school, Book 1 page 158 exercise 12d Nos 1 – 6.

AREA OF SEGMENTS OF CIRCLES

Area of segments = area of sector – area of triangle

∴ **Area of segment** $= \frac{\theta}{360}$ **x** $\pi r^2 - \frac{1}{2} r^2 \text{ Sin } \theta$

Example:

The arc AB of a circle, radius 6.5cm, subtends an angle of 45^0 at the centre O. Find the area of the minor segment cut off by the chord AB (Take $\pi = \frac{22}{7}$)

Solution:

Area of segment = Area of sector AOB – Area of ΔAOB

Area of sector AOB $= \frac{\theta}{360}$ x πr^2

θ = 45^0, r = 6.5, π = $\frac{22}{7}$

Area of sector AOB = $\frac{45}{360}$ x $\frac{22}{7}$ x $(6.5)^2$$cm^2$

= $\frac{1}{8}$ x $\frac{22}{7}$ x 6.5 x 6.5

= $\frac{1}{8}$ x $\frac{22}{7}$ x 42.25

= 16.598cm^2

Area of ΔAOB = $\frac{1}{2}AO$ X OB $Sin45^0$

= $\frac{1}{2}$ X 0.5 x 6.5 $Sin45^0$

= $\frac{42.25}{2}$ x $\frac{1}{\sqrt{2}}$

= $\frac{42.25\sqrt{2}}{4}$

= $\frac{42.25\ x\ 1.414}{4}$

= 14.935375cm^2

∴ Area of segment = 16.598 – 14.935

= 1.66cm^2 (2 d.p)

Class Activity:

1. An arc AB of a circle radius 4.8cm subtends an angle of 158^0 at the centre O. Find
   a. The area of the sector AOB
   b. The area of the minor segment cut off by the chord AB (π = 3.142)
2. A sector of a circle radius 16cm subtends an angle of 84^0 at the centre O. Calculate the area of the shaded segment of the circle. (Take $\pi = \frac{22}{7}$)

PRACTICE EXERCISE:

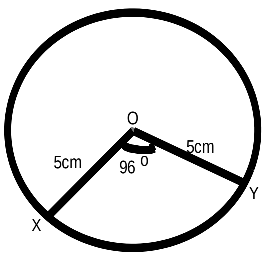
1. A chord of a circle subtends an angle of 60^0 at the centre of a circle of radius 14cm. Find the length of the chord.
2. A rope of length 18m is used to form a sector of a circle of radius 2.5m on a school playing field. What is the size of the angle of the sector? Correct to the nearest degree?
3. An arc of length 21.34cm subtends an angle 101^0 at the centre of a circle. Find the diameter of the circle.

4. The perimeter of a sector is 75.43cm. If the angle – subtended by the sector at the centre is 135^0. Find the radius of the sector.
5. XY is a chord of a circle centre O and radius 7cm. The chord XY which is 8cm long subtends an angle of 120^0 at the centre of the centre of the circle. Calculate the perimeter of the minor segment.

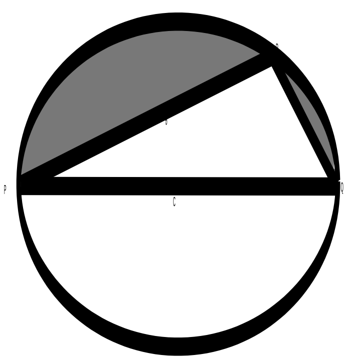
   (Take $\pi = \frac{22}{7}$)

   ASSIGNMENT:

1. XOY is a sector of a circle centre O of radius 3.5cm which subtends an angle of 144^0 at the centre. Calculate, in terms of π, the area of the sector.
2. In the diagram, XY is a chord of a circle of radius 5cm. The chord subtends an angle 96^0 at the centre. Calculate, correct to 3.s.f; the area of the minor segment cut-off by XY.



3.



In the diagram, C is the centre of the circle of radius 5cm, and /PR/ = 8cm. Find the area of the shaded region.

WEEK 5

SUBJECT: MATHEMATICS

CLASS: SS 1

TOPIC: MENSURATION

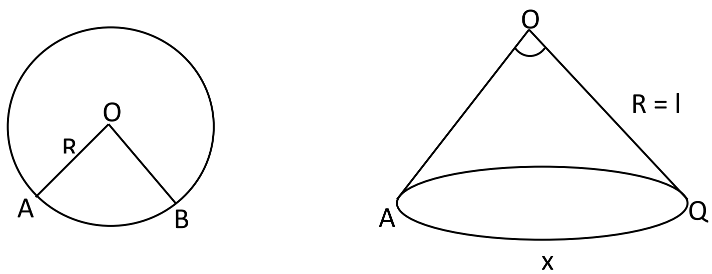
CONTENTS:

(e) Relation between the sector of a circle and the surface area of a cone.

(f) Surface area and volume of solids; (i) Cube, cuboids (ii) Cylinder (iii) Cone (iv) Prisms (v) Pyramids.

Relation between the sector of a circle and the surface area of a cone.

If a sector of a circle AOB is cut and folded into a come as shown in the diagram below.



The arc AB becomes the circumference of the base of the cone. The radius R becomes the slant edge l of the cone.

$\therefore$ Arc AB = $2\pi r$

From the above diagram.

The area of the sector = area of the curved surface of the cone

Length of arc AB = circumference of the circular base of the cone

Curved surface area of cone = $\frac{\theta}{360^0}$ x πl^2

Also, $\frac{\theta}{360^0}$ x $2\pi l = 2\pi r$

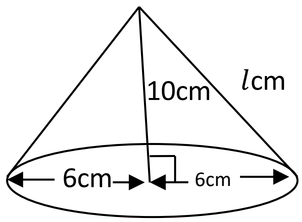
$\frac{\theta}{360^0} = \frac{r}{l}$

Total surface area of cone = $\pi rl + \pi r^2$

= $\pi r(l + r)$.

Example 1: Calculate in terms of π, the total surface area of a cone of base diameter 12cm and height 10cm.

Solution



Using Pythagoras rule,

$l^2 = 10^2 + 6^2$

$l^2 = 100 + 36$

$= 136$

$l = \sqrt{136}$

Total surface area

$= \pi rl + \pi r^2$

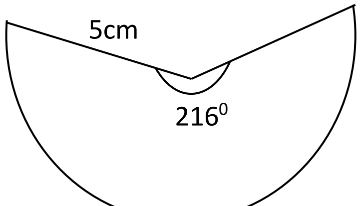
$= \pi r(l + r)$.

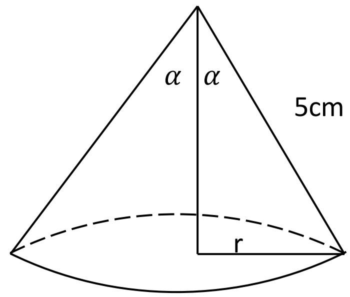
$= 6\pi(\sqrt{136} + 6)cm^2$.

Example 2:

A 216^0 sector of a circle of radius 5cm is bent to form a cone. Find the radius of the base of the cone and its vertical angle.

Solution





Radius = r

Vertical angle = 2α

Circumference of base of cone = length of arc of sector

Substituting values

$\frac{216}{360^0}$ x $2\pi \ x \ 5 = 2\pi r$

r = $\frac{216}{360^0}$ x $2\pi \ x \ 5$ x $\frac{1}{2\pi}$

r = $\frac{216}{72}$= 3

∴ r = 3cm

Sin α = $\frac{3}{5}$ = 0.6000

α = 36.87^0

2α = 36.87^0 x 2

= 73.74^0

Radius of base = 3cm

Vertical angle = 73.7^0 (to 0.1^0)

Class Activity:

Find the curved surface and total surface areas of a closed cone of height 4cm and base radius 3cm. (Take $\pi = \frac{22}{7}$)

SURFACE AREA OF CUBES, CUBOIDS AND CYLINDER

CUBE: since a cube is solid with six faces

Area of one face is a x a = a^2.

Total area = a x a x 6 = $6a^2$.

Surface Area = $6a^2$ units

Example 1:

What is the surface area of a cube of edges 12cm.

Solution

A = 6 x 12 x 12

= 6 x 144

= $864cm^2$

Example 2: Calculate the surface area of a cube of edge 11cm.

Solution

A = $6a^2$

= 6 x 11 x 11

= 6 x 121

= $726cm^2$

CUBOIDS

Surface area of a cuboids = 2(lb + bh + lh)

Example 3: Given a cuboid of edges 3cm, 5cm, 8cm. calculate the surface area.

Solution

Surface area of a cuboids = 2(lb + bh + lh)

= 2(8 x 5 +8 x 3 + 5 x 3)

= 2(40 + 24 + 15)

$= 2 \times 79$

$= 158cm^2$

CYLINDER

Area of cylinder with one end closed = $2\pi rh + \pi r^2$

$= \pi r(2h + r)$

If two ends are closed.

Area = $2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$, where r = radius of cylinder, h = height of cylinder

Example 4:

Find the surface area of a cylinder with height 14cm and base radius of 7cm, consider all the three cases namely;

(a) Open ended cylinder
(b) One end open cylinder
(c) Both ends closed cylinder

Solution

(a) Open ended cylinder

Area = $2\pi rh$

r = 7cm, h = 14cm

$A = 2 \times \frac{22}{2} \times 7 \times 14$

$= 44 \times 14$

$= 616cm^2$

(b) One end open cylinder
Area = $2\pi rh + \pi r^2$
$= (616 + \frac{22}{2} \times 7^2)\ cm^2$

$= (616 + 154)\ cm^2$

$= 770cm^2.$

(c) Both ends closed cylinder

Area = $2\pi r(h + r)$

$= 2 \times \frac{22}{2} \times 7(14 + 7)cm^2$

$= \frac{44}{2} \times 7 \times 21$

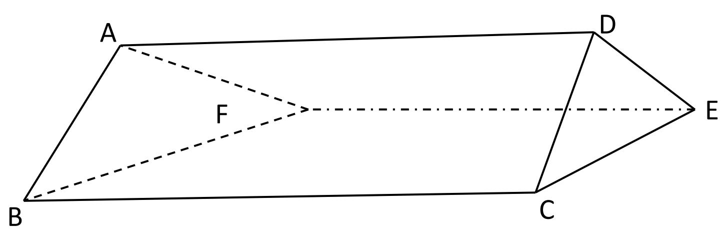
$= 924cm^2.$

Class Activity:

1. Calculate the surface area of a hollow cylinder which is closed at one end, if the base radius is 3.5cm and the height is 8cm. Take (Take $\pi = \frac{22}{7}$)
2. A Cylindrical container, closed at both ends, has a radius of 7cm and height 5cm. Find the total surface area of the container (Take $\pi = \frac{22}{7}$)

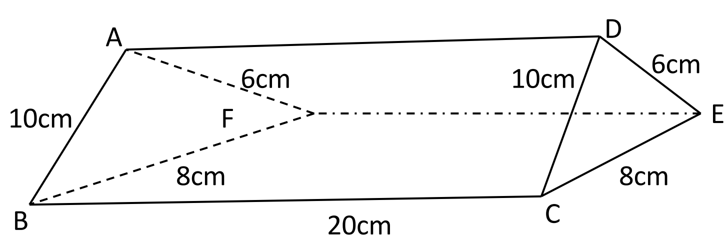
SURFACE AREA OF PRISMS AND PYRAMIDS

PRISMS: The total surface area is the sum of the surface of the five faces as shown below



Total surface area of prism = Areas of (ABCD + AFED + BCEF) + Areas of ($\Delta S(ABF + DCE$)

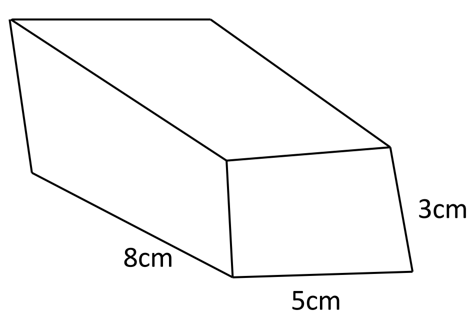
Example 1:
Find the total surface area of the prism shown below



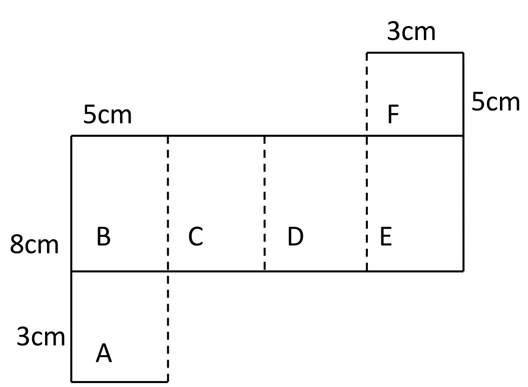
Solution
Area of ABCD
$=20 \text{ x } 10\text{cm}^2$
$= 200\text{cm}^2$
Area of AFED $= 20 \text{ x } 6\text{cm}^2$
$= 120\text{cm}^2$
Area of BCEF $= 20 \text{ x } 8\text{cm}^2$
$= 160\text{cm}^2$
Area of ΔABF = area of ΔDCE
$= \frac{1}{2} \text{ X } 8 \text{ X } 6\text{cm}^2$
$= 24\text{cm}^2$
Total surface area of prism $= (200 + 120 + 160 + 24 + 24)\text{cm}^2$
$= 528\text{cm}^2$

In the case of a rectangular prism or cuboid, the total surface area is determined by finding and summing up the areas of the four rectangular faces and the two end faces.

Examples 2: Find the total surface area of the solid.



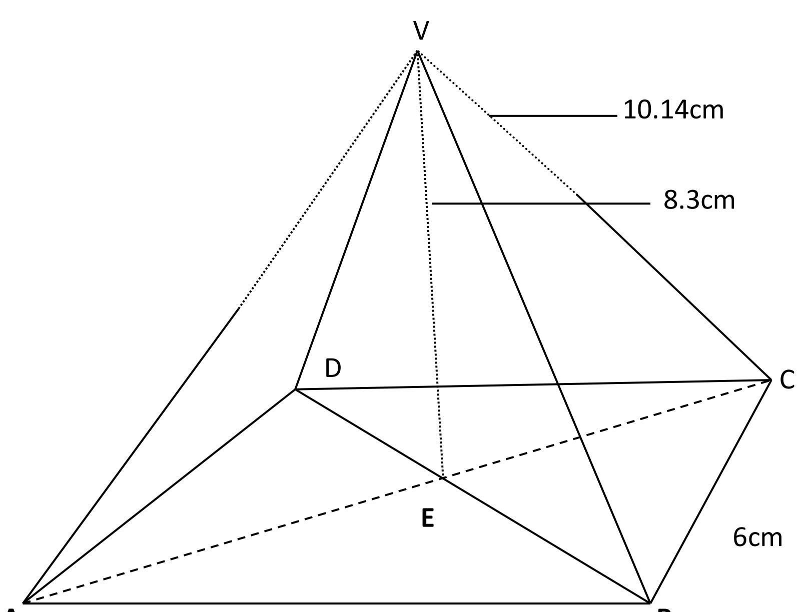
Solution
The net of the solid gives a clearer picture of the shape



Area of A = 5 x 3cm^2 = 15cm^2
Area of B = 8 x 5cm^2 = 40cm^2
Area of C = 8 x 3cm^2 = 24cm^2
Area of D= 8 x 5cm^2 = 40cm^2
Area of E = 8 x 3cm^2 = 24cm^2
Area of F = 5 x 3cm^2 = 15cm^2
∴ Total surface area = (15 + 40 + 24 + 40 + 24 + 15)cm^2
= 158cm^2.

PYRAMIDS: The total surface area of a pyramid is found by summing up areas of the common shapes that make up the pyramid.

Example 3:
Find the total surface area of a right pyramid with a rectangular base 6cm by 10cm, a height of 8.3cm and a slant edge of 10.14cm.



Solution

The total surface area is the sum of the surface areas of the five faces:

ABCD, VAB, VDC, VBC and VAD

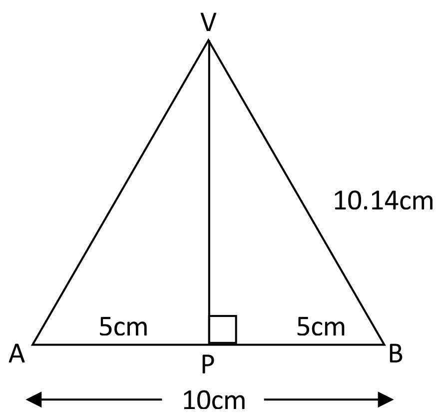
ABCD is a rectangle so AB = DC and AD = BC

If AB = 10cm, then DC = AB = 10cm

If AD = 6cm, then BC = AD = 6cm

Hence, area of ABCD = 10 x 6 = 60cm^2

Area of ΔVAB = Area of ΔVDU and Area of ΔVBC = Area of ΔVAD



Area of ΔVAB = $\frac{1}{2}$ x AB x VP

By using Pythagoras rule

VP^2 = VB^2 – PB^2

= (10.14)^2 - 5^2

= 102.8 – 25

= 77.8

VP = $\sqrt{77.8}$

$= 8.82cm$

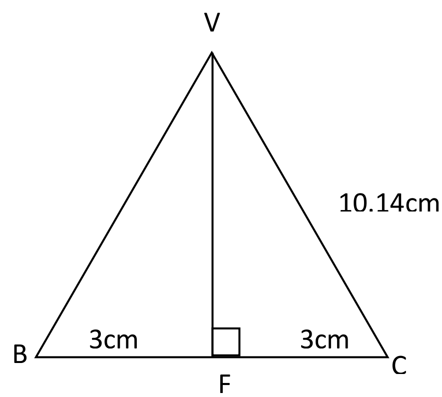
Area of ΔVAB = $\frac{1}{2}$ x AB x VP

= $\frac{1}{2}$ x 10 x 8.82

= 5 x 8.82

= 44.10cm^2

Area of ΔVDC = Area of ΔVAB



Area of ΔVBC = $\frac{1}{2}$ x AB x VF

But VF^2 = VC^2 – FC^2

= (10.14)^2 – 3^2

= 102.8 – 9

= 93.8

VF = $\sqrt{93.8}$

= 9.69cm

Area of ΔVBC = $\frac{1}{2}$ x BC x VF

= $\frac{1}{2}$ x 6 x 9.69

= 3 x 9.69

= 29.07cm^2

Area ofΔVAD = Area of ΔVBC

Area of rectangular base ABCD = 10 x 6 = 60cm^2.

Total surface area of the pyramid = [(41.10 + 41.10 + 29.07 + 29.07 + 60)]
= (82.20 + 58.14 + 60) cm^2
= $200.34cm^2$

Class Activity:

1. Find the length of the slant edge of a right pyramid with
   a. A rectangular base 3cm by 5cm and a height of 4.2cm
   b. A rectangular base 6cm by 4.5cm and a height of 3.4cm
2. Find the surface area of a triangular prism 10.8cm long and having a triangular face of dimensions 8.8cm by 5.7cm by 6.8cm.

VOLUME OF CUBES, CUBOIDS, CYLINDER, CONE, PRISMS AND PYRAMIDS

The Volume of cube = s^2 x s = s^3

Example 1:

Find the volume of a cube whose edges are 7cm each.

Solution

Volume = s^2 x s = s^3
= 7 x 7 x $7cm^3$
= 49 x $7cm^3$
= $343cm^3$

Volume of cuboids

The volume of a cuboid = Area x height
= lb x h
= lbh cm^3

Example 2:

What is the volume of a cuboid if

(a) Height = 6cm, breadth = 10cm, length = 13cm?
(b) Area of cross – section is $105m^2$ and height = 5cm?
(c) Area of cross-section has the same numerical value with the height = 16cm?

Solution

(a) Volume (V) = lbh = 13 x 6 x 10 = $780cm^3$
(b) Volume = Ah = 105 x 5 = $525cm^3$
(c) V = A x h
= 16 x 16

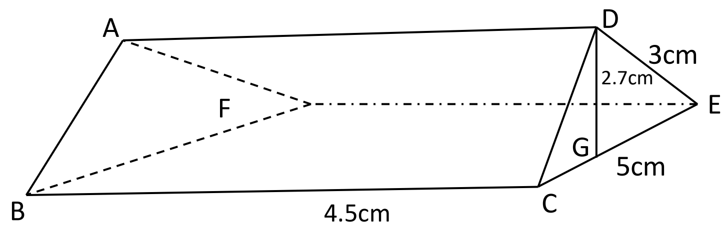
= $256cm^3$

Volume of cylinder

With the Cylinder the cross section is a circle.

Area = πr^2, where is the radius of the circular base. Height = h

∴ Volume = Area x height
= πr^2 x h
= πr^2hcm3
Example 4:
What is the volume of a cylinder if the radius is 5cm and height is 20cm?
Solution
Volume = area of circular base x height
= πr^2 x h
= $\frac{22}{7}$ X 5^2 X $20cm^2$
= $\frac{22}{7}$ X 5 X x 5 x $20cm^2$
= $1571.43cm^3$
Volume of Cone
The volume of cone = $\frac{1}{3}$ x volume of cylinder
= $\frac{1}{3}$ πr^2h where r = radius of the cone and h = height of the cone.
Example 5:
Find the volume of a cone if the perpendicular height is 9cm and radius 4cm.
Solution
Volume of cone = $\frac{1}{3}$ πr^2h
r = 4cm, h = 9cm
V = $\frac{1}{3}$ x $\frac{22}{7}$ x 4 x 4 x $9cm^3$
= $\frac{3168}{21}cm^3$
= $150.86cm^3$.
Volume of prisms
Volume of prisms = area of cross – section x distance between the end faces.
Example 6:
Find the volume of the triangular prism in the diagram below.



Solution

Volume of prisms = area of cross – section x distance between the end faces.

Here, end face = ΔCDE

Area of ΔCDE = $\frac{1}{2}$ CE x DG

= $\frac{1}{2}$ x 5 x 2.6cm^2

= $\frac{13.5}{2}$$cm^2$

= 6.75cm^2

Volume of prism = Area of ΔCDE x BC

= 6.75 x 4.5cm^3

30.375cm^3

Volume of triangular prism

= 30.4cm3

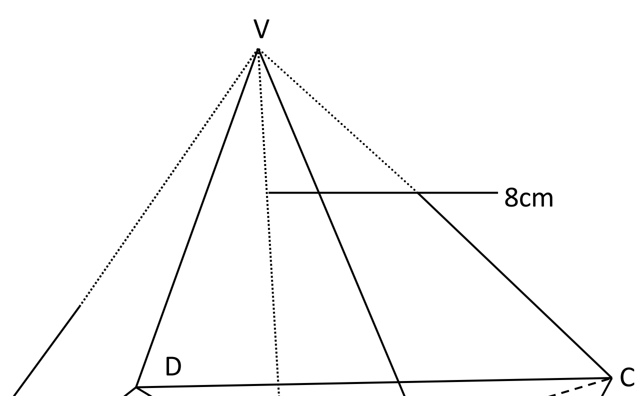
Volume of pyramids

Volume of pyramid = $\frac{1}{3}$ x base area x perpendicular height

Example 7:

Find the volume of right pyramid with vertex V and a rectangular base measuring 5.4cm by 4cm and a height of 8cm.

Solution



Volume of pyramid = $\frac{1}{3}$ x base area x perpendicular height

= Volume of pyramid = $\frac{1}{3}$ x 5.4 x 4 x 8cm^3

Volume of pyramid = $\frac{1}{3}$ x 5.4 x 32cm3

Volume of pyramid = $\frac{172.8}{3}$cm^3

= 57.6cm^3

PRACTICE EXERCISE:

1. Find the total surface area of a solid circular cone with base radius 3cm and slant height 4cm

[Take $\pi = \frac{22}{7}$]

A. $37\,{}^5/_7$cm^2 B. $75\,{}^3/_7$cm^2 C. 66cm^2 D. 88cm^2 E. $78\,{}^2/_7$cm^2
(SSCE 1995)

2. A hollow sphere has a volume of k cm^3 and a surface area of k cm^3. Calculate the diameter of the sphere.

A. 3cm B. 6cmnd base C. 9cm D. 12cm E. more information is needed

(SSCE 1995)

3. Calculate the total surface area of a solid cone slant height 15cm and base radius 8cm in terms of π.

   A. $64\pi cm^2$ B. $120\pi cm^2$ C. $184\pi cm^2$ D. $200\pi cm^2$ E. $320\pi cm^2$ (SSCE 1997)

4. The cross-section of a prism is a right-angled triangle 3 cm by 4 cm by 5 cm. the height of the prism is 8cm. Calculate its volume.

   A. $48cm^3$ B. $60cm^3$ C. $96cm^3$ D. $120cm^3$ E. $240cm^3$ (SSCE 1997)

5. Find the curved surface area of a cone of radius 3cm and slant height 7cm. [Take $\pi = \frac{22}{7}$]

   A. $22cm^2$ B. $44cm^2$ C. $66cm^2$ D. $132cm^2$ E. $198cm^2$ (SSCE 1998)

6. The height of a pyramid on a square base is 15cm. If the volume is 80cm, find the area of the square base.

   A. $8cm^2$ B. $9.6cm^2$ C. $16cm^2$ D. $25cm^2$ (SSCE 2000)

7. A right pyramid is on a square base of side 4cm. The slanting side of the pyramid is $\sqrt[2]{3}$cm. Calculate the volume of the pyramid.

   A. $5^1/_3 cm^2$ B. $10^2/_3 cm^2$ C. $16cm^2$ D. $32cm^2$ (SSCE 2001)

8. The height of a right circular cone is 4cm. The radius of its base is 3cm. Find its curved surface area.

   A. $9\pi cm^2$ B. $15\pi cm^2$ C. $16\pi cm^2$ D. $20\pi cm^2$ (SSCE 2001)

9. A sector of a circle of radius 7 cm subtending an angle of 270^0 at the centre of the circle, issued to form a cone.

a. Find the base radius of the cone
b. Calculate the area of the base of the cone correct to the nearest square centimeter. (Take $\pi = \frac{22}{7}$) (SSCE 1988)

10. A solid metal cone of height 20cm and radius 12cm is melted down to form a cylinder of the same height. What is the radius of the cylinder?

ASSIGNMENT:

1. A right pyramid has a square base of side 8cm. The height of the pyramid is half the side of the square. Find the length of the sloping edge.
2. A rectangular tank is 76cm long, 50cm wide and 40cm high. How many litres of water can it hold? (WAEC)
3. Calculate in terms of π the total surface area of a cylinder of radius 3cm and height 4cm. (WAEC)
4. A conical container has radius 7cm and height 5cm. Calculate the volume of the container (WAEC)
5. The volume of a tank is 4.913cm^3, what is the length of its edge.
6. What is the edge of a cube whose volume is 729cm^3
7. A rectangular tank is 76cm long, 50cm wide and 40cm height. How many litres of water can it hold? (WAEC)
8. Find the volume of a triangular prism of height 10cm whose cross-section is an equilateral triangle of side 4cm.
9. An open cylindrical container has base radius of 3.5cm. if the ratio of the area of its base to that of its curved surface is 1:6, what is the height of the container?
10. A cylindrical container of radius 10cm and height 20cm is filled to the brim with water. A steel ball of radius 6cm is now dropped into the cylinder so that the excess water flows out. How much water is left in the container? (leave your in terms of π)

WEEK 6

SUBJECT: MATHEMATICS

CLASS: SS 1

TOPIC: MENSURATION

CONTENTS:

(g) Surface areas and volume of frustum of a cone and pyramid.

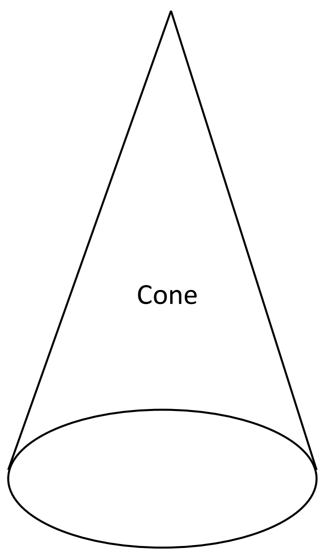
(h) Surface area and volume of compound shapes.

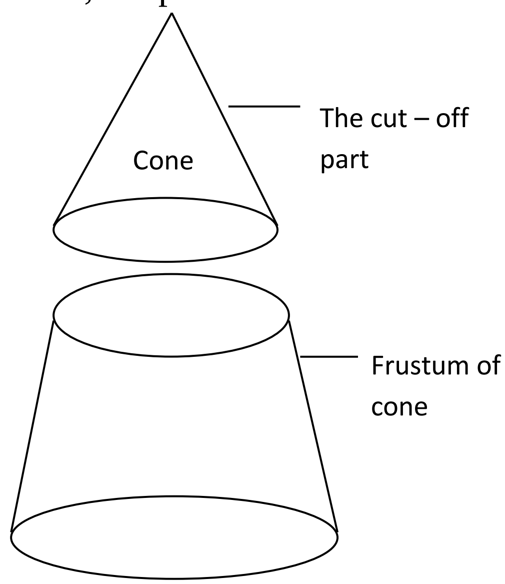
1. Surface area of frustum of a cone and pyramids

2. Volume of frustum of a cone and pyramid
3. Surface area and volume of compound shapes.

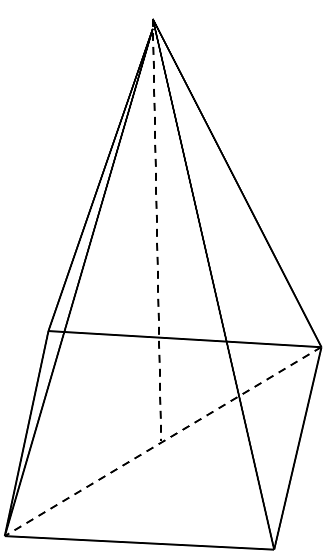
TOTAL SURFACE AREA OF FRUSTUM OF CONE AND PYRAMIDS

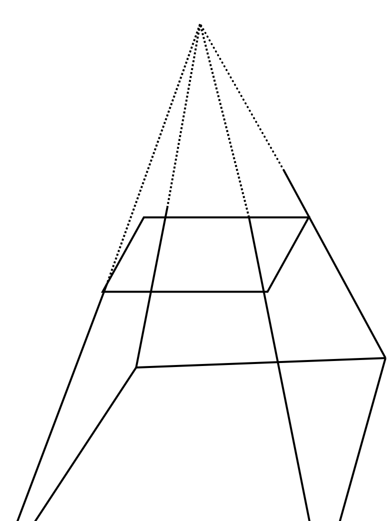
A frustum is the remaining part of cone or pyramid when the top part is cut off as shown below. Daily examples of frustums are buckets, lamps shades e.t.c





Frustum of a cone





Rectangular pyramid

Frustum of a pyramid

For Surface area of the frustrum of a pyramid, we sum up all areas of the faces that make up the frustum.

For Surface area of the frustum of a cone,

Total surface area of a Closed frustum = π(height x sum of radii) + area of top and base circles.

Total surface area of a Open frustum (bucket) = π(height x sum of radii) + area of circle.

Example 1:

Find the total surface area of a bucket 36cm in diameter at the top and 24cm at the bottom.

The depth of the bucket is 30cm.

Solution

The total surface area of bucket = sum of curved part + area of bottom circle

$= \pi$ (height x sum of radii) + area of bottom

$= \pi \times 30 \times (18 + 12) + \pi 12^2$

$= \pi \times 30 \times 30 + \pi(144)$

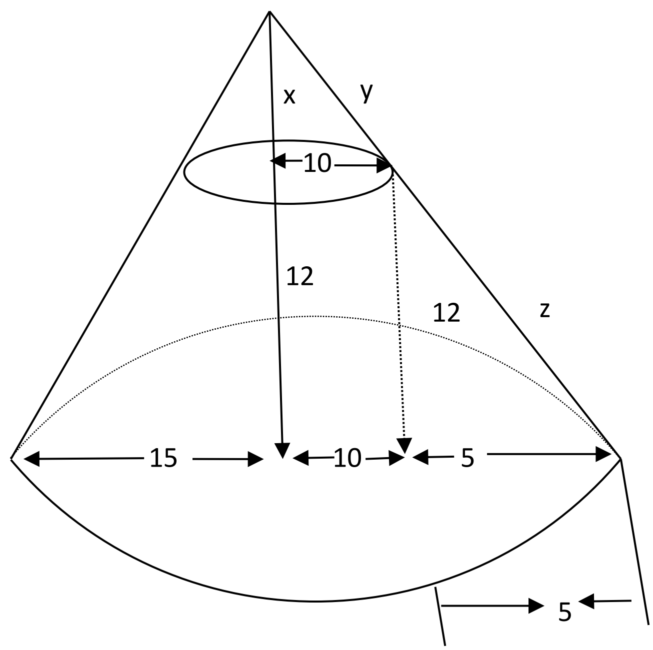
$= 900\pi + 144\pi$

$= 1044\pi \text{cm}^2$

Example 2:

Find in cm^2, the area of material required for a lamp shade in the form of a frustum of a cone of which the top and bottom diameters are 20cm and 30cm respectively and the vertical height is 12cm.

Solution



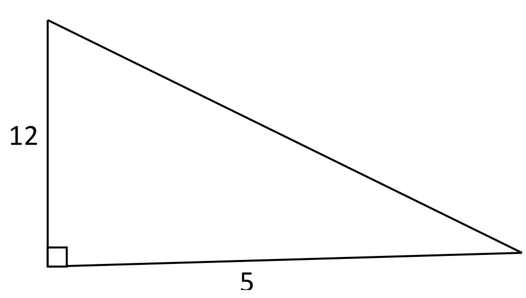
By similar triangles

$\frac{x}{10} = \frac{12}{5}$

5x = 24

By Pythagoras's theorem, y = 26 and z = 13

i.e from



$z^2 = 12^2 + 5^2$

= 144 + 25

= 169

Z = $\sqrt{169}$

= 13

Surface area of frustum

= π x 15 x 39 - π x 10 x 26cm^2

= 13π(45 - 20)cm^2

= 13π x 25cm^2

= 1021cm^2

Area of material required = 1021cm^2. (3 s. f)

Class Activity:

A bucket is 12cm in diameter at the top, 8cm in diameter at the bottom and 4cm deep. Calculate its volume in cm^3 in terms of π. (JAMB)

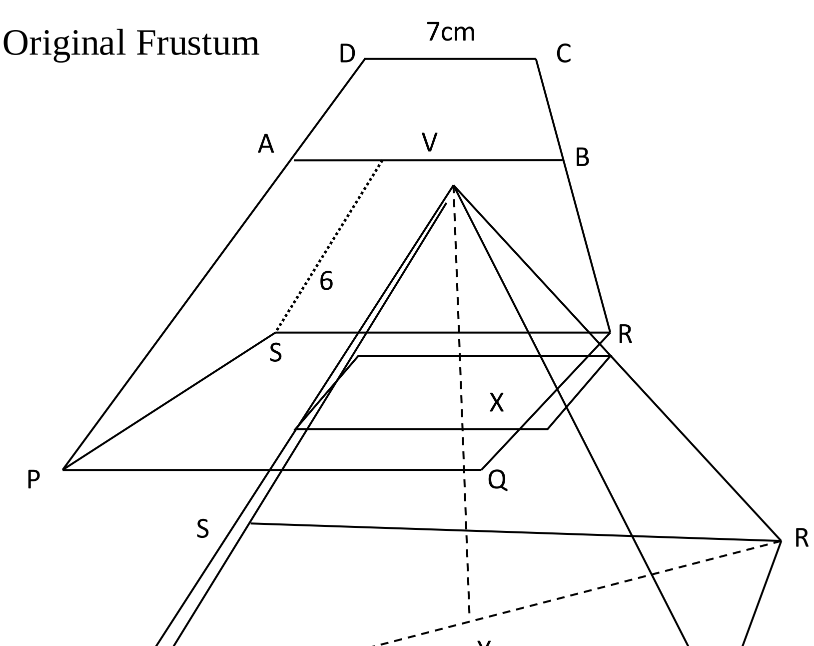
SURFACE AREA OF FRUSTRUM OF PYRAMIDS

Example:

A frustum of a pyramid consists of a square base of length 10cm and a top square of length 7cm, height of the frustum is 6cm. Calculate to the nearest whole number, the surface area of frustum of the pyramid.

Solution

Original Frustum



Completed to pyramid

1. Diagonals of the top and bottom
   $IPRI^2 = IPQI^2 + IQRI^2$
   $= 10^2 + 10^2$
   $= 100 + 100$
   $= 200$
   $IPRI = \sqrt{200}$
   $= 14.14$
   $IYRI = \frac{14.14}{2}$
   $= 7.07$cm
   $= 7.1$cm
   Diagonal AC
   $IACI^2 = IABI^2 + IBCI^2$
   $= 7^2 + 7$
   $= 49 + 49$
   $= 98$
   $IACI^2 = \sqrt{98}$
   $= 9.89$
   BUT $CX = \frac{9.89}{2}$
   $= 4.945$
   $= 4.95$cm

   To calculate the height of the added pyramid.
   Let the heights of the added pyramid be h, by similar Δ
   $\frac{h}{4.95} = \frac{h+6}{7.07}$

$7.07h = 4.95h + 29.7$

$7.07h - 4.95h = 29.7$

$2.12h = 29.7$

$= \frac{29.7}{2.12}$

= 14cm

∴ height of bigger pyramid

= 14 + 6cm

= 20cm

To calculate the slant side of the smaller and bigger Δs

Bigger Δ

$IVRI^2 = 7.1^2 + 20^2$

= 50.41 + 400

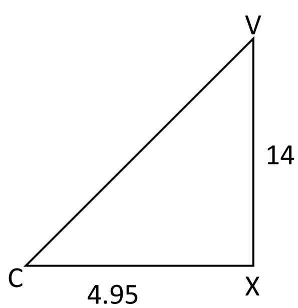
= 450.41

∴ $IVRI^2 = \sqrt{450.41}$

= 21.22cm

= 21cm

Smaller Δ:



$IVCI^2 = ICXI^2 + h^2$

$= 4.95^2 + 14^2$

= 24.50 + 196

= 220.50

$IVCI = \sqrt{220.50}$

= 14.8493cm

= 14.85cm

To find the surface area of frustum of pyramids using Hero's formula for finding the area of the triangular face.

A = $\sqrt{(s(s-a)(s-b)(s-c)}$

Where s = $\frac{1}{2}$(a + b + c) and a, b and c are sides of the triangle

s = $\frac{1}{2}$(21 + 21 + 10)

= $\frac{1}{2}$ x 52

= 26cm

Area of a Δface of big pyramid = $\sqrt{(s(s-a)(s-b)(s-c)}$

A = $\sqrt{(26(26-21)(26-21)(26-10)}$

= $\sqrt{(26(5)(5)(16)}$

= $\sqrt{10400}$

= 101.98

= 102cm^2

Area of 4 Δfaces of the pyramid

= 4 x 101.98cm^2

= 407.92cm^2

= 408cm^2

Area of 4 Δfaces of the small pyramid

s = $\frac{1}{2}$(a + b + c)

s = $\frac{1}{2}$(14.85 + 14.85 + 7)

$s = \frac{1}{2}$ x 36.7

= 18.35cm

Area of a face = $\sqrt{(s(s-a)(s-b)(s-c)}$

= $\sqrt{(18.35(18.35-14.85)(18.35-14.85)(18.35-7)}$

= $\sqrt{(18.35(3.5)(3.5)(11.35)}$

= $\sqrt{2551.338}$

= 50.51cm^2

Area of a 4face = 4 x 50.51

= 202.04cm^2

= 202cm^2

Area of the base of the frustum = l x b

= 10 x 10

= 100cm^2

Area of the top of the frustum = l x b

= 7 x 7

= 49cm^2

Total surface area of the frustum = [(Area of the 4∆faces of the big pyramid – Area of the 4∆faces of the small pyramid)] + Bottom area + Top area

= [(408 – 202) + 100 + 49]

= 206 + 149

= 355cm^2

Class Activity:

A pyramid is on a square base of 25m side and 25m high. The top of the pyramids 10m high was cut off. Find the surface area of the frustum formed.

VOLUME OF FRUSTUM OF CONE AND PYRAMIDS

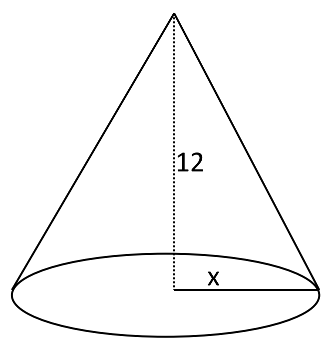
Volume of frustum = volume of the full cone/pyramid – volume of the part cut off.

Example:

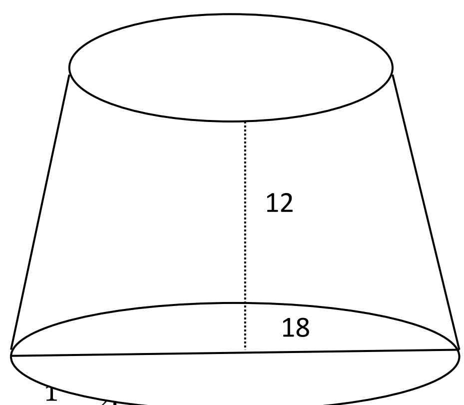
A circular cone 24cm high and 18cm in diameter at its base is cut off at half of its height. Calculate the volume of the remaining frustum. Leave π in your answer.

Solution

(i)



(ii)



Volume of original cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \text{ x } 9^2 \text{ x } 24$

$= \frac{1}{3}\pi \text{ x } 81 \text{ x } 24$

$= \pi \text{ x } 27 \text{ x } 24$

$= 648\pi \text{cm}^3$

To find the radius r of the base of the cut off part, we use similarity of Δs = $\frac{12}{r} = \frac{24}{9}$

r = $\frac{12 \times 9}{24} = \frac{9}{2}$ = $4^1/_2$cm

Volume of small cone = $\frac{1}{3}\pi(\frac{9}{2})^2 \times 12$

= $81\pi cm^3$

Hence, Volume of frustum (i) – (ii)

= 27 x 24π - 81π

= (81 x 8) π - 81π

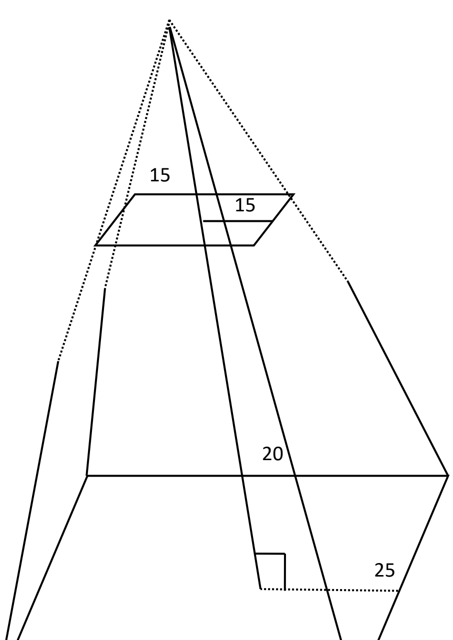
= 81π(8 - 1)

= $567\ \pi cm^3$

Example 2:

Find the volume of the frustum of a pyramid with 30cm square top and 50cm square base and height 20cm.

Solution



If x is the height of the cut – off part and 15cm is half the side of its square base,

Then

$\frac{x}{x+20} = \frac{15}{25}$

i.e 25x = 300 + 15x

10x = 300

x = 30cm.

Volume of entire pyramid

$= \frac{1}{3}\,(base\ area \text{ x } height)$

$= \frac{1}{3}$(base area x height)

$= \frac{1}{3}$(50 x 50) x (20 + 30)

$= \frac{1}{3}$(2500 x 50)

$= \frac{1}{3}(125{,}000)cm^3$

But volume of top pyramid

$= \frac{1}{3}$(base area x height)

$= \frac{1}{3}$(30 x 30 x 30)

$= \frac{1}{3}(27,000)$

= Volume of frustum = $\frac{1}{3}(125,000 - 27,000)$

$= \frac{1}{3}(98,000)cm^3$

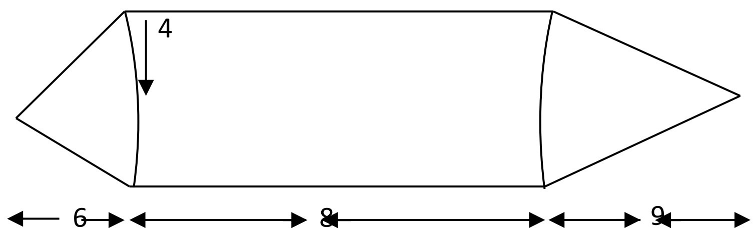
$=32,666\ \frac{2}{3}cm^3$

Class Activity:

What is the capacity of a bucket that is 42cm deep and inner radii of the base and topmost part of the bucket are 12cm and 20cm respectively?

Further Example:

A machine part is made up of a cylinder and a cone on each end, the dimension of which are shown below. Calculate the total surface area of the machine.



Solution

Surface area of 1st cone = πrl

By Pythagoras rule.

$L^2 = (6^2 + 4^2)cm^2$

$= (36 + 16)cm^2$

= 52cm

$l = \sqrt{52}$

the surface area of the come

$= \pi \times 4cm \times \sqrt{52}cm$

$= 4\pi\sqrt{52}cm^2$

Surface area of 2nd cone = πrl

$l^2 = 9^2 + 4^2$

$= 81 + 16$

$= 97cm$

$L = \sqrt{97}cm^2$

Area $= 4\pi\sqrt{97}cm^2$

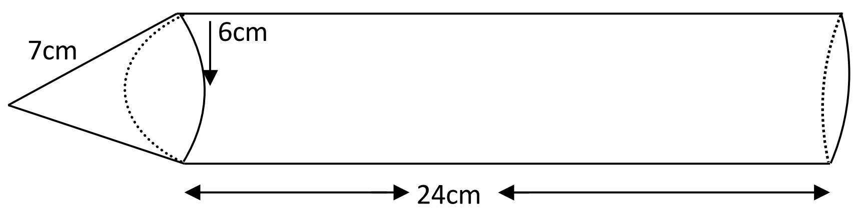
Therefore total surface area of the machine = surface area of cylinder + surface area of 1st cone + surface area of 2nd cone

$= 64\pi cm^2 + 4\pi\sqrt{52}cm^2 + 4\pi\sqrt{97}cm^2$

$= 4\pi(16 + \sqrt{52} + \sqrt{97})cm^2$

Class Activity:

Find the surface area of the bullet in the figure below, using those dimensions



PRACTICE EXERCISE:

1. A cone of radius 7cm is 42cm deep. If the cone is $\frac{3}{4}$ filled with water. How deep is the water in the cone (Take $\pi = \frac{22}{7}$)
2. A square base of a pyramid of side 3cm has height 8cm. If the pyramid is cut into two parts by a plane parallel to the base midway between the base and the vertex, calculate the volumes of the two sections to the nearest centimetre.
3. What is the height of a plastic bucket whose two radii are 20cm and 29cm and a slant height of 41cm?
4. A cone of height 6cm and radius of base 4cm has its top cut-off by plane parallel to its base and 4cm from it. Find the volume of the remaining frustum.
5. A bucket full of water is in the form of a frustum of a cone. The bottom and top radii of the frustum are 18cm and 28cm respectively and the vertical depth is 30cm. If the water in the bucket is then poured into an empty cylindrical container with base radius 20cm, find the depth of the water in the container. (Take $\pi = \frac{22}{7}$)

ASSIGNMENT:

1. The internal and external radii of a water pipe are 9cm and 10cm respectively. If the pipe is 35cm long, find, in cm^3, the volume of material used in making it. (Take $\pi = \frac{22}{7}$)
2. The internal and external radii of a cylindrical bronze pipe are 1.5cm and 2cm respectively. If the pipe is 10cm long, calculate the volume of bronze used. (Take $\pi = \frac{22}{7}$)
3. A cylindrical pipe is 28metres long. Its internal radius is 5cm. calculate :
   (a) The volume of water, in litres that the pipe can hold when full;

(b) The volume in cm^3 of metal used in making the pipe. (Take $\pi = \frac{22}{7}$)

4. A Cylindrical vessel open at one end is made of metal. The internal diameter is 7cm, the internal depth 10cm and the thickness of the metal is 0.5cm. Calculate: (a) the internal volume of the vessel (b) the volume of the metal. (Take $\pi = \frac{22}{7}$)
5. A rectangular block of metal, 6cm long by 3cm wide and 3cm high has a cylindrical hole of radius 1.2cm and depth 2.5cm bored out in the centre of the top surface. Calculate the volume of the remaining part of the block.

WEEK 7

MID-TERM BREAK

WEEK 8

SUBJECT: MATHEMATICS

CLASS: SS 1

TOPIC: DATA PRESENTATION

CONTENTS:

(a) Revision on collection, tabulation and presentation of data.

(b) Frequency distribution (grouped data).

Revision on collection, tabulation and presentation of data.

Statistics can be considered as a way or method used in the collection and organisation of data in order to interprets, predict and get other information required out of the data.

This data could be information about some people e.g.. ages, weights, heights, examination scores, etc

Collection of Data

Let us remind ourselves that there are two kinds of data namely:

(a) Discrete data (b) continuous data

Discrete data are those which are obtained by direct counting e.g numbers of persons born on a particular days of the week, month of the year, etc.

Continuous data are those which require measurement before counting e.g number of persons of some age, height, weight etc.

Class Activity:

Collect and tabulate data on the days of the Week each member of the class was born.

Tabulation and Presentation of data.

For easy access to information, data are normally presented using frequency tables. This table marches each data with the number of times it appeared. The frequency table is prepared as follows:

1. Draw the columns
2. Represent the data given in the first column in ascending or descending order.
3. Represent each data with the use of a tally.

Raw data and frequency tables

When data is first collected and has not been organised in any way, it is called **Raw Data**.

Example 1:

In a game, a die was thrown several times. Below are the results of the scores.

2	3	4	4	2	1	3	2	6	5
3	2	1	1	2	5	2	1	4	4
6	5	6	1	6	5	4	5	4	3
6	5	5	3	5	2	1	4	5	2
4	5	4	6	3	1	5	6	6	5

The above is an example of raw data. One way we can organise the above data is to present it in a frequency distribution table (or frequency table for short) as shown below.

No of throws	Tally	Frequency
1	~~1111~~ 11	7
2	~~1111~~ 111	8
3	~~1111~~ 1	6
4	~~1111~~ 1111	9
5	~~1111~~ ~~1111~~ 11	12
6	~~1111~~ 111	8
	TOTAL	**50**

Note – Frequency table can be given with or without the tally column

Example 2

In a test marked out of 10, a group of pupils obtained the following marks.

3	4	6	3	4	3	5	6	7	6
8	9	5	9	10	7	8	2	6	5
4	10	5	6	7	3	8	9	4	2

Prepare a frequency table for the distribution

Solution

Marks x	Tally	Frequency
2	11	2
3	1111	4
4	1111	4
5	1111	4
6	~~1111~~	5
7	111	3

8	111	3
9	111	3
10	11	2
	Total	**30**

Class Activity:

The weight to the nearest kilogram, of a group of 50 students in a college of technology is given below. Prepare a grouped frequency table with class intervals 45 – 49, 50 – 54, 55 – 59, e.t.c

65	70	60	46	51	55	59	63	68	53
47	53	72	58	67	62	64	70	57	56
73	56	48	51	58	63	65	62	49	64
53	59	63	50	48	72	67	56	61	64
66	52	49	62	71	58	53	69	63	59

PRACTICE EXERCISE:

1. The score of 50 students in a mid-term test were as shown below. Construct a frequency table using the class intervals 1 – 5, 6 – 10, 11 – 15, e.t.c

11	20	30	24	13	28	33	40	23	28
40	8	30	13	15	34	8	34	32	22
26	21	25	18	26	10	19	3	27	18
18	24	26	25	27	29	28	13	35	24
9	24	14	28	27	38	40	32	33	34

2. The age in years of 50 teachers in a school are given below. Form a frequency distribution of the data using the internals 21 – 25, 26 – 30, 31 – 35, e.t.c

21	37	49	27	49	42	26	33	46	40
50	29	23	24	29	31	36	22	27	38
30	26	42	39	34	23	21	32	41	46
46	31	33	29	28	43	47	40	34	44
26	38	34	49	45	27	25	33	39	40

3. Using class interval 30 – 34, 35 – 39, 40 – 44, …… Construct the frequency distribution table.

62	54	53	44	46	55	46	56	68	63
59	61	66	54	39	48	47	53	59	57
50	35	40	30	46	44	36	49	54	51
57	56	45	33	38	41	40	45	53	58
51	45	48	34	36	46	43	49	63	52

WEEK 9

SUBJECT: MATHEMATICS

CLASS: SS 1

TOPIC: DATA PRESENTATION

CONTENTS:

(i) Line graph, (ii) Bar graph (iii) Histograms (iv) Pie Chart (v) frequency polygon (vi) Deductions and Interpretations.

LINE GRAPH

Line graphs are used to show trends over a period of time and have the advantage that they can be extended. To draw a line graph, plot the given data as a series of points and then join the points together by straight lines. The lines can be drawn vertically or horizontally and have no thickness.

Example 1

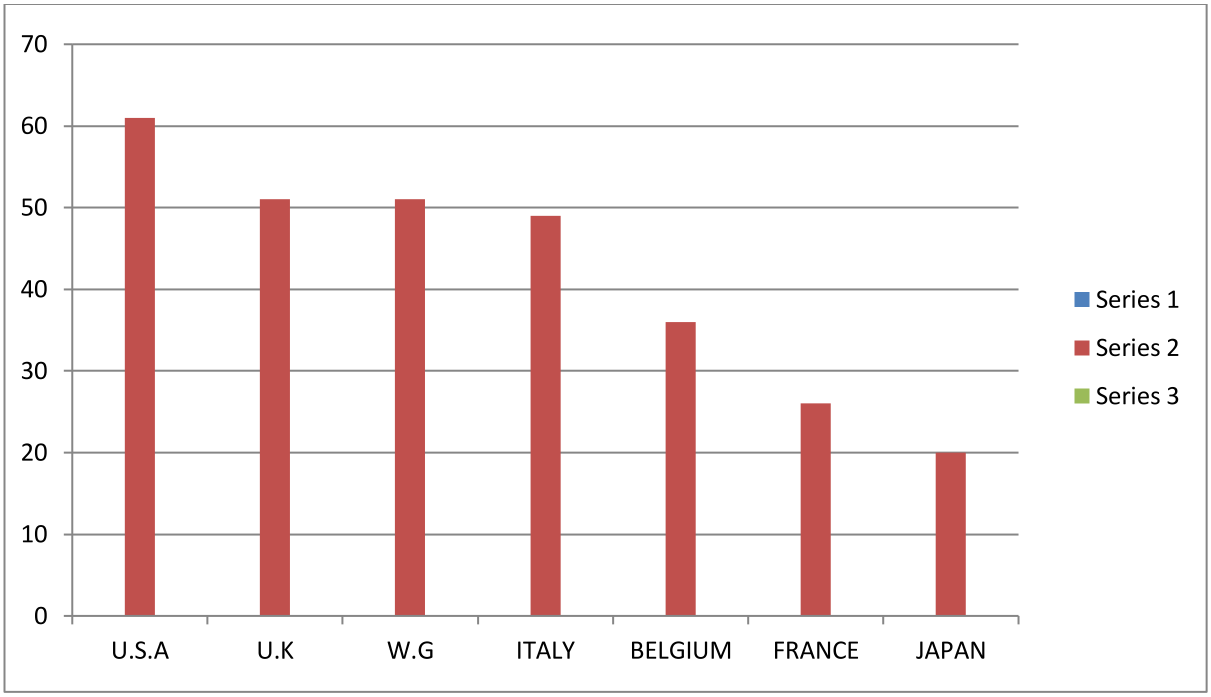
A comparison of traffic densities in a number of countries produced the following results.

Country	**Number of Vehicles**
U. S.A	61
U.K	51
West Germany (W.G)	51
Italy	49
Belgium	36
France	26
Japan	20

Present the data in a line graph

Solution

USA ITALY



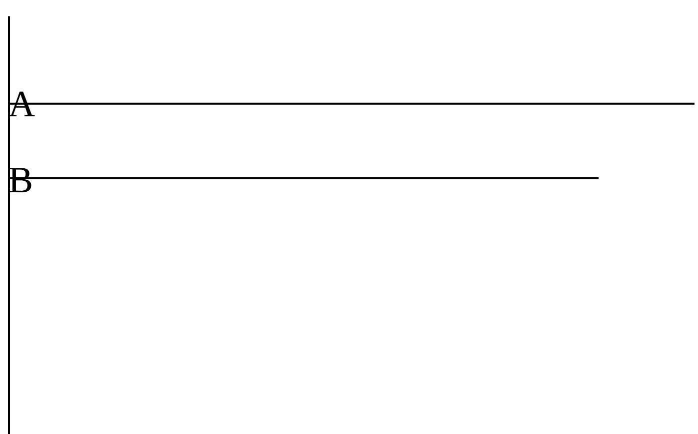
Example 2:

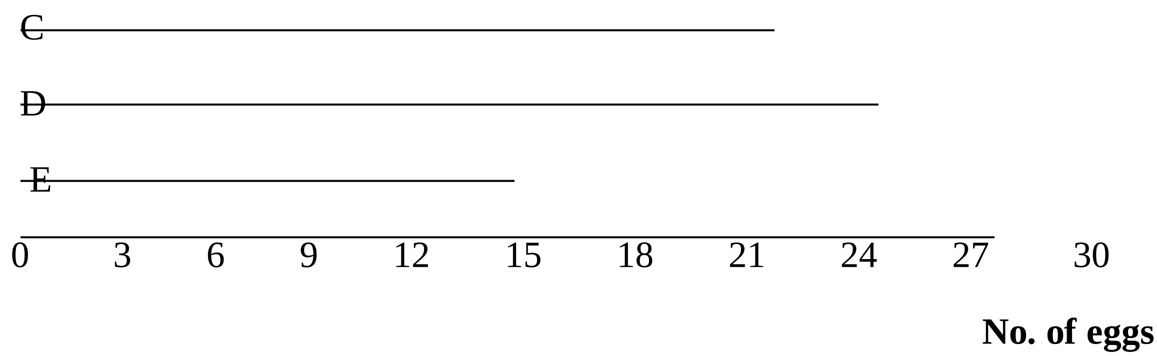
The table below shows the number of eggs laid in one month by 5 hens. Draw a horizontal line graph for the data.

Hen	A	B	C	D	E
No of Eggs	24	21	27	30	18

`Solution

The number of eggs laid in one month by 5 hens can be presented in the following horizontal line graph.





Class Activity:

1. The student population of the Federal Government college Kano was recorded as follows.

Class	1	2	3	4	5
No of Students affected	30	29	30	28	27

Plot the line graph of the above data.

2. The table below shows the expenditure of a company in Benue State in a Certain year.

Description	**Expenditure/₦1000**
Wages and Salaries	25
Fuel and Power	15
Raw Materials	65
Maintenance	5
Miscellaneous	10

Represent the information on a line graph

BAR GRAPH/CHART

Bar charts are rectangular shapes of equal widths but different lengths drawn to represent the frequency. It is uniform in thickness. Bars can be drawn vertically or horizontally.

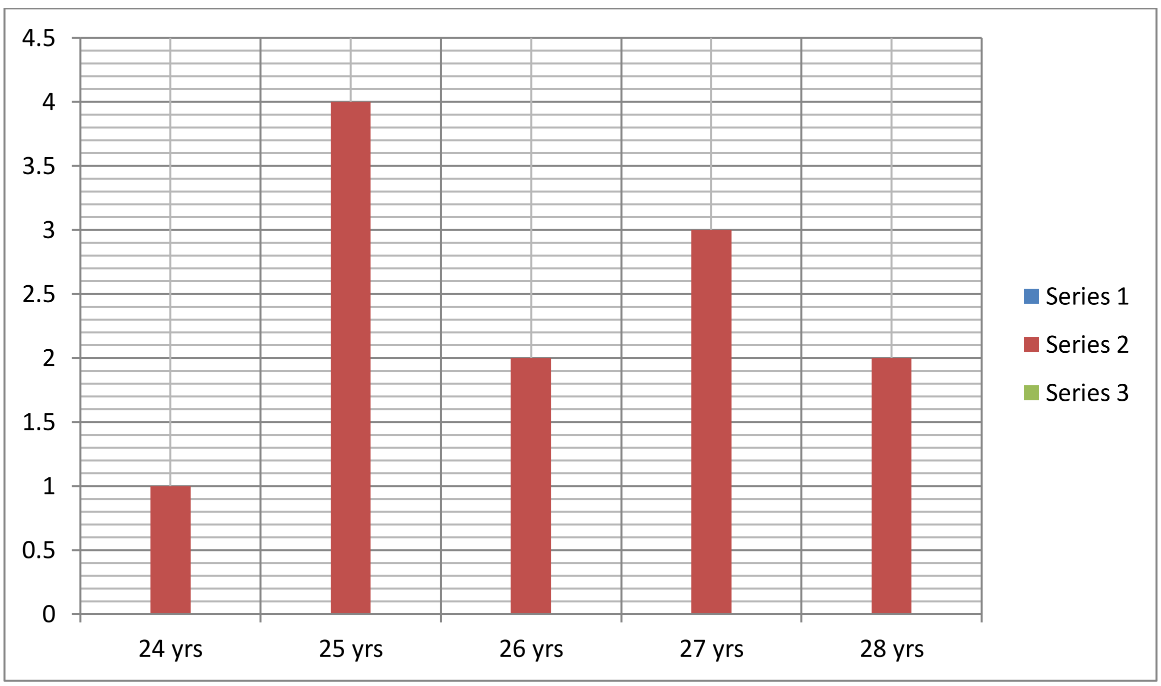
Example 1:

The table below shows the number of babies born to a number of women within a given age range to a number of women within a given age range.

Women ages	24	25	26	27	28
No. of babies	1	4	2	3	2

Draw a bar chart to illustrate the above distribution.

Solution



Example 2:

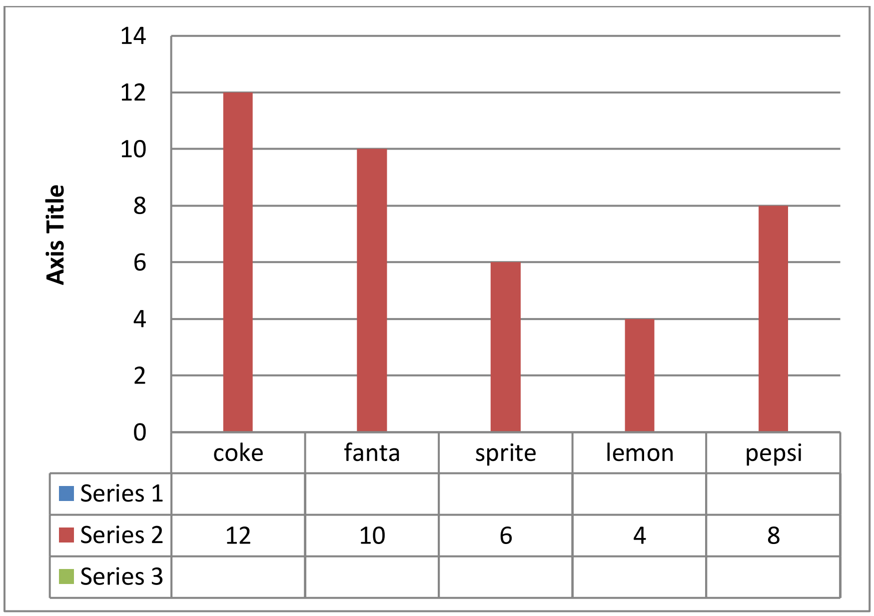
The number of bottles of soft drinks sold in a restaurant one evening is given by the data in the table below.

Type of soft drink	**No of bottles**
Coke	12
Fanta	10
Sprite	6
Lemon	4

Pepsi	8

Draw a bar chart to display this information

Solution



HISTOGRAMS

Histogram is a rectangular block graph representing the frequency distribution of the analysed items. The blocks are joined to one another.

Example 1: The marks scored by 40 students in a particular subject are as follows:

38	74	28	32	10	31	49	34	50	19
30	92	50	42	38	64	24	65	91	77
18	35	12	87	41	27	8	90	22	21
42	43	52	59	72	70	90	91	29	28

(a) Prepare a frequency table, using class interval: 1 – 20, 21 – 40, 41 – 60, ...

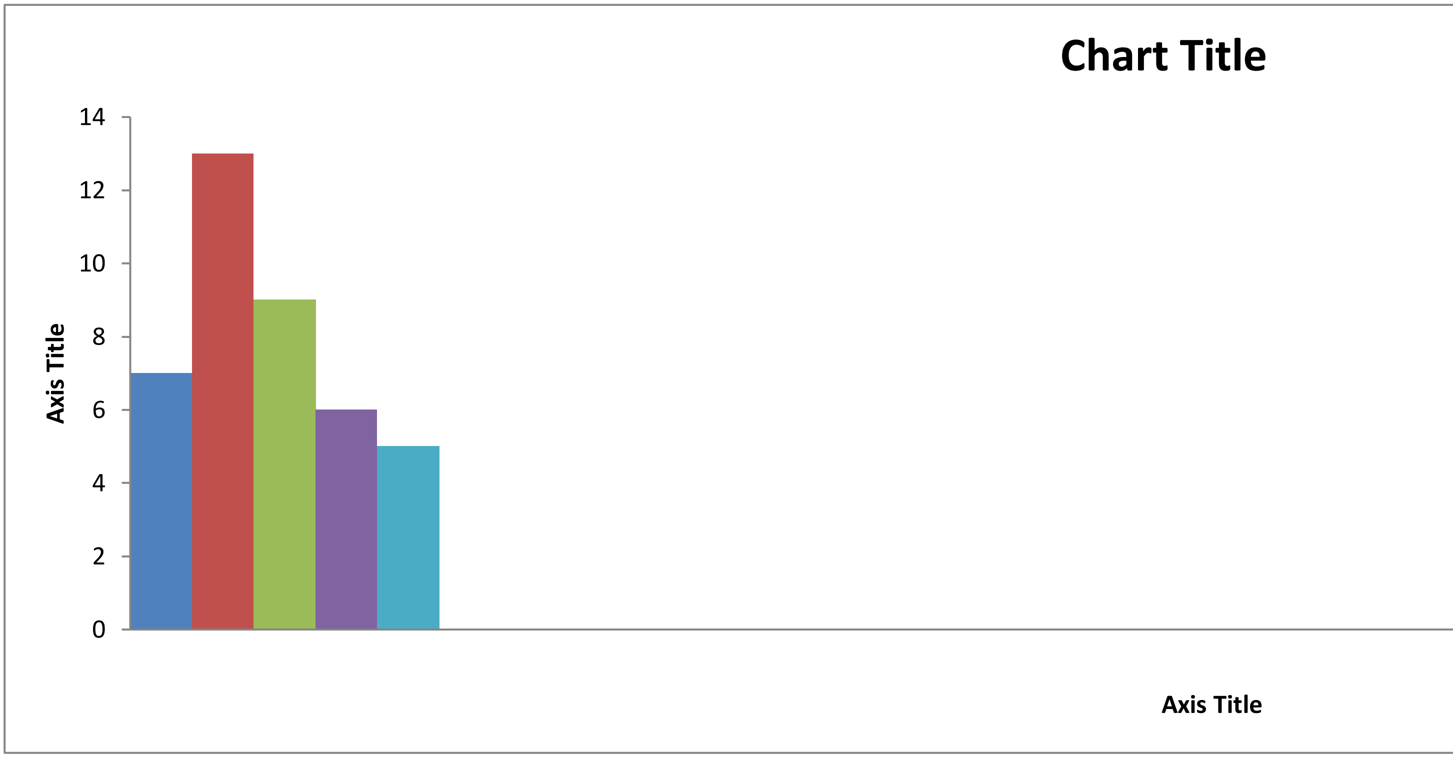
(b) Use the table to draw a histogram.

Solution

(a)

Class interval	**Tally**	**Frequency**
1 – 20	1111 11	7
21 – 40	~~1111~~ 1111 111	13
41 – 60	1111 1111	9
61 – 80	1111 1	6
81 – 100	1111	5
		$\sum f = 40$

(b)



Class Activity:

1. Drawing histogram using class boundary, draw a histogram for the frequency distribution in the table below. (Use class boundaries to plot against the frequency).

NB: The use of class boundaries to plot against the frequency is important in using histogram to estimate the mode.

Class	1 - 5	6 - 10	11 - 15	16 - 20	21 - 25
Frequency	2	4	6	5	3

2. The table below shows the marks obtained by forty pupils in a Mathematics test.

Marks		10 -	20 -	30 - 39	40 - 49	50 - 59

0 - 9	19	29			
No of pupils 4	5	6	12	8	5

Draw a histogram for the mark distribution

PIE CHART.

A pie chart is a circular shape which is divided into sections whose angles are proportional to the frequencies of the items.

Steps to Drawing a Pie Chart

Step 1.

Calculate the angles of sector of the pie chart using x/y x 360^0. Where x is the frequency of the total frequencies of all the items.

Step 2.

Using a pair of compasses, draw a circle with a suitable radius.

Step 3.

Partition it into the various angles of sectors obtained in step 1 using a protractor.

Step 4.

Label each sector showing the information which relates to the sector.

Step 5

Give a title to your pie chart, by stating the information the pie chart is representing.

Example 1:

The table below shows the expenditure of a Bachelor in a given month

Items	**Expenditure ₦1000**
Food	10
Rent	5
Bills	3
Clothing	2
Savings	6

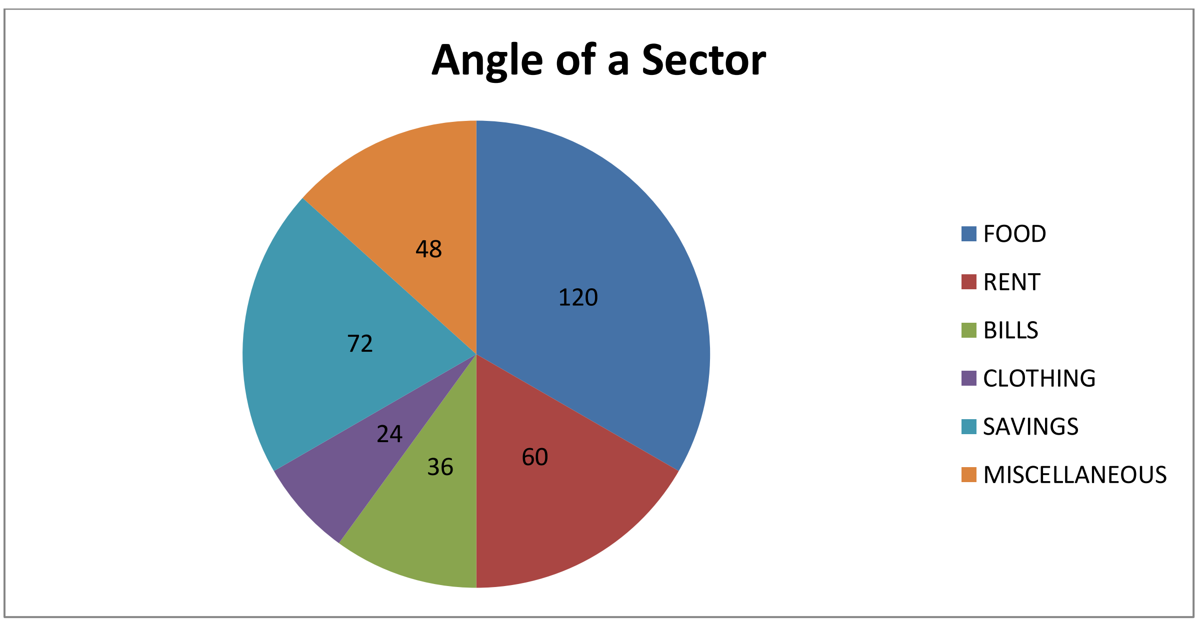
Miscellaneous	4

Draw a pie chart to illustrate the information

Solution

First, we shall calculate the angle of sector for each item as follows.

Items	**Expenditure in ₦1000**	**Angle of Sectors**
Food	10	$\frac{10}{30}$ X $\frac{360}{1}$ = 120^0
Rent	5	$\frac{5}{30}$ X $\frac{360}{1}$ = 60^0
Bills	3	$\frac{3}{30}$ X $\frac{360}{1}$ = 36^0
Clothing	2	$\frac{2}{30}$ X $\frac{360}{1}$ = 24^0
Savings	6	$\frac{6}{30}$ X $\frac{360}{1}$ = 72^0
Miscellaneous	4	$\frac{4}{30}$ X $\frac{360}{1}$ = 48^0
Total	30	$=360^0$



Title: Pie chart showing the expenditure of Bachelor in a certain month.

Example 2:

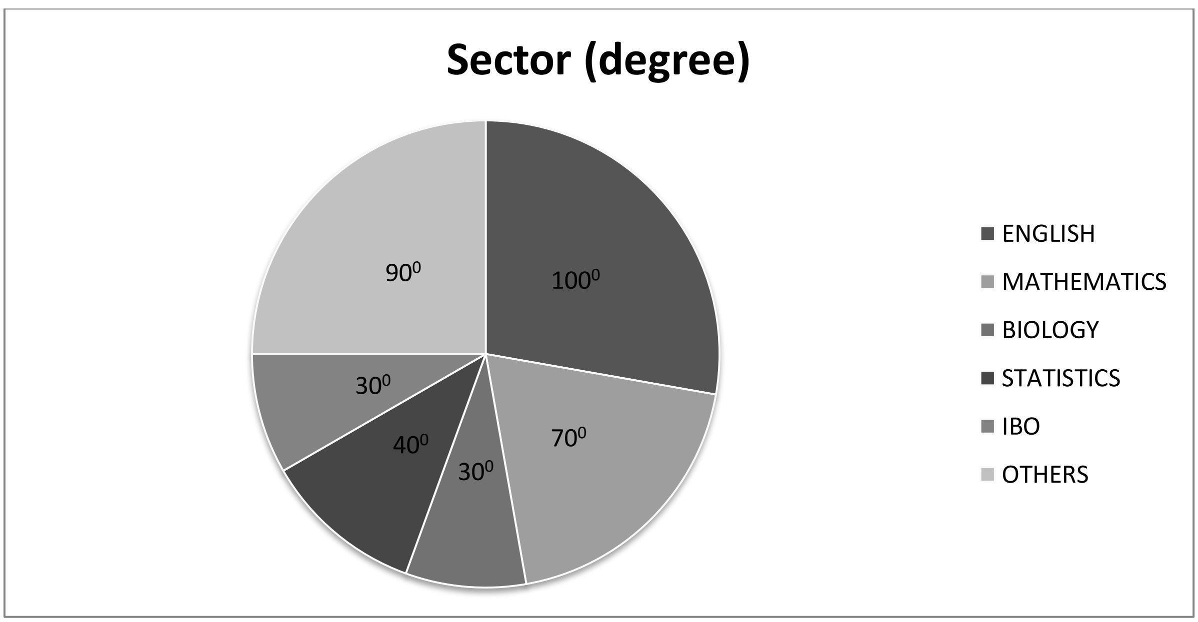
In a certain school, the lesson periods for each week are as itemised below.

English 10, Mathematics 7, Biology 3, Statistics 4, Ibo 3, others 9.

Draw a pie chart to illustrate this information

Solution

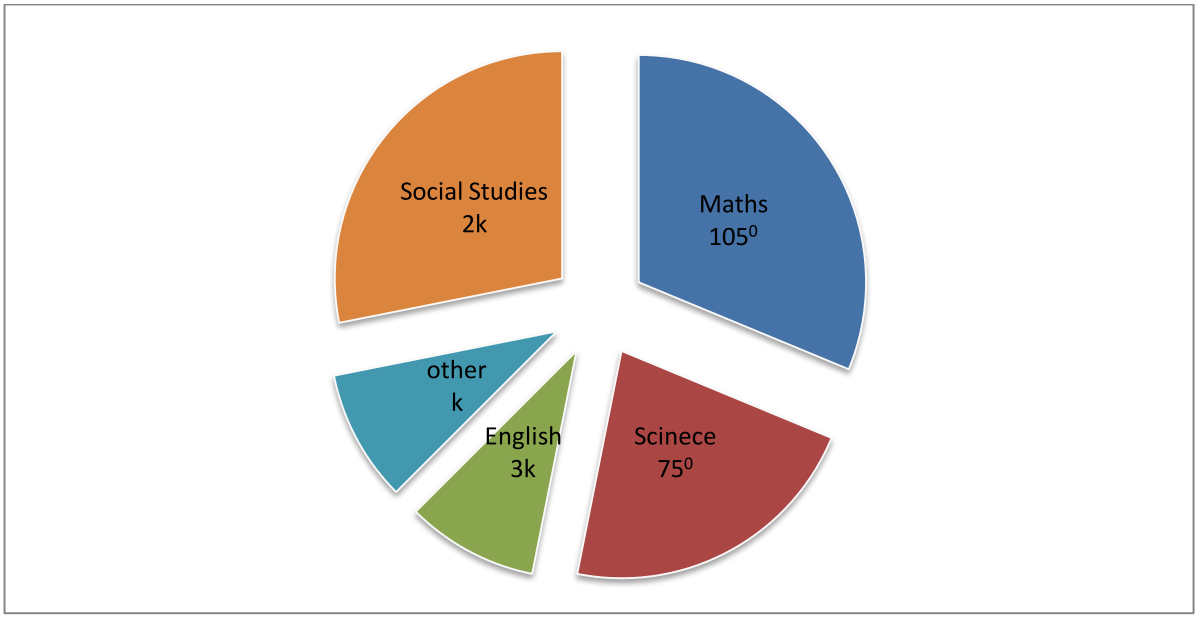
Subjects	**Periods**	**Sector (degrees)**
English	10	$\frac{10}{36}$ X $\frac{360}{1}$ = 100^0
Mathematics	7	$\frac{7}{36}$ X $\frac{360}{1}$ = 70^0
Biology	3	$\frac{3}{36}$ X $\frac{360}{1}$ = 30^0
Statistics	4	$\frac{4}{36}$ X $\frac{360}{1}$ = 40^0
Ibo	6	$\frac{3}{30}$ X $\frac{360}{1}$ = 30^0
others	9	$\frac{9}{36}$ X $\frac{360}{1}$ = 90^0
Total	36	$=360^0$



Title : pie Chart showing the lesson periods for each week in a certain school.

Example 3:

The pie chart illustrates the amount of private time a student spends in a week studying various subjects. Find the value of k.



Solution

Since the total angles of all sectors in a given circle is 360^0.

$105^0 + 75^0 + (2k)^0 + k^0 + (3k)^0 = 360^0$

$105^0 + 75^0 + (6k)^0 =360^0$

$1800 + 6k =360^0$

$6k = 360^0 -180^0$

$6k = 180^0$

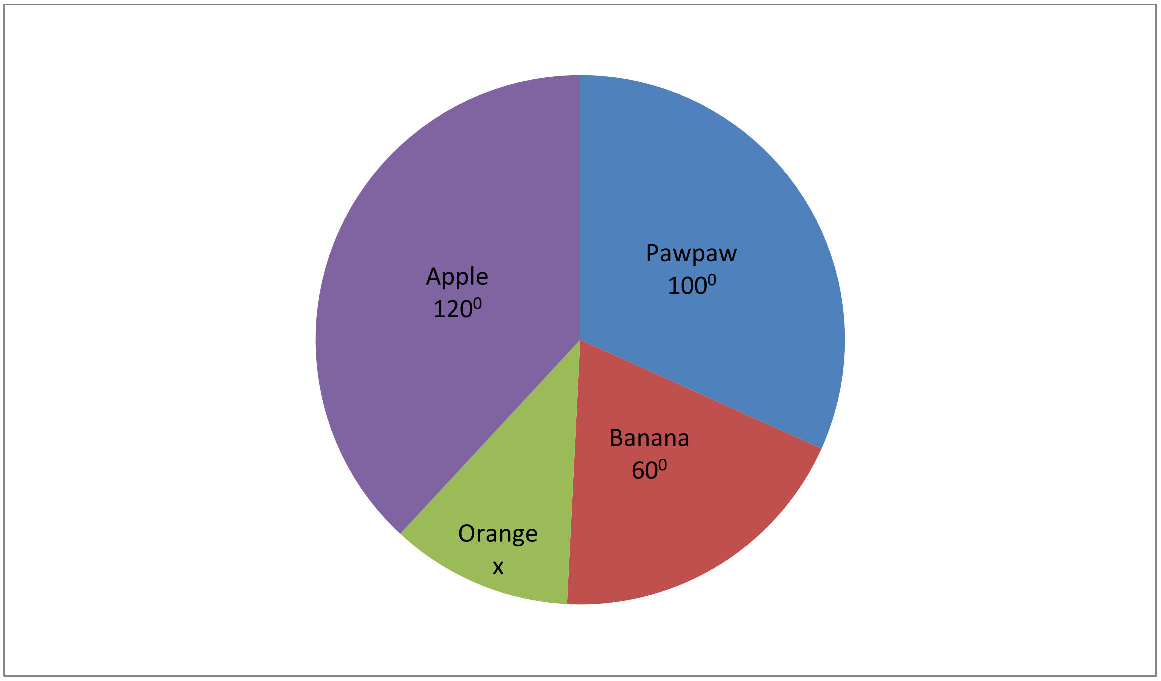
Divide through by 6

$6k/6 = 180^0/6$

$K = 30^0$.

Example 4:

The pie chart represents the fruits on display in grocery shop. If there are too oranges on display. How many apples are there?



Solution

The only sector that has useful information is that of oranges.

Number of oranges = 60

Sector angle of orange = 360 – (60 + 100 + 120)

= $360^0 - 280^0$

= 80^0.

But $\frac{60}{total\ fruits}$ x $\frac{360}{1}$ = 80^0.

$\frac{60}{T.f}$ x $\frac{360}{1}$ = 80^0

$\frac{21600}{total\ fruits}$ = 80/1

Cross multiplying

21600 = 80(total fruit)

Dividing both sides by 80

$\frac{21600}{80} = \frac{80(total\ fruit)}{80}$

= 270

∴ Total number of fruit is 270.

Let the number of apples be x, then;

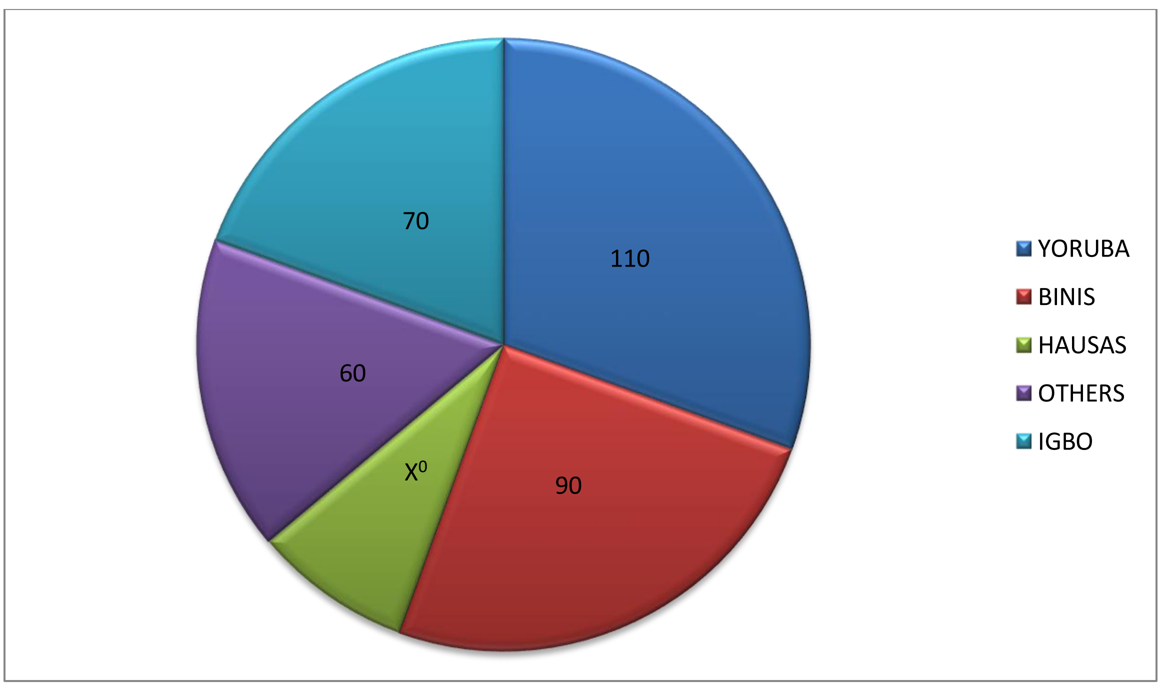
$\frac{x}{270}$ x $\frac{360}{1}$ = 120^0

$x = \frac{120 \text{ x } 270}{3600}$

= 90^0

Example 5:

The Pie chart below shows the distribution of students in a certain school into some major ethnic groups in Nigeria.



(a) In its simplest form, what fraction of the students are Igbos?
(b) What percentage of the students are Hausas?
(c) What is the ratio of the Binis to Hausas in the school in its simplest terms.

Solution

(a) The angle of the sector representing Igbos is 70^0.

= 70/360 of the students are Igbos.

= 70/360 = 7/36

(b) The angle of sector for Hausa is x^0 and this can be obtained using

$110^0 + x^0 + 70^0 + 60^0 + 90^0 = 360^0$

$330^0 + x^0 = 360^0$

$x = 30^0$.

Taking the percentage of the fraction, the students who are Hausas use

= 30/ 300 x 100/1

= 1/12 x 100/1

= 100/12

8.3%

(c) The angle of sector for binis = 90^0

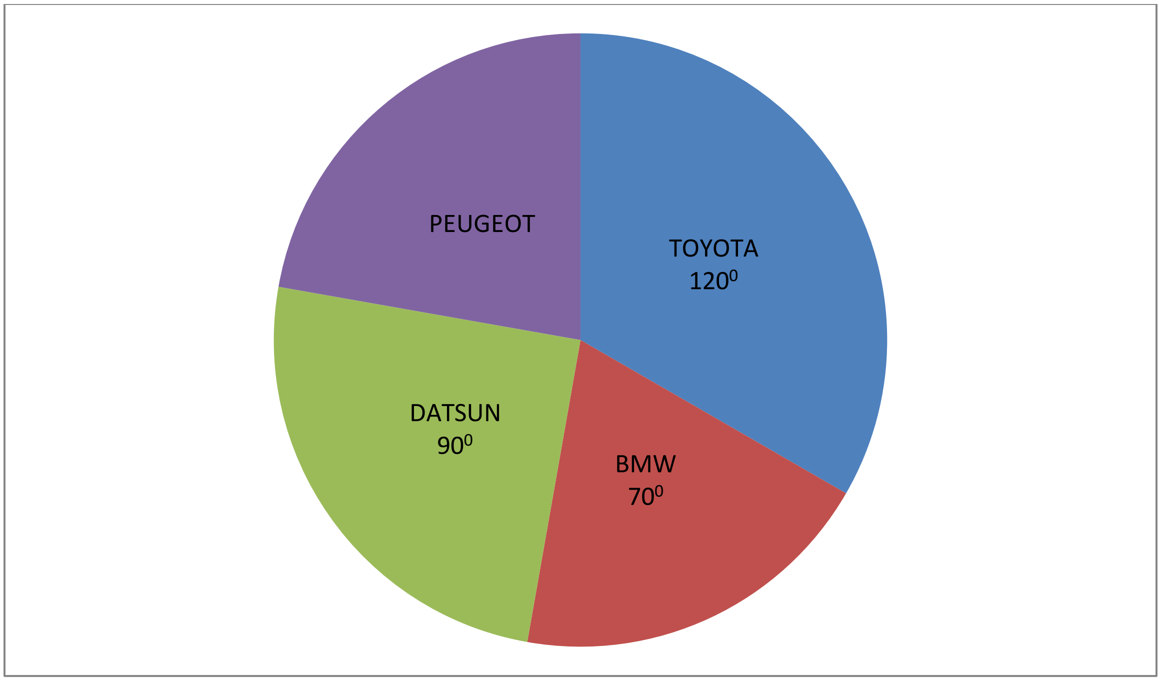
The angle of sector for Hausas= 30^0

ratio of Binis to Hausas $90^0 : 30^0$

= 3 : 1

Example 6:

The Pie chart below shows the weekly sales of a motor dealer in Lagos in 1999.



(a) What fraction of the cars were Toyota?
(b) What percentages of the Cars were Datsun?
(c) If the dealer sold 16 Peugeots, How many BMW did he sell in a week?

Solution

(a) The angle of the sector representing Toyota is 120^0.
= 120/ 360
= 12/36
= 1/3

(b) Taking the percentage of the fraction of the Datsun Cars
= 90/ 360 x 100/1
= 1/4 x 100/1
= 25%

(c) To get the total number of Cars, we will first get the angle of sector for Peugeot.
70 + 120 + 90 + x + 360
280 + x = 360
X = 360 – 280
x = 800

Assuming the total number of Cars to be y, then

$16/y \times 360/1 = 80^0$

$5760/y = 80/1$

Cross multiplying

$5760 = 80y$

Divide both sides by 80

$5760/80 = 80y/80$

$y = 72^0$.

To get the number of BMW sold in a week, let the number of the BMW sold be m

$m/72 \times 360/1 = 70^0$

$360m/72 = 70$

Cross multiplying

$360m = 5040$

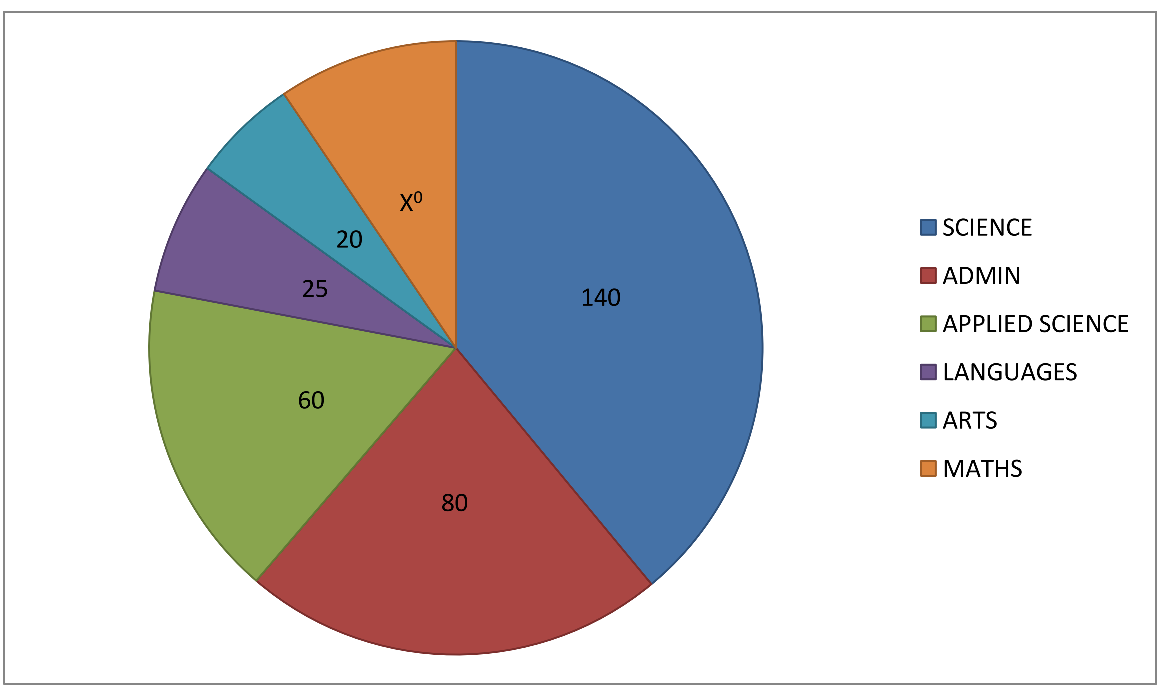
Dividing both sides by 360

$360m/360 = 5040/360$

$m = 14$

Class Activity:

(1) The pie chart below shows the allocation of money to the different departments in a secondary school.



If applied Science Department were allocated the sum of ₦120,000.00. What was the total allocation to Mathematics Department?

2. In a certain year, government bought 240 Cars; 110 were Peugeots, 54 were Datsuns, 35 were Fords, 28 were Hondas and the rest were Volkswagens. Draw a pie chart to represent the above information.

Frequency polygon

The frequency polygon is obtained by joining the midpoints of the tops of the histogram rectangles with straight lines. Since the polygon is a closed figure, we take one interval below the lowest internal on the base axis. Then we join the ends of the polygon to the midpoints of the new intervals as shown below. **Class Activity:**

The table shows the distribution of the masses of fifty logs exported in September 1981 by a timber and plywood company.

Mass (kg)	**No of Logs**
150 -154	1
155 – 159	4
160 – 164	8
165 - 169	13
170 – 174	12
175 – 179	8
180 – 184	3
185 – 189	1

Construct the frequency polygon for the distribution

PRACTICE EXERCISE:

1. The body temperatures of some patients in a hospital ward one morning are as follows: Illustrate this data in a bar chart.

No. of beds	P	Q	R	S	T
Temperature	98^0	100^0	101.1^0	78^0	105^0

2. A dice is thrown 27 times. The table below shows the distribution of the number of times each number showed up. Present this data in a bar chart.

Face	1	2	3	4	5	6
Frequency	4	6	8	5	3	1

3. A class of students had a test. The following table gives the number f of students obtaining various marks x

x	0	1	2	3	4	5	6	7	8	9	10
f	2	3	3	4	6	8	5	4	3	2	0

Illustrate the information on a histogram.

4. The table below shows how a company's sales manager spent his 2017 annual salary:

Food	30%
Rent	18%
Car Maintenance	25%
Savings	12%
Taxes	5%
Others	10%

(a) Represent this information on a pie chart.
(b) Find his savings at the end of the year if his annual salary was #60,000.00

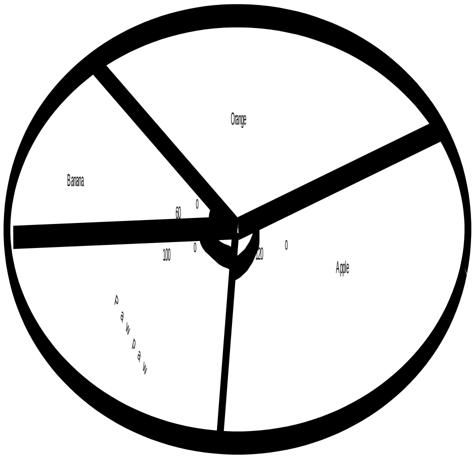
5. The student intake at a University for a particular year was distributed among its faculties as follows:

(a) Illustrate the data below on a pie chart
(b) What is the percentage of students admitted into the faculty of science?

Faculties	**Numbers**
Agriculture	200
Arts	372
Law	56
Science	540
Technology	272

ASSIGNMENT:

1. The pie chart represents the fruits on display in a grocery shop. If there are 60 oranges on display, how many apples are there?



2. The table below shows how a man spends his income in a month.

Items	Amount spent
Food	N4500
House rent	N3000
Provisions	N2500
Electricity	N2000
Transportation	N5000
Others	N3000

(a) Represent this information on a pie chart.

(b) What percentage of his income is spent on transportation?

3. The following data shows the marks of 40 students in a history examination.

41	52	37	56	63	48	65	46
54	32	51	66	74	23	35	61
58	44	49	53	45	57	56	38

59 28 50 49 67 56 36 45

76 68 43 56 26 47 55 71

i. Form a grouped frequency table with the class intervals 20 – 29, 30 – 39, 40 – 49 etc.
ii. Draw the histogram of the distribution.

4. The distribution of junior workers in an institution is as follows;

Clerk	78
Drivers	36
Typists	44
Messengers	52
Others	30

Represent the above information by a pie chart.

5. The table below shows the marks obtained by forty pupils in a mathematics test.

Marks	0-9	10-19	20-29	30-39	40-49	50-59
Number of pupils	4	5	6	12	8	5

(a) Draw a histogram for the mark distribution
(b) Use your histogram to estimate the mode