

SS1 SECOND TERM: E-LEARNING NOTES

SCHEME SECOND TERM

WEEK	TOPIC	CONTENT
1	MODULAR ARITHMETIC 1	(a) Revision of addition, subtraction, multiplication and division of integers. (b) Concept of module arithmetic.
2	MODULAR ARITHMETIC 2	(a) Addition, subtraction, multiplication and division operations in module arithmetic. (b) Application to daily life.
3	QUADRATIC EQUATION 1	(a) Revision of factorization of quadratic expressions. (b) Solution of quadratic equation of the form: $ax^2 + bx + c = 0$ i.e. $a \neq 0$ or $b = 0$. (c) Formation of quadratic equation with given roots.
4	QUADRATIC EQUATION 2	(a) Drawing quadratic graph. (b) Obtain roots from a quadratic graph. (c) Application of quadratic equation to real life situations.
5	CONSTRUCTIONS 1	Revision of (i) Construction of triangles with given sides. (ii) Bisection of an angle; 30° , 45° , 60° and 90° .
6	CONSTRUCTIONS 2	Construction of (i) An angle equal to a given angle. (ii) 4-sided plane figure given certain conditions. (iii) Locus of moving points equidistance from 2 lines, 2 points, and constant distance from a point, etc.
7	MID-TERM BREAK	
8	PROOFS OF SOME BASIC THEOREMS 1	Proofs of (i) sum of a triangle is 180° (ii) The exterior angle of a triangle is equal to the sum of two interior opposite angles. (iii) Congruency and similarity of triangles.
9	PROOFS OF SOME BASIC THEOREMS 2	Riders including – (i) angles of parallel lines (ii) angles in a polygon (iii) congruent triangles (iv) properties of parallelogram (v) intercept.
10	REVISION	
11	EXAMINATION	

WEEK1

DATE.....

SUBJECT: MATHEMATICS

CLASS: SS 1

TOPIC: Modular Arithmetic

CONTENT:

- Revision of addition and subtraction of integers

- Revision of multiplication and division of integers
- Concept of modular arithmetic/Cyclic events

Revision of addition, subtraction, multiplication and division of integers

Recall: Integer is a counting whole numbers, either positive or negative.

Examples 1,5,20, -1, -5 etc

These numbers can be added, subtracted, divided or multiplied.

- (i) Addition of integers;
 - (a) $486 + 289 = 775$
 - (b) $-25 + (-78) = -103$
- (ii) Subtraction of integers;
 - (a) $582 - 328 = 254$
 - (b) $902 - 437 = 465$
- (iii) Multiplication of integers;
 - (a) $181 \times 42 = 7602$
 - (b) $208 \times 5 = 1040$
- (iv) Division of integers;
 - (a) $972 \div 27 = \frac{972}{27} = 36$
 - (b) $1008 \div 12 = \frac{1008}{12} = 84$

Class Activity:

Solve the following;

- (i) $3092 + 216 + 1801 = \cdot$
- (ii) $2968 - 989 = \cdot$
- (iii) $318 \times 2 = \cdot$
- (iv) $420 \div 12 = \cdot$

Concept of Modular Arithmetic

The word Modular implies consisting of separate parts or units which can be put together to form something, often in different combinations.

Arithmetic– the science of numbers involving adding, subtracting, multiplying and dividing of numbers

Modular Arithmetic is the type of arithmetic that is concerned with the remainder when an integer is divided by a fixed non-zero integer. The word remainder as used in definition practically refers to the excess in number, after full cycles have been completed.

Examples;

1. Reduce 65 to its simplest form in:

(a) modulo 3 (b) modulo 4 (c) modulo 5 (d) modulo 6

Solution

(a) $65 \div 3 = 21, \text{remainder } 2$

$$65 = 2(\text{mod } 3)$$

(b) $65 \div 4 = 16, \text{remainder } 1$

$$65 = 1(\text{mod } 4)$$

(c) $65 \div 5 = 13, \text{remainder } 0$

$$65 = 0(\text{mod } 5)$$

(d) $65 \div 6 = 10, \text{remainder } 5$

$$65 = 5(\text{mod } 6)$$

2. If 20 oranges are to be put into bags that can contain a maximum of eight oranges in each, calculate

- i. The number of bags that will be filled with the oranges
- ii. The number of oranges in the bag with some space left.

Solution

$$20 = 2 \times 8 + 4$$

- i. 2 bags will be filled with oranges
- ii. 4 oranges will be in the bag with space left

Class Activity:

1. Reduce 72 to its simplest form

(a) Modulo 3

(b) Modulo 4

(c) Modulo 5

(d) Modulo 6

(e) Modulo 7

Cyclic Events: Cyclic means happening in cycles.

Just as you ride your bicycle, the wheel rotates from a point to another. There are events that have constant intervals of three days, four, five or a week.

Examples: If ice cream is served every three days. If you are served on Thursday, the next serving will be $\text{thursday} + 3\text{days} = \text{sunday}$

Find the number which results from the following additions on the number cycle below of ice cream

(a) $2 + 9 = 11$

$$11 \div 3 = 3, \text{remainder } 2$$

$$\therefore 2 + 9 \equiv 2$$

(b) Simplify $3 + 14$ in 4 cyclic events,

$$17 \div 4 = 4, \text{ remainder } 1$$

$$\therefore 3 + 14 \equiv 1$$

Class Activity:

Use the number cycle 5 to simplify

(a) $1 + 6$

(b) $2 + 32$

(c) $3 + 35$

PRACTICE EXERCISE

1. Thirty nine oranges are to be put into bags that have a capacity of 7 oranges. If the bags are to be filled in turn, how many oranges would be required to fill the last bag. How many bags will be needed to contain all the oranges
2. State the quotient on division of
 - i. 6 by 3
 - ii. 15 by 7
3. Use the cyclic 4 to simplify the following
 - i. $3 + 13$
 - ii. $1 + 3$
4. Reduce 35 to simplest form
 - i. Module 3
 - ii. Module 4
5. Arrange the days of the week, Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday on a circle using the number code 0 for Sunday, 1 for Monday, 2 for Tuesday, 3 for Wednesday, 4 for Thursday, 5 for Friday and 6 for Saturday. If Thursday, which day will it to be in,
 1. 5 day's time
 2. 10 day's time

ASSIGNMENT

1. Arrange the days of the week, Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday on a circle using the number code 0 for Sunday, 1 for Monday, 2 for Tuesday, 3 for Wednesday, 4 for Thursday, 5 for Friday and 6 for Saturday. If Thursday, which day will it to be in,

- I. 7 day's time
- II. 39 day's time
- 2. Use the cyclic 4 to simplify the following
 - I. $0 + 12$
 - II. $2 + 32$
- 3. State the quotient on division of
 - i. 6 by 8
 - ii. -13 by 5
 - iii. 15 by -7

- 4. Reduce 35 to simplest form
 - I. Module 5
 - II. Module 6
- 5. Find the simplest form of the following in given moduli
 - i. $-5(\text{mod } 6)$
 - ii. $-52(\text{mod } 11)$
 - iii. $-75(\text{mod } 7)$
 - iv. $-50(\text{mod } 4)$

KEYWORDS: Arithmetic, Modular, cyclic, events e.t.c

WEEK2

DATE.....

SUBJECT: MATHEMATICS

CLASS: SS 1

TOPIC: Modular Arithmetic

CONTENT:

- Addition, Subtraction, Multiplication and Division operations in module arithmetic
- Application to daily life

ADDITION AND SUBTRACTION:

In modular arithmetic, addition and subtraction are symbolized by \oplus and \ominus respectively. In the table we enter only the remainders when a pair of elements in the \oplus on the set $\{0,1,2,3,4\}$ modulo 5

\oplus	0	1	2	3	4
0	0	1	2	3	4

1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

Addition \oplus in mod 5

1. Simplify Find $39 \oplus 29 \pmod{6}$

$$\begin{aligned} \text{Solution: } 39 \oplus 29 &= 68 \\ &= (6 \times 11 + 2) \\ &= 2 \pmod{6} \end{aligned}$$

$$\begin{aligned} \text{N.B } 68 \div 6 &= 11, \text{ remainder } 2 \\ &= 2 \pmod{6} \end{aligned}$$

2. Calculate the following in the given moduli (a) $12 \ominus 5 \pmod{4}$ (b) $38 \ominus 42 \pmod{7}$

$$\begin{aligned} \text{Solution: (a) } 12 \ominus 5 &= 7 \\ 7 &= 4 + 3 \\ &= 3 \pmod{4} \end{aligned}$$

$$\begin{aligned} \text{(b) } 38 \ominus 42 &= -4 \\ -4 &= -7 + 3 \\ &= 3 \pmod{7} \end{aligned}$$

Class Activity:

- (1) Find the following additions modulo 5

- (a) $3 \oplus 9$
- (b) $65 \oplus 32$
- (c) $41 \oplus 52$
- (d) $8 \oplus 17$

- (2) Find the simplest positive form of each of the following numbers modulo 5

- (a) -9
- (b) -32
- (c) -75
- (d) -256

Multiplication of modulo

Examples: Evaluate the following modulo 4

- (a) $2 \oplus 2$
- (b) $5 \oplus 7$
- (c) $6 \oplus 73$

Solution:

$$\begin{aligned} \text{(a) } 2 \oplus 2 &= 4 \\ &= 4 + 0(\text{mod } 4) \\ &= 0(\text{mod } 4) \end{aligned}$$

$$\begin{aligned} \text{(b) } 5 \oplus 7 &= 35 \\ &= 4 \times 8 + 3 \\ &= 3(\text{mod } 4) \end{aligned}$$

$$\begin{aligned} \text{(c) } 6 \oplus 73 &= 438 \\ &= 4 \times 109 + 2 \\ &= 2(\text{mod } 4) \end{aligned}$$

Class Activity:

Find the values in the moduli written beside them

- (a) $16 \otimes 7(\text{mod } 5)$
- (b) $21 \otimes 18(\text{mod } 10)$
- (c) $8 \otimes 25(\text{mod } 3)$
- (d) $27 \otimes 4(\text{mod } 7)$
- (e) $80 \otimes 29(\text{mod } 7)$

DIVISION OF MODULO

Examples: Find the values of the following;

- (a) $2(\div)3(\text{mod } 4)$
- (b) $7(\div)2(\text{mod } 5)$
- (c) $2(\div)2(\text{mod } 4)$

Solution:

(a) If $2(\div)3 = x$

$$\Rightarrow \frac{2}{3} = \frac{x}{1}$$

Cross-multiply , $3x = 2$

Add 4 to RHS

$$3x = 2 + 4(\text{mod } 4)$$

$$3x = 6(\text{mod } 4)$$

Divide both sides by 3

$$x = 2(\text{mod } 4)$$

$$(b) 7(\div)2 = x$$

$$\frac{7}{2} = \frac{x}{1}$$

$$2x = 7(\text{mod } 5)$$

$$2x = (5 \times 1) + 2(\text{mod } 5)$$

$$2x = 2$$

$$x = 1$$

$$(c) 2(\div)2 = x$$

$$\frac{2}{2} = \frac{x}{1}$$

$$2x = 2(\text{mod } 4)$$

Divide both sides by 2

$$x = 1$$

Or

$$2x = 2 + 4(\text{mod } 4)$$

$$2x = 6(\text{mod } 4)$$

$$x = 3(\text{mod } 4)$$

N.B If $3(\div)2 = x$, then $2x = 3$

No multiple of 4 can be added to 3 to make it exactly divisible by 2. There are no values of $3(\div)2$ in modulo 4.

Class Activity:

Calculate the following division in modulo 5

(a) $28(\div)7$

(b) $29(\div)2$

(c) $58(\div)4$

(d) $74(\div)7$

N.B Educators should also solve various examples.

PRACTICE EXERCISE :

(1) Copy and complete the table for addition (mod 5)

\oplus	0	1	2	3	4
0					4
1					
2					

		3			
3					
4	4				

(2) Copy and complete the table for subtraction modulo 6

\ominus	0	1	2	3	4	5
0						
1						
2						
3						
4						

(3) Complete the multiplication modulo 5 in the table below

\otimes	0	1	2	3	4	5
0	0	0	0	0		
1	0					
2	0					
3	0		1			
4					1	
5	0				0	

(4) Simplify the following

- i. $8-5+2$ in mod 6
- ii. $5 \times 4 \div 3$ in mod 11
- iii. $6 \times 13^{-1} + 5 \times 7$ in mod 12
- iv. $9 \times (3 + 6)$ in mod 8

(5) Evaluate $4 \div 3$ in (mod 5)

APPLICATION OF MODULAR ARITHMETICS TO DAILY LIFE

Time plays crucial role in indicating how often events occur or qualities vary. Whenever the time rate of occurrence of events is constant, the order of occurrence is repeated.

Lets consider these basic facts:

- i. 1 rotation of hour of hand of the clock or record 12hrs (half day)
 - ii. 1 rotation of the minute hand of the clock or watch record 60mins or 1hr
 - iii. 1 rotation of the second hand of the clock or watch records 60 secs or 1 minute.
 - iv. 1 rotation of 7days record 1 week
 - v. 1 rotation of 24 hours record is 1 day
- Etc.

Examples

1. If the hour hand of a clock is at 2:00 a.m. What time of the day will it indicate after 20 rotations

Solution

We apply mod 12 since the movement is that of the hour hand
We then have

$$\begin{aligned}0 \text{ hr} &\equiv \text{the time at midnight} \\20 \text{ rotations} &\equiv 20 \text{ hours} \\ \text{but } 20 &\equiv 1 \times 12 + 8 \\20 &\equiv 8\end{aligned}$$

Therefore the day is 8hrs from 2'0 clock into its second phase
The require time $2 + 8 = 10\text{pm}$

2. The market in a village holds every 6days. If the current market is on Wednesday when will the next market held?

Solution

We operate in mod 7

If $0 \equiv \text{sunday}$

Then Wednesday $\equiv \text{day } 3$

Next market holds 6 days after

Total number of days from Sunday $\equiv 9$

$$9 \equiv 1 \times 7 + 2$$

$$\text{i.e } 9 \equiv 2$$

the next market is 2 days after Sunday

therefore the expected market day is Tuesday

ASSIGNMENT:

1. Construct an addition table in mod 5. Use it to evaluate:
 - i. 4×3 Hint: $3 \times 2 = 3 \times 3$
 - ii. $3 + 2$
 - iii. $4 + 3 + 3$
2. The minute of a stop watch is 3. Where will it be if it were round

- I. $3\frac{1}{2}$ rotations clockwise
 - II. $\frac{1}{2}$ rotation anti-clockwise
 - III. $2\frac{1}{2}$ rotations anti-clockwise
3. Find the complete set of solutions to the following
 - i. $4x \equiv 3 \pmod{7}$
 - ii. $x + 1 \equiv 3 \pmod{7}$
 4. Dayo attends a sports club as a member every five days. If he made his fifteenth attendance on a Thursday, when did he first attend the club as a member.
 5. Construct an operation table for multiplication and another for addition in module 7

KEYWORDS: Arithmetic, Modular, cyclic, events e.t.c

WEEK 3

Date.....

SUBJECT: MATHEMATICS

CLASS: SS 1

TOPIC: QUADRATIC EQUATIONS

CONTENT:

- Revision of linear and quadratic expressions
- Solution of quadratic expression of the form $ax^2 + bx + c = 0$, $a \neq 0$ or $b \neq 0$
- Formation of quadratic equation with given roots

REVISION OF LINEAR AND QUADRATIC EXPRESSIONS

Any expression in which highest power of the unknown is 1 is called a linear expression. Some examples of linear expressions are; (a) $x + 1$ (b) $2y + 3$ (c) $p - \frac{1}{2}$

In general, linear expressions are expressions of the form $ax + b$, where a & b are constants and x is a variable.

A quadratic expression is that whose highest power of the unknown is 2. Examples are; (a) $x^2 + 3x$ (b) $2x^2 - 6x + 10$

Factorization of quadratic expressions;

Examples; (i) Factorize the following quadratic expressions

(a) $x^2 + 4x$ (b) $2x^2 - 8x$

Solution:

(a) $x^2 + 4x$, x is a common factor of the terms x^2 & $4x$. Hence $x^2 + 4x$ can be written as $x \cdot x + 4x$ isolating common factors, we have $x(x + 4)$

(b) $2x^2 - 8x$, the common factor of the terms $2x^2$ & $8x$ is $2x$
 $\therefore 2x^2 - 8x$ can be written as $2x \cdot x - 2x \cdot 4$, hence we obtain $2x(x - 4)$

(ii) Factorize the following;

(a) $x^2 + 8x - 20$ (b) $6a^2 + 15a + 9$ (c) $7 - 22x + 3x^2$

Solution: (a) $x^2 + 8x - 20$, find the product of the first and last terms $x^2 \times (-20) = -20x^2$. Find two terms such that their product is $-20x^2$ and their sum is $+8x$

Factors of $-20x^2$	sum of factors
(a) $-20x$ and $+x$	$-19x$
(b) $+20x$ and $-x$	$+19x$
(c) $-10x$ and $+2x$	$-8x$
(d) $+10x$ and $-2x$	$+8x$
(e) $-5x$ and $+4x$	$-x$
(f) $+5x$ and $-4x$	$+x$

Of these, only (d) gives the required result. Replace $+8x$ with $+10x$ and $-2x$ in the given expression. Then factorize by grouping the terms.

$$\begin{aligned}
 x^2 + 8x - 20 &= x^2 + 10x - 2x - 20 \\
 &= x(x + 10) - 2(x + 10) \\
 &= (x + 10)(x - 2)
 \end{aligned}$$

(b) $6a^2 + 15a + 9$, 3 is common factor, first take out the common factor.

$$3(2a^2 + 5a + 3)$$

$$2a^2 \times 3 = 6a^2$$

Factors of $+6a^2$	Sum of factors
+6a and +a	+2a
+3a and +2a	+5a

$$\begin{aligned}
 6a^2 + 15a + 9 &= 3(2a^2 + 5a + 3) \\
 &= 3(2a^2 + 3a + 2a + 3) \\
 &= 3[a(2a + 3) + 1(2a + 3)] \\
 &= 3(2a + 3)(a + 1)
 \end{aligned}$$

(c) $7 - 22x + 3x^2$, find the product of the first and last terms i.e $7 \times (+3x^2) = +21x^2$

Find two terms such that their sum is $-22x$ and their product is $+21x^2$. Since the middle term is negative, consider negative factors only. The terms are $-21x$ and $-x$, replace $-22x$ with $-21x - x$ in the given expression.

$$\begin{aligned}
 7 - 22x + 3x^2 &= 7 - 21x - x + 3x^2 \\
 &= 7(1 - 3x) - x(1 - 3x) \\
 &= (1 - 3x)(7 - x)
 \end{aligned}$$

Class Activity:

Factorize the following quadratic expressions

i. $6a^2 - 18a$

ii. $3x^2y - xy^2$

- iii. $x^2 - 4x + 3$
- iv. $1 - 3x + 2x^2$
- v. $3m^2 + 5mn - 2n^2$
- vi. $p^2 - p + \frac{1}{4}$
- vii. $x^2 - 16$
- viii. $25x^2 - 1$

**SOLUTION OF QUADRATIC EXPRESSION OF THE FORM $ab=0$, $a=0$
OR $b=0$**

If the product of two numbers is 0, then one of the numbers (or possibly both of them) must be zero. For example, $3 \times 0 = 0$, $0 \times 5 = 0$ and $0 \times 0 = 0$.

In general, if $a \times b = 0$, then either $a = 0$ or $b = 0$ or both a & b are zero

Examples 1. Solve the equation $(x - 2)(x + 7) = 0$

Solution: $(x - 2)(x + 7) = 0$

If $(x - 2)(x + 7) = 0$, then either $(x - 2) = 0$ or $(x + 7) = 0$

$\Rightarrow x = 2$ or $x = -7$

2. Solve the equation $a(a + 3) = 0$

If $a(a + 3) = 0$, then either $a = 0$ or $a + 3 = 0$

$\Rightarrow a = 0$ or $a = -3$

3. Solve the equations (i) $(2m - 5)^2 = 0$ (ii) $d(d - 4)(d + 6)^2 = 0$

Solution: (i) if $(2m - 5)^2 = 0$

Then, $(2m - 5)(2m - 5) = 0$

$(2m - 5) = 0$ twice

$\Rightarrow m = \frac{5}{2}$ twice

(ii) if $d(d - 4)(d + 6)^2 = 0$, then any one of the four factors of LHS may be 0

$$\text{i.e } d = 0, d - 4 = 0, (d + 6)^2 = 0$$

$$\Rightarrow d = 0, d = 4 \text{ or } d = -6 \text{ twice}$$

Class Activity:

Solve the following equations;

$$(1) (a - 3)(a + 5) = 0$$

$$(2) 2y \left(y - \frac{1}{3} \right) = 0$$

$$(3) \left(m - \frac{2}{3} \right)^2 (m - 1) = 0$$

$$(4) x^2(x + 5)(x - 5) = 0$$

$$(5) (8 - v)(8 - v)$$

FORMATION OF QUADRATIC EQUATION WITH GIVEN ROOTS

The roots of a quadratic equation are the solutions of that equation. Suppose the roots of a quadratic equation in x are a & b , then we can write; $x = a$ & $x = b$

Examples; (1) Find the quadratic equation whose roots are -2 and $+2$

Solution: let $x = -2$ or $x = 2$, then

$$x + 2 = 0 \text{ or } x - 2 = 0$$

$$(x + 2)(x - 2) = 0$$

$$\text{On careful expansion, we obtain } x^2 - 4 = 0$$

2. Find the quadratic equation whose roots are $2\frac{1}{2}$ and -1

Solution: If the roots are $2\frac{1}{2}$ and -1

$$\text{Let } x = 2\frac{1}{2} \text{ and } x = -1$$

$$x = \frac{5}{2} \text{ and } x = -1$$

$$2x = 5 \text{ and } x = -1$$

$$2x - 5 = 0 \text{ or } x + 1 = 0$$

$$\Rightarrow (2x - 5)(x + 1) = 0$$

$$2x^2 + 2x - 5x - 5 = 0$$

$$2x^2 - 3x - 5 = 0$$

Class Activity:

Form the quadratic equations whose roots are;

- i. 3 and -4
- ii. $\frac{1}{9}$ and $\frac{2}{3}$
- iii. -5 and $\frac{1}{10}$
- iv. $\frac{-1}{5}$ and $\frac{-1}{6}$
- v. $\frac{-3}{8}$ and 0
- vi. $\frac{4}{7}$ and 7

PRACTICE EXERCISE

1. Find the quadratic equation whose roots are $x = -2$ or $x = 7$.
A. $x^2 + 2x - 7 = 0$ B. $x^2 - 2x + 7 = 0$ C. $x^2 + 5x + 14 = 0$
D. $x^2 - 5x - 14 = 0$
E. $x^2 + 5x - 14 = 0$. (SSCE 1988)
2. Find the roots of the equation $2x^2 - 3x - 2 = 0$.
A. $x = -2$ or $1\frac{1}{2}$ B. $x = -2$ or 1 C. $x =$
2 or 2 D. $x = 1$ or 2
E. $x = -\frac{1}{2}$ or 2 (SSCE 1988)
3. Solve the following equation: $6x^2 - 7x - 5 = 0$
A. $x = \frac{1}{2}$ or $x = -2\frac{1}{2}$ B. $x = \frac{1}{3}$ or $x = -2\frac{1}{2}$ C. $x = \frac{1}{3}$ or $x = -\frac{1}{2}$
D. $x = -\frac{1}{3}$ or $x = \frac{1}{2}$
E. $x = \frac{5}{6}$ or $x = -1$ (SSCE 1989)
4. Solve for x: $(x^2 + 2x + 1) = 25$

- A. $-6, -4$ B. $6, -4$ C. $6, 4$ D. $-6, 4$ E. $5, 5$
 (SSCE 1989)

5. Solve the quadratic equation $3x^2 + 4x + 1 = 0$

ASSIGNMENT

1. Factorize $32x^2 - 8xy^2$
 A. $4(4x + y)(2x - y)$ B. $(16x - y)(2x + y)$ C. $8x(2x - y)$
 D. $8x(2x + y)(2x - y)$ E. $4(2x + y)(4x - y)$ (SSCE 1996)

2. Factorize $6x^2 + 7x - 20$
 A. $(6x - 5)(x + 4)$ B. $2(3x - 5)(x + 2)$ C. $(3x + 4)(2x - 5)$
 D. $(3x - 4)(2x + 5)$ (SSCE 2001)

3. Which of the following is not a quadratic expression?
 A. $y = 2x^2 - 5x$ B. $y = x(x - 5)$ C. $y = x^2 - 5$ D. $y = 5(x - 1)$

4. Factorize $m(2a - b) - 2n(b - 2a)$.
 A. $(2a - b)(2n - m)$ B. $(2a + b)(m - 2n)$ C. $(2a - b)(m + 2n)$
 D. $(2a - b)(m - 2n)$ (SSCE 2002)

5. One of the factors of $(mn - nq - n^2 + mq)$ is $(m - n)$. The other factor is:
 A. $(n - q)$ B. $(q - n)$ C. $(n + q)$ D. $(q - m)$.

(SSCE 2011) **KEYWORDS: EXPRESSION, EQUATION,**

QUADRATIC, LINEAR, ROOTS

WEEK 4

Date.....

SUBJECT: MATHEMATICS

CLASS: SS 1

TOPIC: QUADRATIC EQUATION

CONTENT:

- Revision of linear graph and drawing quadratic graph
- Obtaining roots from a quadratic graph
- Finding an equation from a given graph

➤ Application of quadratic equation to real life situations

LINEAR GRAPHS

Recall that any equation whose highest power of the unknown is 1 is a linear equation. To draw the graph of a linear equation, we need to

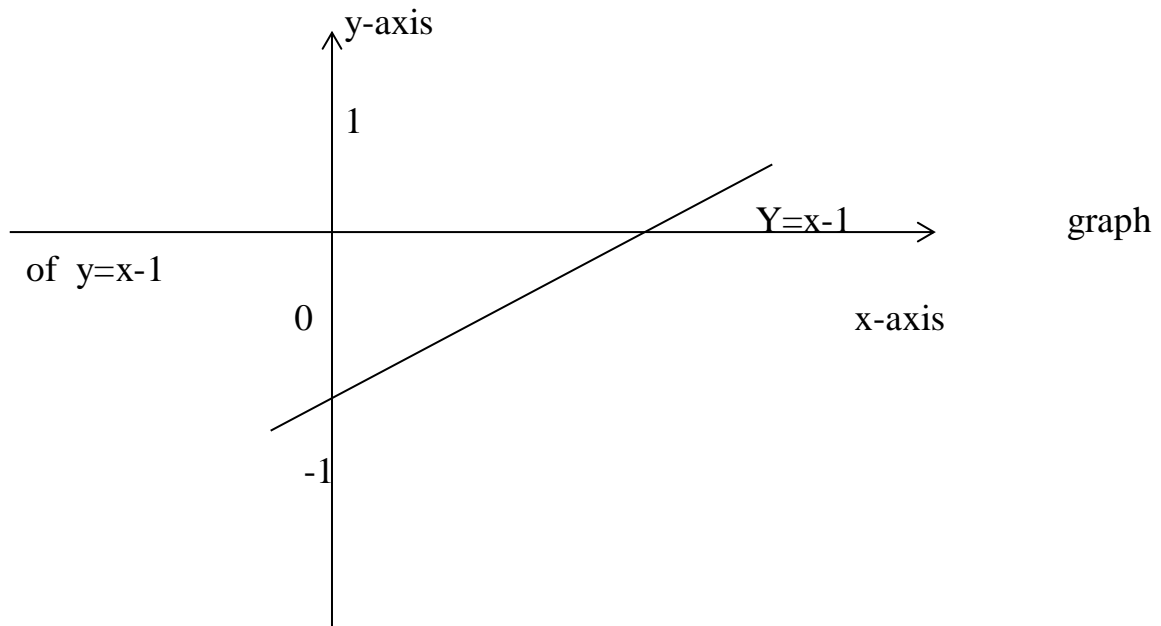
1. Make a table of value for the equation
2. Plot the graph of the linear equation

Example: Draw the graph of $y = x - 1$

Solution; $y = x - 1$

x	-2	0	2
-1	-1	-1	-1
y	-3	-1	1

Scale: 2cm to 1unit on both axes



DRAWING QUADRATIC GRAPH: To draw a quadratic graph, we need to also follow the same process of drawing linear graph

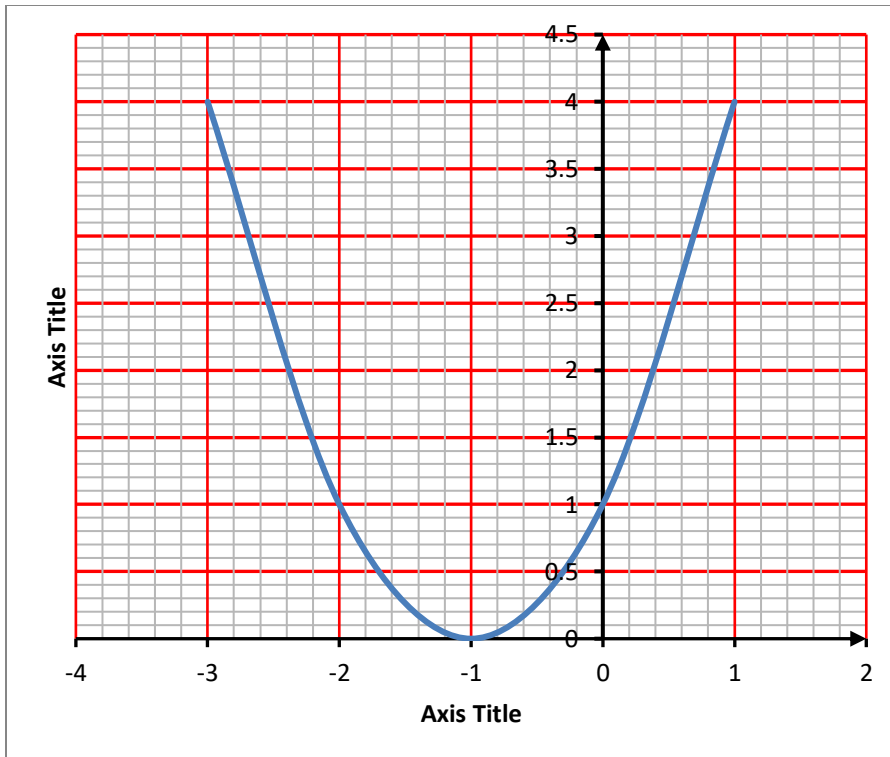
Example: Draw the graph of $y = x^2 + 2x + 1$

Solution; since $y = x^2 + 2x + 1$, we shall now make a table for the values of x & y .

x	-3	-2	-1	0	2	3
x^2	9	4	1	0	4	9
$2x$	-6	-4	-2	0	4	6
1	1	1	1	1	1	1
y	4	1	0	1	9	16

Note: when plotting the graph,

- a. we choose a scale such that our graph is as large as possible and also occupies the centre of the graph sheet. This will enable us to obtain the point where the graph cuts the x -axis more easily.
- b. We join the points in the graph by a smooth curve



Class Activity:

Draw the graph of $y = 2x^2 - 5x + 3$

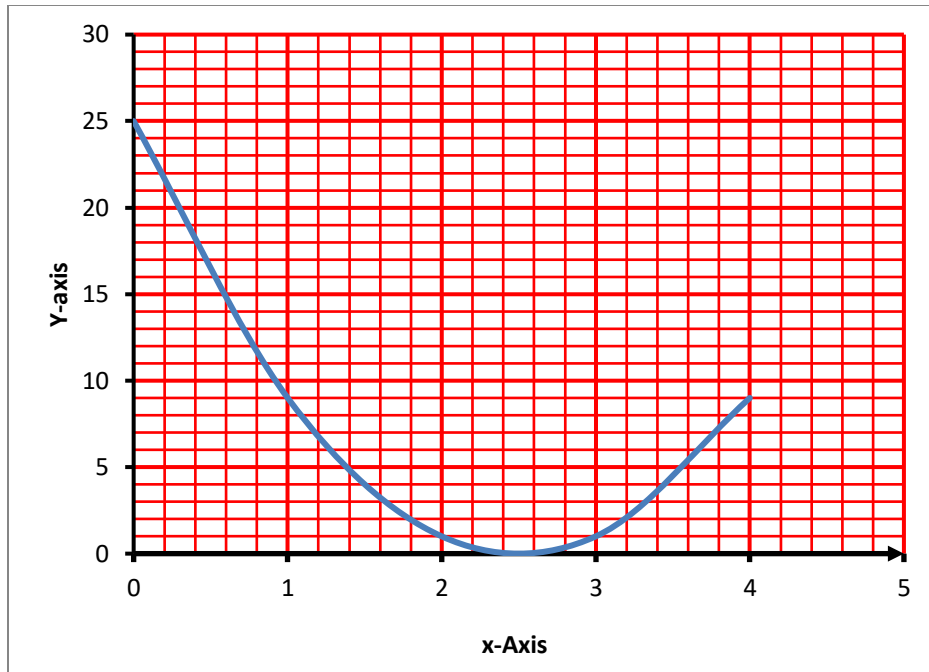
OBTAINING ROOTS FROM A QUADRATIC GRAPH

To obtain the roots of a quadratic equation from a quadratic graph, we need to first plot the graph of the expression and then obtain the roots by reading the two values of x where the graph cuts or touches the x -axis, i.e. where $y=0$

Example: Draw a graph to find the roots of the equation $y = 4x^2 - 20x + 25$

Solution; $y = 4x^2 - 20x + 25$

x	0	1	2	3	4	5
$4x^2$	0	4	16	36	64	100
$-20x$	0	-20	-40	-60	-80	-100
25	25	25	25	25	25	25
y	25	9	1	1	9	25



From the graph it is clear that the curve does not cut the x-axis. It appears to touch the x-axis where $x=2.5$. this result can be checked by factorisation.

$$\text{i.e } 4x^2 - 20x + 25 = 0$$

$$(2x-5)(2x-5)=0$$

$$(2x - 5)^2 = 0$$

$\therefore x = 2.5$ twice

Note: when the curve touches the x-axis, the roots are said to be **coincident**

Class Activity:

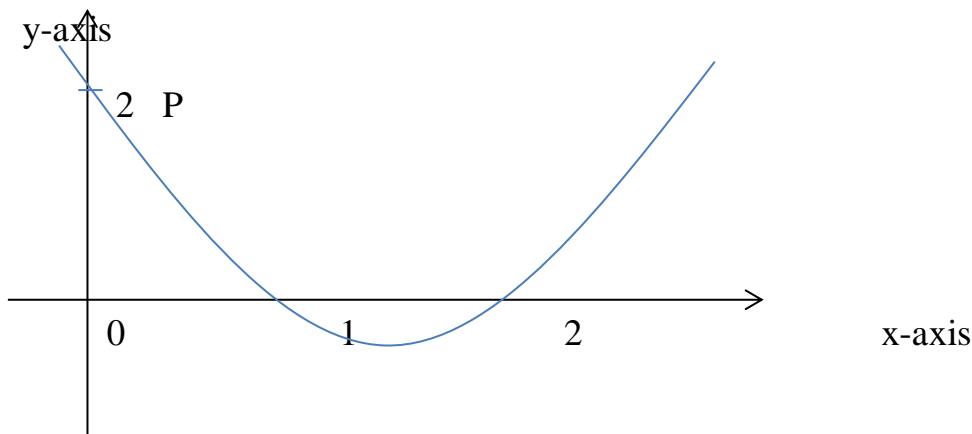
Use the table of values below to solve the equation $y = x^2 + x - 8$ graphically for $-4 \leq x \leq 3$

x	-4	-3	-2	-1	0	1	2	3
y				-8				4

FINDING AN EQUATION FROM A GIVEN GRAPH

It is possible to find the equation of a curve from its graph. The graph of $y = x^2 - 2x - 3$ cuts the x-axis (i.e the line $y=0$) at the points $x=-1$ and $x=3$. This implies that -1 and 3 are the roots of the equation $x^2 - 2x - 3 = 0$. Therefore in general if a graph cuts the x-axis at points a & b, it satisfies the equation $(x-a)(x-b)=0$

Example 1: Obtain the equation of the graph below



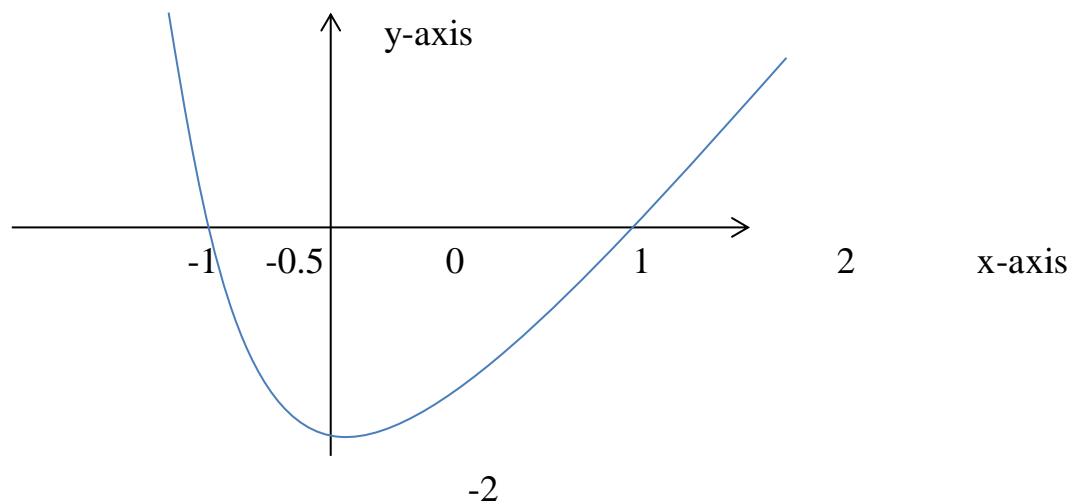
Solution; From the graph when $y=0$, $x=1$ and $x=2$,

Then $(x-1)(x-2)=0$

$.x^2 - 3x + 2 = 0$, at point P, $y=2$ when $x=0$

\therefore the equation of the curve is $y = x^2 - 3x + 2$

Example 2: obtain the equation of the graph below (WAEC)



Solution: In the graph above, where $y=0$, $x=-\frac{1}{2}$ and $x=2$

$$\left(x - -\frac{1}{2}\right)(x - 2) = 0$$

$$\left(x + \frac{1}{2}\right)(x - 2) = 0$$

$$x^2 - 1\frac{1}{2}x - 1 = 0 \quad \dots \dots (i)$$

Second, in the curve above, at point P, $y=-2$ when $x=0$. However the constant term in equation (i) is only -1. So we multiply both sides of the equation (i) by 2

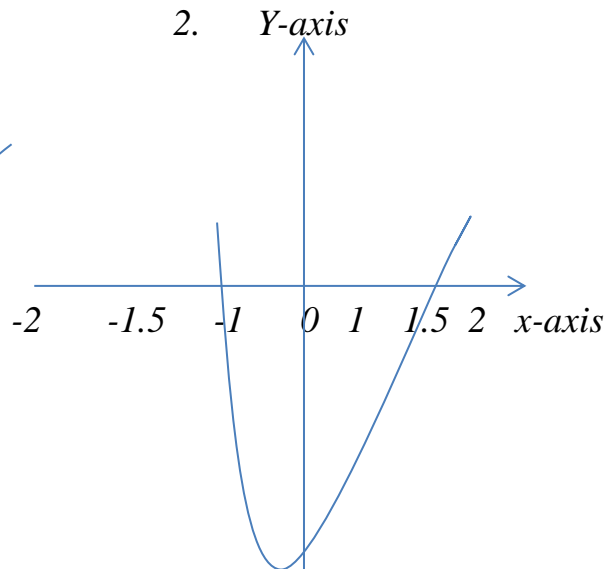
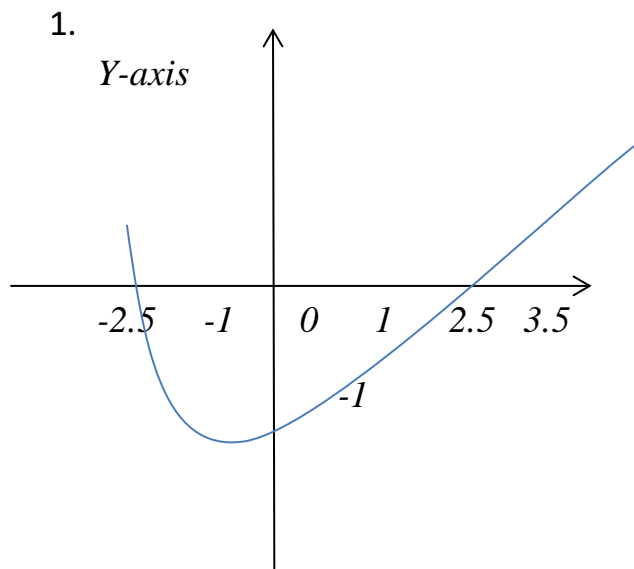
$$2x^2 - 3x - 2 = 0 \quad \dots (ii)$$

Equation (ii) satisfies $\left(x - -\frac{1}{2}\right)(x - 2) = 0$ and the requirement that the constant term should be -2

\therefore The equation of the curve is $y = 2x^2 - 3x - 2$

Class Activity:

Find the equation of the graphs below



APPLICATION OF QUADRATIC EQUATION TO REAL LIFE SITUATIONS

Example 1: The area of a rectangle is 60cm^2 . The length is 11cm more than the width. Find the width.

Solution:

Let the width be $x\text{cm}$, length = $\frac{\text{area}}{\text{width}}$

\therefore length will be $\frac{60}{x}$ cm.

The length is 11cm more than the width gives $\frac{60}{x} = x + 11$

Simplifying; *we have* $60 = x^2 + 11x$

i.e $x^2 + 11x - 60 = 0$

factorizing completely we have, $x = -15$ or $x = 4$

\therefore the width is 4cm since it cannot be negative.

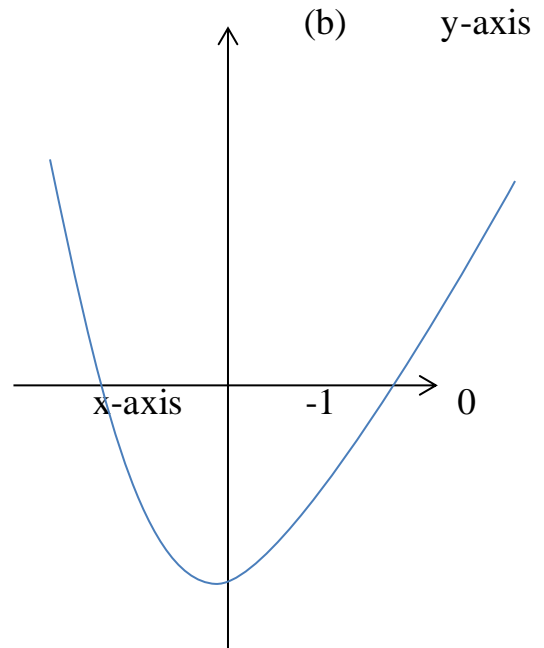
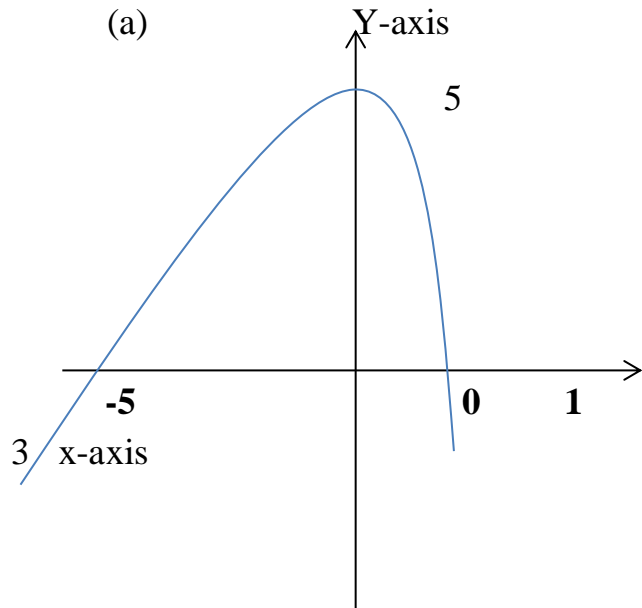
Check: length = $\frac{60}{4} = 15 = 4 + 11$ and $15 \times 4 = 60$

Class Activity:

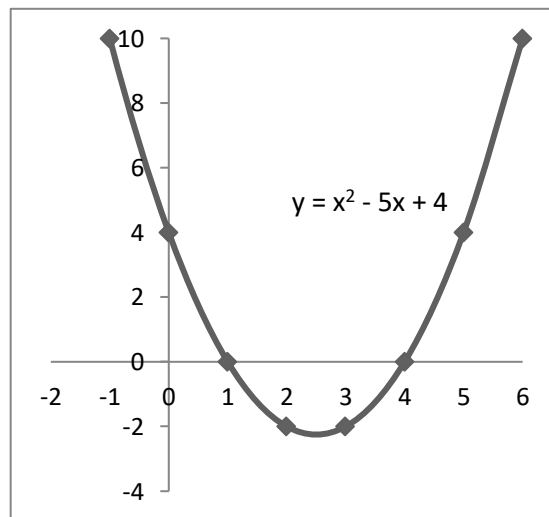
1. When 11 times a certain integer is subtracted from twice the square of the integer, the result is 21. What is the integer?
2. A rectangular lawn is 4cm longer than its width. If its area is 165cm^2 , calculate its width
3. Musa is 60years old and Joy is 25years old. How many years from now will the product of their ages be 2244years?

PRACTICE EXERCISE:

1. Solve the following equations graphically and obtain the least value of y
 - (a) $y = x^2 - 4x + 3 = 0$
 - (b) $y = 2x^2 - 3x + 1$
 - (c) $y = 3x^2 - 4x + 1$
2. The sum of the ages of a mother and her child is 63. If the product of their ages four years ago was 484. What are their ages now?
3. Find the equations of the graphs below



-3



4. The following is a graph of a quadratic function. Use it to answer question 3 and 4.

1. Find the co-ordinates of point **P**.

- A. (0, 4) B. (1, 4) C. (0, -4) D. (-4, 0) (SSCE2009)

2. Find the values of x when $y = 0$.

A. 1, 3

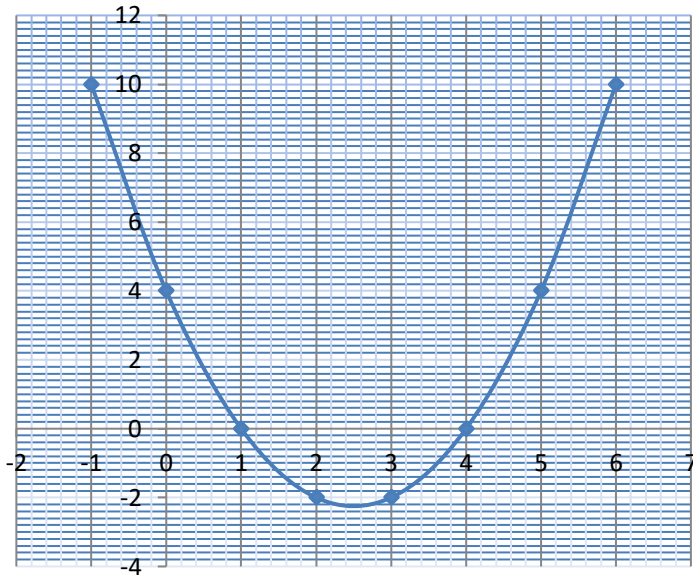
B. 1, 4

C. 2, 3

D. 1, 6

(SSCE 2009)

5.



(SSCE 2006)

The figure is the graph of a quadratic equation. Use the information to answer questions 1 and 2.

1. What are the roots of the equation?

A. -1 and 1

B. 4 and -1

C. 1 and 4

D. 4 and 4

(SSCE 2005)

2. What is the equation of the curve?

A. $y = x^2 + 6x + 4$

B. $y = x^2 - 5x - 4$

C. $y = x^2 - 5x + 4$

D. $y = x^2 + 5x - 4$

ASSIGNMENT

1. Draw the graph of $y = 4x^2 + 25$ and $y = 3x + 2$ for $-3 \leq x \leq 3$.

Use the graph(s) to:

- i. find the roots of the equation $y = 4x^2 + 25$ and $y = 3x + 2$
- ii. determine the line of symmetry of the curve $y = 4x^2 + 25$.

2. (a) Copy and complete the table of values for the relation $y = -x^2 + x + 2$ for $-3 \leq x \leq 3$.

x	-3	-2	-1	0	1	2	3
y		-4		2			-4

(b) Using scales of 2cm to 1 unit on the x-axis and 2cm to 2 units on the y-axis.

Draw a graph of the relation $y = -x^2 + x + 2$

(c) From the graph find the:

- i. minimum value of y
- ii. roots of the equation $x^2 - x - 2 = 0$
- iii. gradient of the curve at $x = 0.5$

(SSCE 2010)

KEYWORDS: EXPRESSION, EQUATION, QUADRATIC, LINEAR, ROOTS, MINIMUM VALUE, MAXIMUM, INTERCEPT ETC

WEEK 5

DATE.....

Subject: Mathematics

Class: SS 1

TOPIC: Constructions

Content:

- Guidelines for constructions
- Constructions of basic angles
- Construction of triangles with given sides and angle.
- Bisection of basic angles

GUIDELINES FOR CONSTRUCTIONS

When making constructions, the following guidelines should be followed.

1. A short pencil of about 3 inches should be fixed on the pair of compasses when constructing to avoid any obstruction when turning your compass round to draw arcs.
2. Ensure that the pivot of your pair of compasses is tight to avoid unwanted shift when carrying out your constructions.
3. To ensure that your lines and points are as fine and accurate as possible make use of a hard pencil with a sharp point.
4. Before making the actual construction, make a rough sketch of the problem under consideration. This will make the construction of the actual problem easy.
5. Leave all your arcs and construction lines visible. Do not clean any arc that leads you to your final result.
6. Double lines and arcs in constructions are not allowed, hence clean up all double arcs and lines neatly and re-draw.

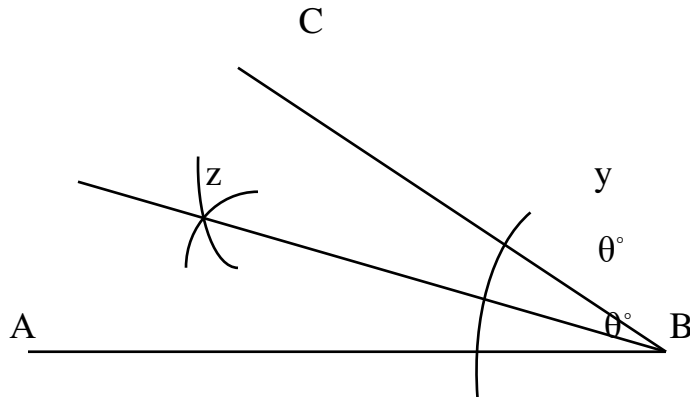
Bisecting Of Angles

To bisect the angle ABC,

Step 1: –Measure a length of about 2.5cm with your pair of compasses. Fix pin at B and draw an arc to cut the two line AB and BC that formed the angle. Cut the arc at x and y.

Step 2: – Fix pin at x and draw an arc and at y and draw another arc both to meet at z.

Step 3: – Join the point zB. $\therefore ABz = zBC$



Bisecting A Line AB:

To bisect the line AB

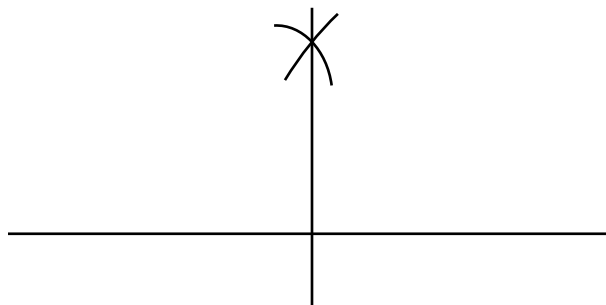
Step 1: – Measure the line AB with your ruler.

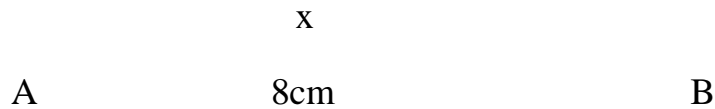
Step 2: – Measure about $\frac{3}{4}$ of the given line with your compasses, use for step 3 and step 4.

Step 3: – Fix pin at A and draw an arc above the line AB and below the line AB.

Step 4: – Fix pin also at B and draw an arc above the line AB to cut the arc drawn in step 3 at x and draw another arc below to cut the other arc drawn in step 3 at y.

Step 5: – Join the points xy with your ruler and produce both ways.





Class Activity:

What are the basic guidelines for constructions?

CONSTRUCTIONS OF THE BASIC ANGLES

To construct the basic angles, we shall make use of the line AB of length 8cm for each case.

NOTE:

Angles constructed at the point A are normally read from the right side of A to left using the protractor and angles constructed at point B are normally read from the left side of B to the right.

Constructions Of The Basic Angles

To construct the basic angles, we shall make use of the line AB of length 8cm for each case.

Note:

Angles constructed at the point A are normally read from the right side of A to left using the protractor and angles constructed at point B are normally read from the left side of B to the right.

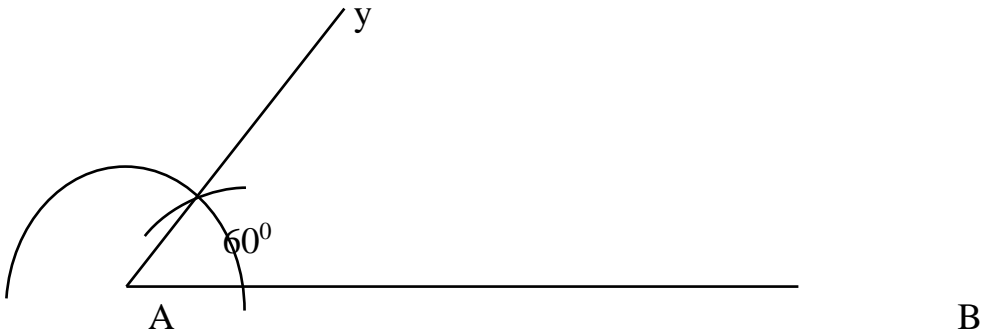
Angle 60°

Step 1: – Measure a length of about 2.5cm with your pair of compasses

Step 2: – Put pin at A and draw an arc in form of a semi-circle to cut the line AB at X.

Step 3: – Using the same length as in step 1 draw an arc putting pin at x cut the arc in step2 at y.

Step 4: – Join Ay and produce. Angle yAB = 60° .



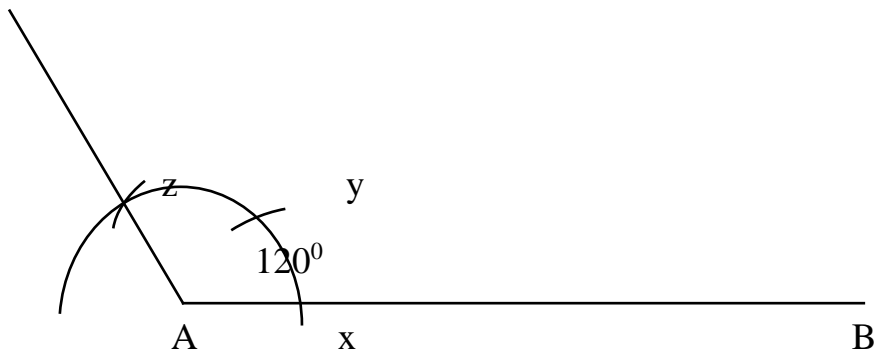
Angle 120° (Obtained by constructing $60^{\circ} + 60^{\circ}$)

The steps for constructing angle 60° above is followed from step 1 to step 3.

Step 5: –Using the same arc length as in step 1 above put pin at point y and draw an arc to cut the arc in step2 at point z.

Step 6: – Join Az and produce

Angle zAB = 120°



Class Activity

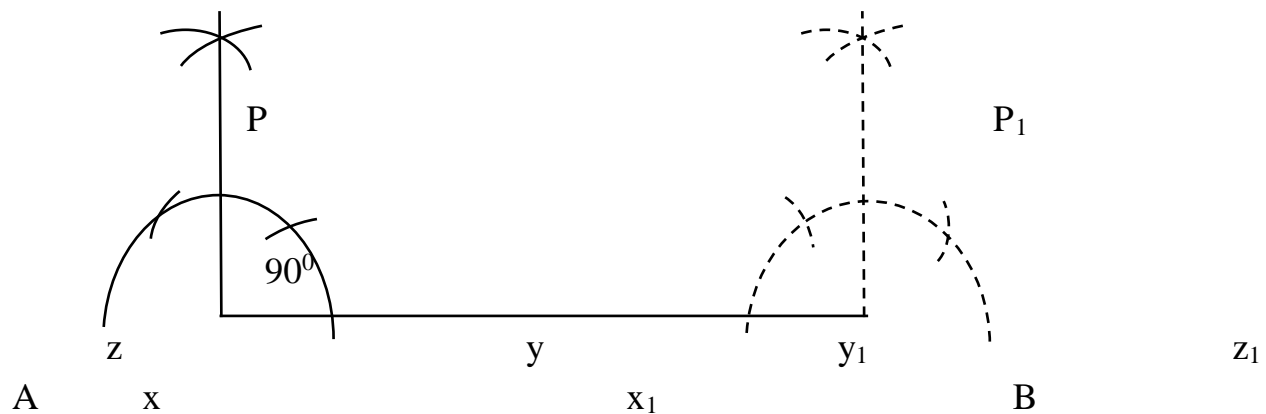
Construct all the basic angles shown above at point B on the line AB.
Where AB = 8cm.

Angle 90° (Obtained by constructing 60° + 30°)

The steps for constructing angle 120° above are followed from step 1 to step 5.

Step 6: – Bisect the angle formed by arc yz to have 30° added to angle yAx = 60°. This is done by drawing an arc with compass pin at point z and another arc with pin at point y, both to meet at point p.

Step 7: – Join Ap produced. Angle PAB = 90°



NB: – To construct angle at point B as shown above, the first arc is drawn by fixing pin at B and cutting arc from x₁.

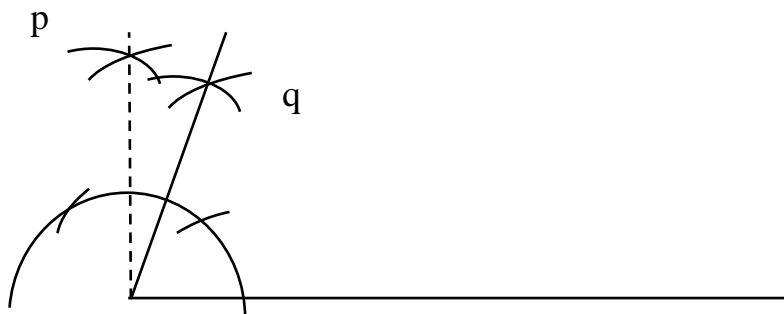
Angle 75° (obtained by constructing 60° + 15°)

The steps for constructing angle 90° above are followed from **step 1** to **step 7**.

As for the step 7, the line Ap is drawn using broken line, since the required angle is not 90°. The angle, 90° will only aid us in constructing 75°.

Step 8: – Fix pin at point where Ap cut arc yz draw an arc and also fix pin at y and draw an arc both to meet at q.

Step 9: – Join Aq and produce ∴ qAB = 75°





Angle 105° (obtained by constructing $90^\circ + 15^\circ$)

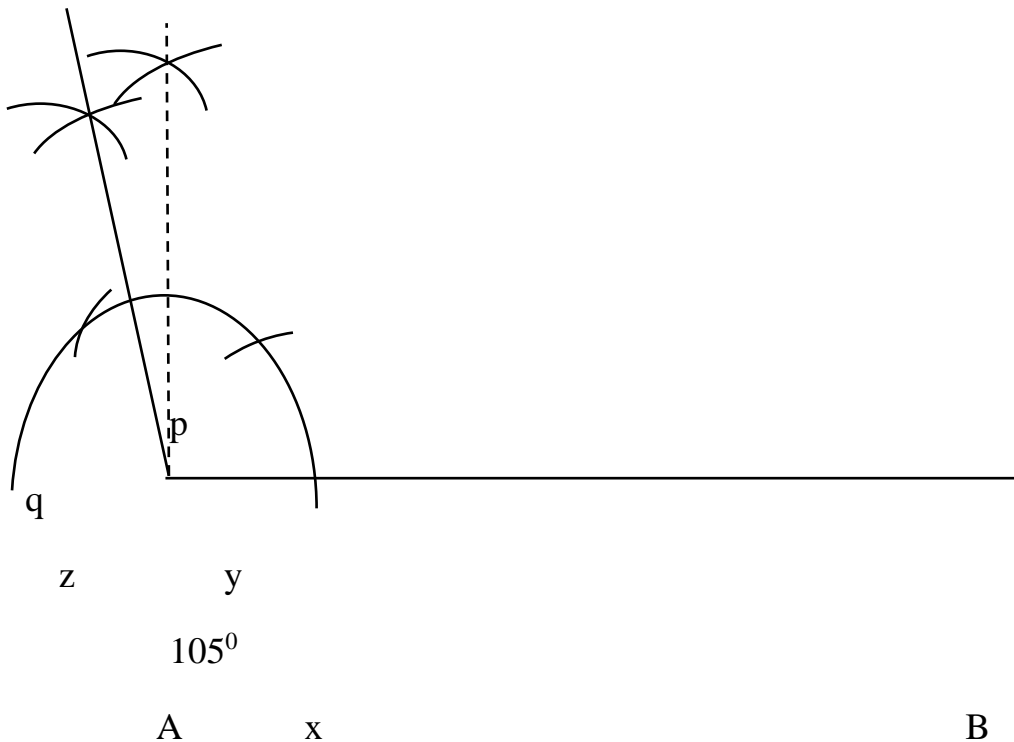
The steps for constructing angle 90° above are followed from step 1 to step 7.

As for step 7, the line Ap is drawn using broken line, since the required angle is not 90° .

The angle 90° will only aid us in constructing 105° .

Step 8: – Fix pin at point where Ap cut arc yz draw an arc and also fix pin at z and draw an arc both to meet at q.

Step 9: – Join Aq and produce $\therefore qAB = 105^\circ$



Class Activity

Construct all the basic angles shown above at point B on the line AB. Where AB = 8cm.

Angle 135° (obtained by constructing 90° + 45°)

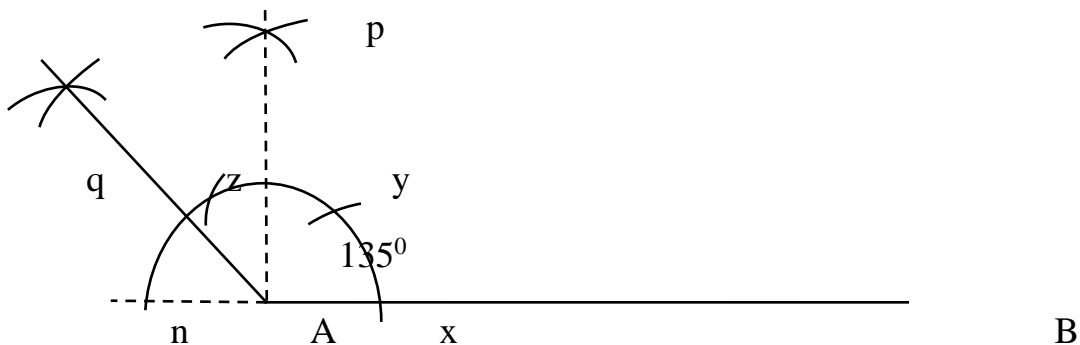
The steps for constructing angle 90° above are followed from step 1 to step 7.

As for step 7, the line Ap is drawn using broken line, since the required angle is not 90°. The angle 90° will only aid us in constructing 135°.

Step 8:-- Produce BA to cut the 1st arc at point n.

Step 9:--Fix pin at point n and draw an arc, fix pin also at the point where Ap cut arc yz and draw an arc both to meet at q.

Step 10:-- Join Aq and produce ∴ qAB = 135°



Angle 45°(obtained by constructing 90° ÷ 2)

The steps for constructing angle 90° above are followed from step 1 to step 7.

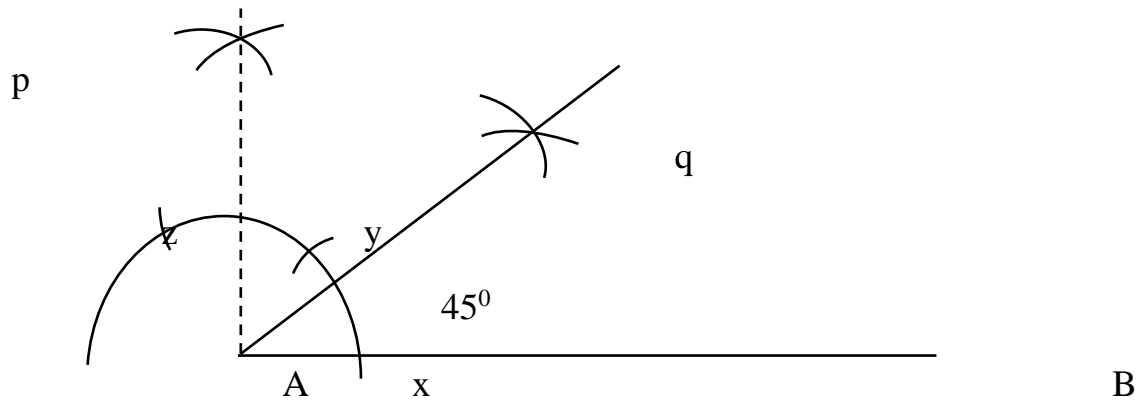
As for step 7, the line Ap is drawn using broken line, since the required angle is not 90°.

Step 8: – Bisect angle pAB (90°) to get 45°.

i.e. Fix pin at x and draw an arc and fix pin at the point where pA cuts arc zy and draw another arc both to meet at q.

Step 9: –Join Aq and produce.

$$\therefore qAB = 45^\circ$$



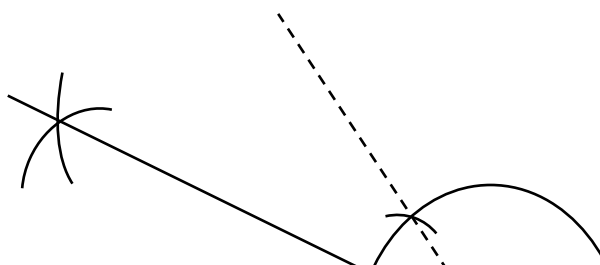
Angle 30°:

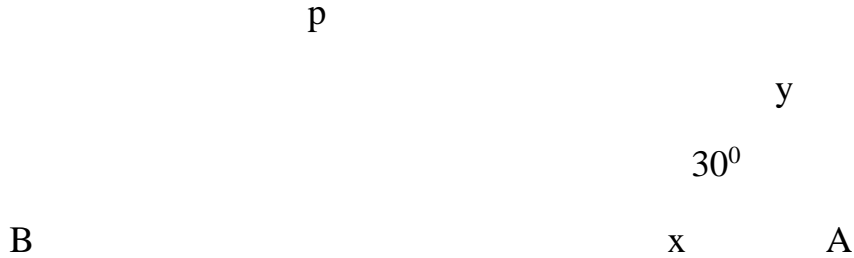
This is obtained by bisecting angle 60°.

Step 1:- Construct angle 60° with broken line since the required angle is not 60°.

Step2: -To bisect the angle, fix pin at x and draw an arc and also fix pin at y and draw another arc both to meet at p.

Step3: –Join Ap and produce. $\therefore pAB = 30^\circ$





Note that:

1. Other angles can be obtained by bisecting these basic angles that have been drawn already.

e.g. - 15° can be obtained by bisecting 30°

- $22\frac{1}{2}$ can be obtained by bisecting 45°
- $52\frac{1}{2}$ can be obtained by bisecting 105°
- $67\frac{1}{2}$ can be obtained by bisecting 135°
- $37\frac{1}{2}$ can be obtained by bisecting 75°

2. In practical construction, the letter x, y, z, p and q are not indicated at the points where the arcs are drawn. We did this in this text to enable you understand the steps in constructing the basic angles. **Also small letters are not used to label points in Geometry.**

3. These basic angles can also be constructed at the point B on the line AB. In this case the arcs x, y, z are drawn from the left side of B. The angles on the protractor are also read from the left side of B to the right.

Class Activity

1. Construct all the basic angles shown above at point B on the line AB. Where AB = 8cm.

2. Using a ruler and a pair of compasses only, construct the following angles at the point B on the line AB = 6.5cm.

- (a) 60° (b) 120° (c) 90° (d) 45° (e) 135° (f) 75° (g) 105° (h) $52\frac{1}{2}^{\circ}$ (i) $22\frac{1}{2}^{\circ}$ (j) $37\frac{1}{2}^{\circ}$
 (k) $67\frac{1}{2}^{\circ}$ (l) 30° (m) 15° (n) 150°

CONSTRUCTION OF TRIANGLES:

Case 1: When only the three sides are given.

Example 1:

Construct a triangle ABC such that $AB = 8\text{cm}$, $BC = 7\text{cm}$ and $AC = 6\text{cm}$.

Solution:

Step 1: – Draw line $AB = 8\text{cm}$ with your ruler.

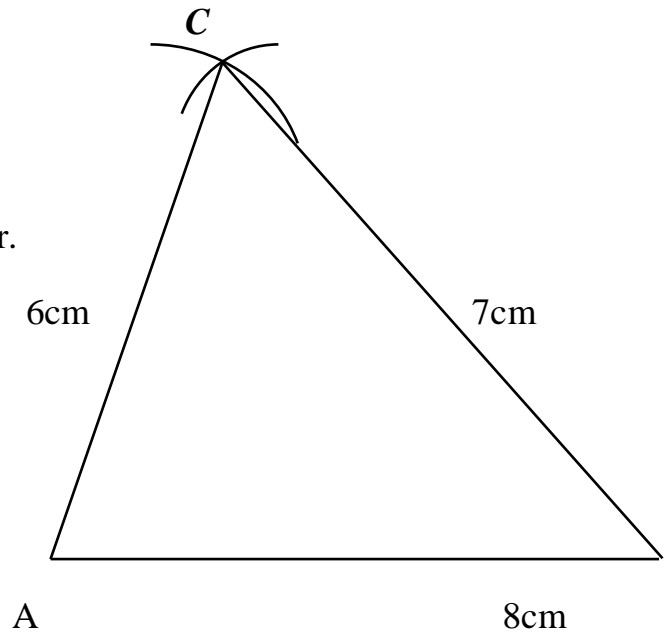
Step 2: – Measure 7cm from your ruler with your pair of compasses, fix pin at B and draw an arc at the suspected position of C.

Step 3: – Measure 6cm from your ruler

With your compasses, fix pin at A and B

draw an arc to cut the arc drawn in step 2 at C.

Step 4: – Join AC and BC and label the vertices and sides.



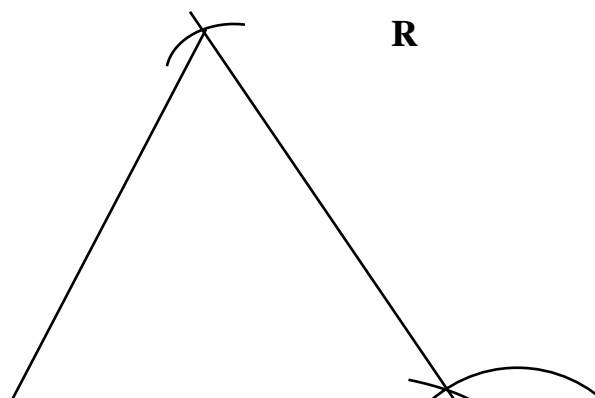
Case 2: When given two sides and the included angle.

Example 2:

Construct a triangle PQR such that $PQ = 7.5\text{cm}$, $\angle PQR = 60^\circ$ and $QR = 6\text{cm}$.

Solution:

Step 1: – Draw the line $PQ = 7.5\text{cm}$ with your ruler.



Step 2: – Construct the angle

6cm

60° at the point Q.

Step 3: – Measure 6cm from your ruler

with your pair of compasses. Fix pin at

point Q and cut an arc on QR at R.

60°

Step 4: – Join PR to form a triangle.

P

7.5cm

Q

Class Activity

(1) Using a ruler and a pair of compasses only, construct the following triangles.

(a) ΔABC , such that $|AB| = 8\text{cm}$, $|BC| = 6\text{cm}$ and $|AC| = 7\text{cm}$. Measure $\angle CAB$

(b) ΔABC , such that $|AB| = 9\text{cm}$, $|BC| = 8\text{cm}$ and $|AC| = 7\text{cm}$. Construct also a circle passing through A, B and C. Find the radius of the circle.

(c) ΔPQR , such that $|PQ| = 7.3\text{cm}$, $|QR| = 5.4\text{cm}$ and $|PR| = 6.5\text{cm}$. Measure $\angle PRQ$.

(d) ΔPQR , such that $|PQ| = 7.5\text{cm}$, $\angle PQR = 75^{\circ}$ and $QR = 6\text{cm}$. Measure $\angle PRQ$.

(e) ΔXYZ , such that $YZ = 7\text{cm}$, $\angle XYZ = 45^{\circ}$ and $XY = 10\text{cm}$. Measure $\angle XZ$.

Case 3: When given one side and two angles.

Example 3:

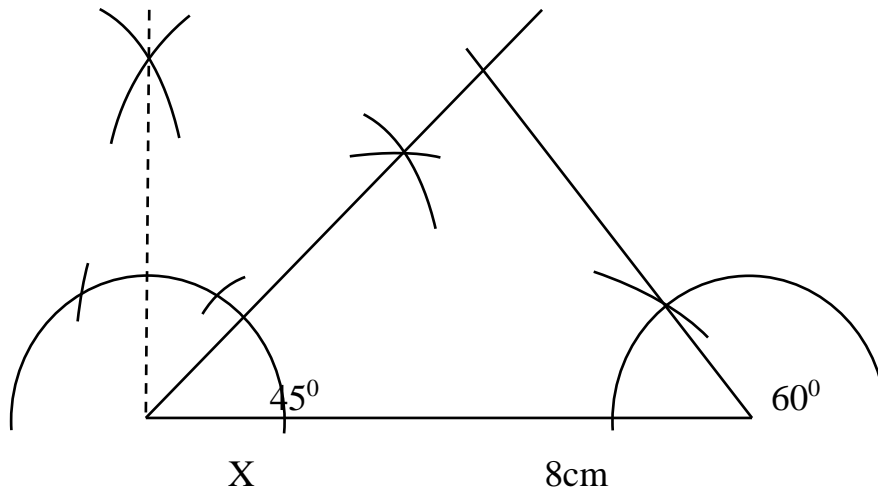
Construct a triangle XYZ such that $XY = 8\text{cm}$, $\angle XYZ = 60^{\circ}$ and $\angle ZXY = 45^{\circ}$

Solution: Step 1: – Draw the line $XY = 8\text{cm}$ using your ruler.

Step 2: – Construct angle 60° at point Y.

Step 3: – Construct angle 45° at point X.

Step 4: – Produce the line for angle 60° from Y and the line for angle 45° from X to meet at point Z



Case 4: When given two sides and an angle.

Example 4:

Construct a triangle ABC such that $AB = 7.5\text{cm}$, $\angle B = 75^\circ$ and $AC = 9\text{cm}$.

Solution:

Step 1: – Draw the line $AB = 7.5\text{cm}$ using your ruler.

Step 2: – Construct angle 75° at B.

Step 3: – Measure 9cm from your ruler with a pair of compasses, fix pin at point A and cut an arc on the line BC at C.

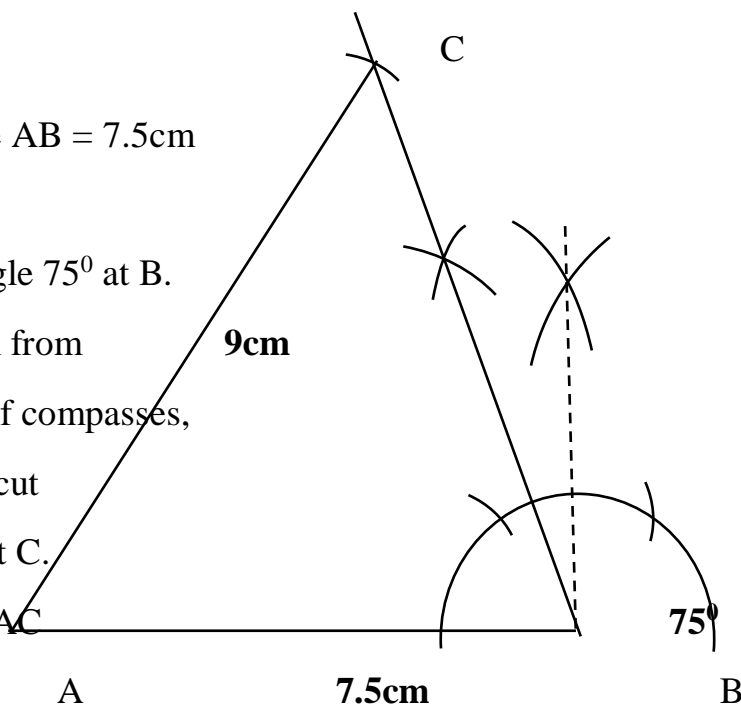
Step 4: – Join the line AC

to form the triangle.

A

7.5cm

B



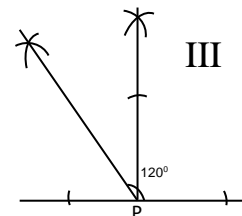
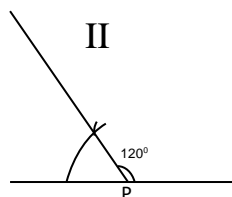
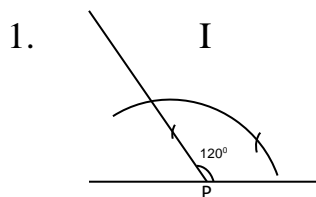
Class Activity

- (i) ΔABC , such that $|AB| = 6.8\text{cm}$, $\angle ABC = 60^\circ$ and $\angle CAB = 75^\circ$. Measure AC.
- (j) ΔXYZ , such that $XY = 9\text{cm}$, $\angle XYZ = 45^\circ$ and $\angle ZXY = 75^\circ$. Construct also a circle passing through the points X, Y and Z. Measure the radius of the circle.
- (k) ΔPQR , such that $PQ = 6.5\text{cm}$, $\angle PQR = 75^\circ$ and $|PR| = 9\text{cm}$. Measure QR.
- (l) ΔXYZ , such that $XY = 7\text{cm}$, $\angle ZXY = 60^\circ$ and $|YZ| = 10\text{cm}$. Measure XZ.
- (m) ΔPQR , such that $|PQ| = 10\text{cm}$, $\angle PQR = 37\frac{1}{2}$ and $\angle RPQ = 60^\circ$. Measure QR.
- (n) ΔPQR , such that $|PQ| = 7\text{cm}$, $\angle PQR = 52\frac{1}{2}$ and $QR = 6\text{cm}$. Measure PR.

PRACTICE EXERCISE

- ΔPQR , such that $|PQ| = 8.4\text{cm}$, $\angle PQR = 90^\circ$ and $QR = 6.5\text{cm}$. Construct a circle also passing through P, Q and R. What is the radius of the circle?
- Construct the ΔABC such that $AB = 6\text{cm}$, $BC = 5.5\text{cm}$ and $\angle ABC = 60^\circ$
- Using a ruler and a pair of compasses only, construct;
a triangle PQR such that $|PQ| = 10\text{cm}$, $|QR| = 7\text{cm}$ and $\angle PQR = 90^\circ$
- Construct a triangle ABC with $AB = 7\text{cm}$, $BC = 5\text{cm}$ and $AC = 6\text{cm}$
- Construct a triangle ABC given $AB = 7\text{cm}$, $\angle A = 75^\circ$, $\angle B = 45^\circ$. Calculate the third angle

ASSIGNMENT

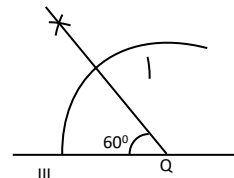
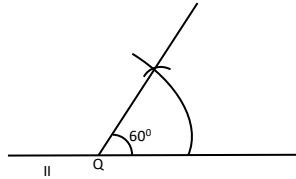
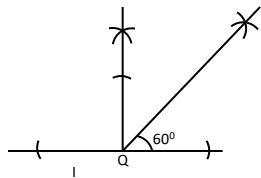


Which of the sketches above give the correct method for constructing an angle of 120° at the point P?

I only B. II only C. III only D. I and II only E. I, II and III.
(SSCE 1990)

2. Which of the following is a correct method for constructing angle 60° at Q?

A. I only B. II only C. III only D. I and II only E. II and III only
(SSCE 1988)



P and Q are two points which are 7cm apart. If the positions of a point R which is 4.2 cm from P and 5.6 cm from Q are constructed, how many possible positions for R are there?

A. 1 B. 2 C. 3 D. 4

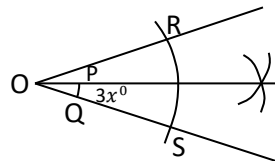
3. Construct triangle ABC in which $AB = 7.6$ cm, $\angle BAC = 45^\circ$, $\angle ABC = 60^\circ$

4. Construct triangle ABC in which $\angle B = 105^\circ$, $BC = 8.2$ cm, $\angle C = 45^\circ$

1. 5. In the diagram, $\angle ROS = 66^\circ$ and $\angle POQ = 3x$. Some construction lines are shown. Calculate the value of x .

A. 10° B. 11° C. 22° D. 30°

(SSCE 2010)



KEYWORD: BISECT, LOCUS, PARALLEL, PERPENDICULAR, EQUIDISTANT ETC

WEEK 6:

Date:.....

Subject: Mathematics

Class: SS 1

TOPIC: Constructions 2

Content:

- Construction Of Perpendiculars
- How To Construct Perpendiculars
- Circumscribed And Inscribed Circles Of A Triangle
- Construction Of Quadrilaterals:
- Locus

CONSTRUCTION OF PERPENDICULARS

How To Construct A Perpendicular To A Given Straight Line AB From A Point P Outside The Line.



Given: A line AB with a point P outside the line AB

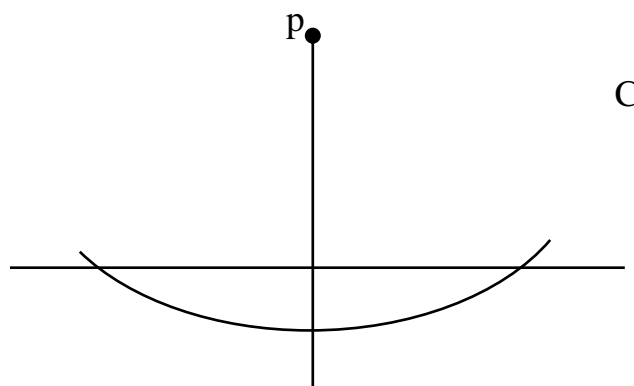
B

Step 1: Fix Compasses Pin at point P. Extend the Compasses wide enough to draw an arc to cut the given line AB at two points x_1 and x_2 .

Step 2: Fix Compasses Pin at x_1 and x_2 with equal radii, draw arcs to cut each other at Q.

Step 3: Join QP cutting AB at C.

Then $\angle ACP = \angle BCP = 90^\circ$



B

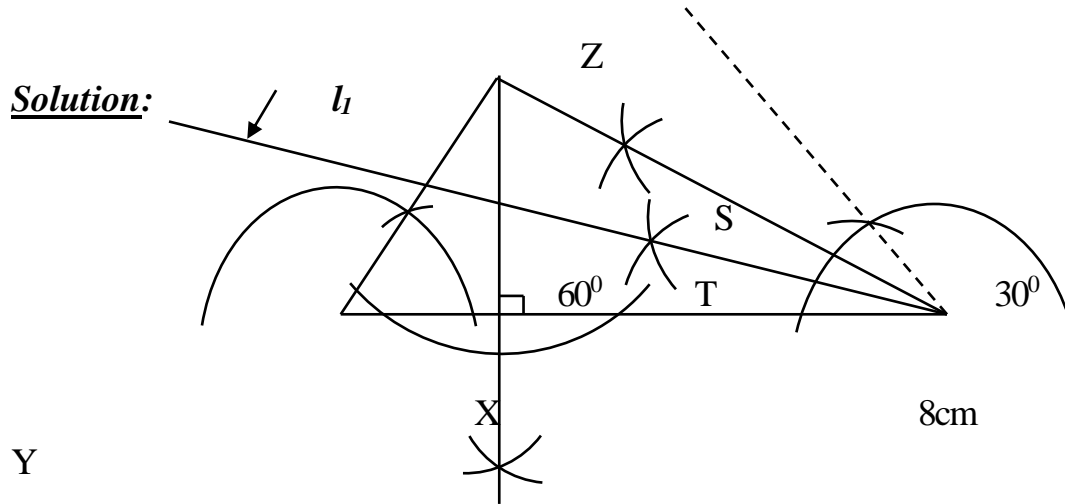
Q

Examples 1:

Using ruler and a pair of Compasses only,

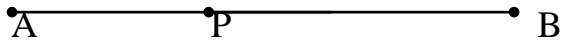
- (a) *Construct*
 (i) triangle XYZ with $XY = 8\text{cm}$, $\angle YXZ = 60^\circ$
 and $\angle XYZ = 30^\circ$
 (ii) the perpendicular ZT to meet XY in T;
 (iii) the locus l_1 of points equidistant from ZY and XY.
 (b) If l_1 and ZT intersect at S, Measure ST
WASSCE, JUNE 2002, No 10.

Solution:



How To Construct A Perpendicular To A Given Straight Line AB From A Point P, On The Line.

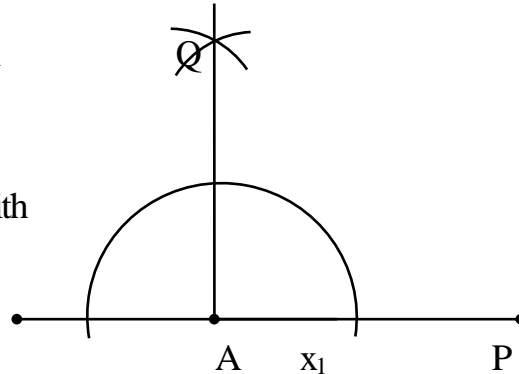
Given: A line AB with a point P on the line AB.



Step 1: Fix Compasses Pin at P and draw an arc to cut AB at X_1 and X_2 .

Step 2: Fix compasses Pin at X_1 and X_2 , with equal radii, draw arcs to cut each other at Q.

Step 3: Join QP
 X_2 B



Class Activity:

New General Mathematics for Senior Secondary School , Book 1, pages 196 to 197, Exercise 16b, Nos. 3, 4, 5 and 7

CIRCUMSCRIBED AND INSCRIBED CIRCLES OF A TRIANGLE

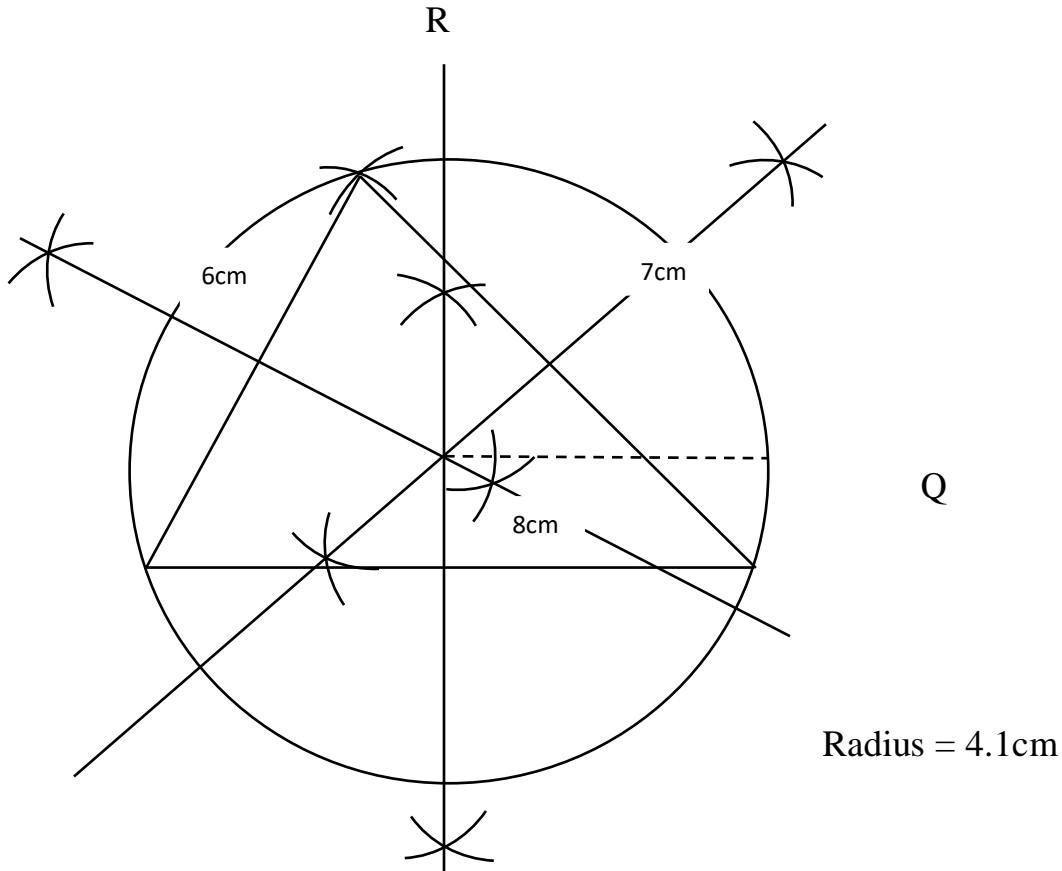
How To Draw A Circle Passing Through The Three Vertices Of A Triangle (Circumscribed Circle Of A Triangle)

Step 1: – Construct the perpendicular bisectors of each of the three sides of the triangle. The lines of bisection will all meet at a point.

Step 2: – Fix pin at the meeting point of the three lines and extend compass to one of the vertex and draw the circle.

Example 2:

Construct a triangle PQR such that $PQ = 8\text{cm}$, $QR = 7\text{cm}$ and $PR = 6\text{cm}$. Construct a circle passing through the points P, Q and R. What is the radius of the circle?



HOW TO CONSTRUCT THE INSCRIBED CIRCLE OF A GIVEN TRIANGLE ABC

Step 1: – Construct an internal bisector of each of the angles A, B and C. The bisectors of the

three angles will meet at a point O.

Step 2: – Construct a perpendicular from O to AB to meet AB at P

Step 3: – Join OP ,(OP is the radius of the circle)

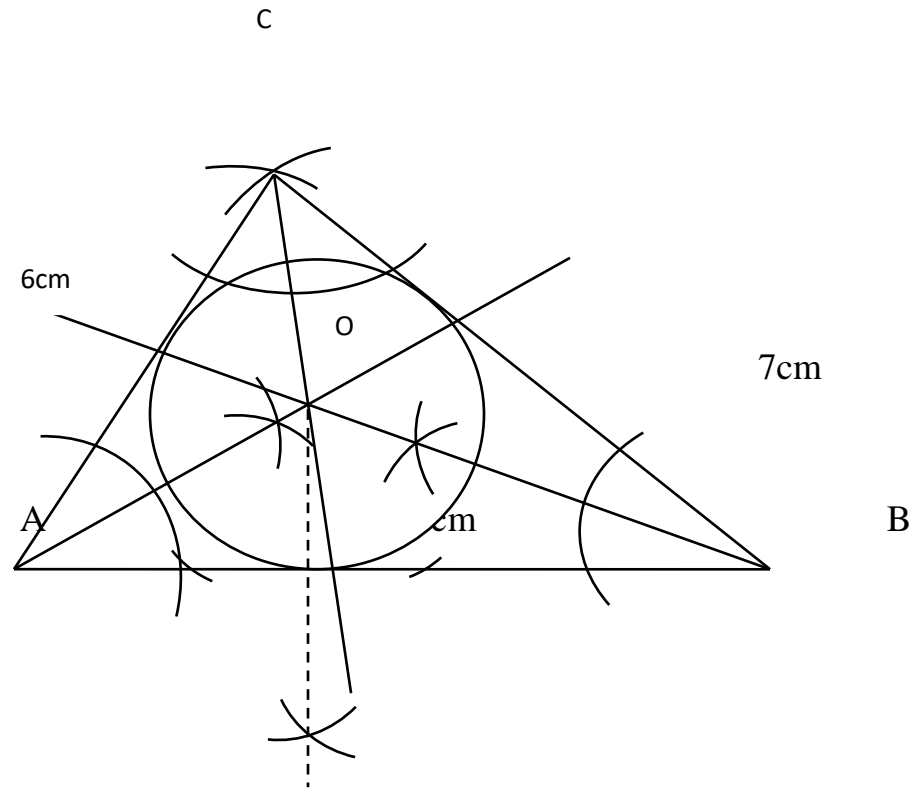
Step 4: – Draw the inscribed circle with center O and radius OP.

Example 3:

Construct a triangle ABC such that

$AB = 8\text{cm}$, $BC = 7\text{cm}$ and $AC = 6\text{cm}$.

Construct an inscribed circle of the triangle .



Class Activity:

New General Mathematics for Senior Secondary School , Book 1, pages 196 to 197 , Exercise 16b, Nos. 1, 2, and 6

Construction Of Quadrilaterals:

Parallelograms:

Example 4:

Using a ruler and a pair of compasses

only construct a parallelogram $RXYZ$

such that $XY = 7\text{cm}$, $\angle XYZ = 120^\circ$ and

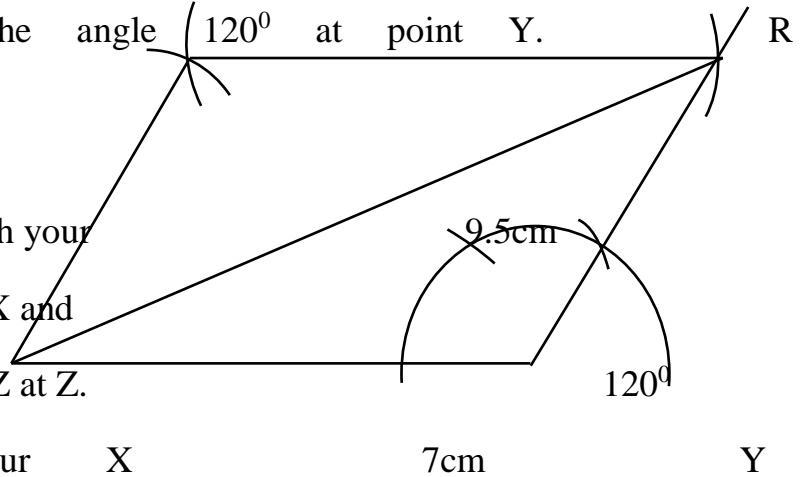
the diagonal $XZ = 9.5\text{cm}$.

Solution:

Step 1: – Draw the line $XY = 7\text{cm}$ with your ruler.

Step 2: – Construct the angle 120° at point Y.

Step 3: – Measure 10.5cm with your pair of compasses. Fix pin at X and draw an arc to cut the line YZ at Z.



Step 4: – Measure YZ with your pair of compasses. Fix pin at the point X and draw an arc at the suspected position of R (Since opposite sides of a parallelogram are equal $XR = YZ$).

Step 5: – Measure 7cm with your pair of compasses. Fix pin at Z and cut an arc at the suspected position of R to cut the first arc in step 4. (Opposite sides of a parallelogram are equal $RZ = XY$). The meeting point of the arcs is R.

Step 6: – Join RZ and XR.

* Put into considerations the properties of a parallelogram

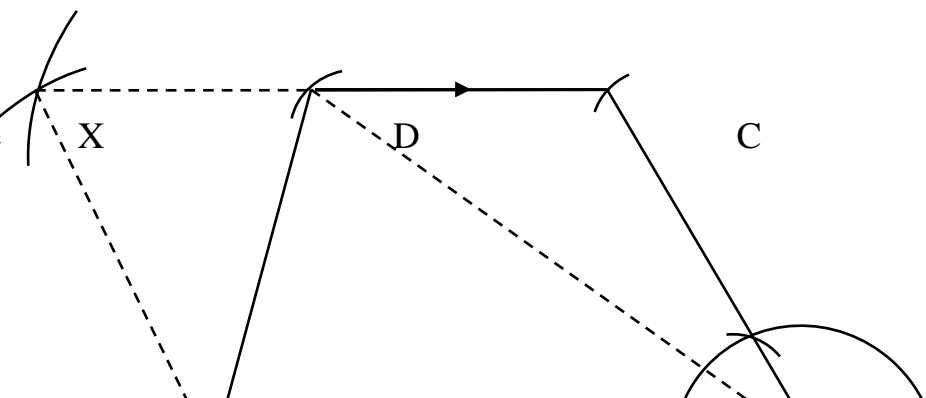
Trapezium:

Example 5:

Using a ruler and a pair of compasses only, construct a trapezium ABCD, in which $AB \parallel DC$, $AB = 8\text{cm}$, $\angle ABC = 60^\circ$, $BC = 5.5\text{cm}$ and $BD = 8.3\text{cm}$.

Solution:

Step 1: – Draw the line



AB = 8cm using a ruler.

Step 2: – Construct

angle 60° at point B.

Step 3:
 60°

–

Measure

5.5cm

with your Compasses,
B

A

8cm

fix pin at B and cut C to get BC.

Step 4: – Using the measurement

of BC = 5.5cm and properties of a parallelogram. Draw AX

parallel to BC. Measure BA = 8cm,

fix pin at C, draw CX parallel to BA (*Use broken lines*).

Step 5: – Measure 8.3cm with your pair of compasses. Fix pin at B and draw an arc to cut the line CX at D.

Step 6: – Join CD with a thick line, Join AD with a thick line.

Class Activity

(1) Using a ruler and a pair of compasses only, construct the following parallelograms.

(a) //gm ABCD, such that AB = 8cm, $\angle ABC = 135^\circ$ and $|BC| = 4.5\text{cm}$. Measure AC.

(b) //gm PQSR, such that $|PQ| = 7\text{cm}$, $\angle SPQ = 120^\circ$ and $|QR| = 5\text{cm}$. Measure SQ.

(c) //gm ABCD, such that $|AB| = 7.5\text{cm}$, $\angle ABC = 105^\circ$ and AD = 4cm. Measure BD.

(d) //gm ABCD, such that $|AB| = 8\text{cm}$, $|AD| = 5\text{cm}$ and $|BD| = 6\text{cm}$.
Measure BCD

PRACTICE EXERCISE

(2) Using a ruler and a pair of compasses only, construct the following trapeziums.

(a) ABCD, such that $|AB| = 8\text{cm}$, $\angle ABC = 75^\circ$, $\angle DAB = 60^\circ$, $AD = 4.5\text{cm}$ and $AB \parallel DC$. Measure BC.

(b) PQRS, such that $|PQ| = 7.6\text{cm}$, $\angle SPQ = 90^\circ$, $PS = 4\text{cm}$, $SR = 5.7\text{cm}$ and $PQ \parallel SR$. Measure QR.

(c) ABCD, such that $AB = 7\text{cm}$, $|BC| = 5\text{cm}$, $\angle ABC = 60^\circ$, $|CD| = 4\text{cm}$ and $DC \parallel AB$. Measure AD.

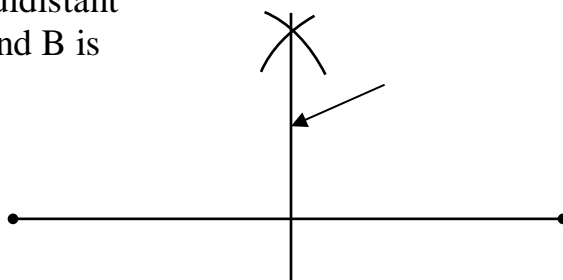
(d) ABCD, such that $|AB| = 6\text{cm}$, $|BC| = 4.3\text{cm}$, $\angle ABC = 120^\circ$, $CD = 8.5\text{cm}$ and $AB \parallel DC$. Measure DAB.

LOCUS:

DEFINITION

The Locus of a point is the set of all possible positions occupied by an object, which varies its position according to some given law. The plural of locus is loci. Below are some examples of common loci.

1. The locus l_1 of points equidistant from two given fixed point A and B is

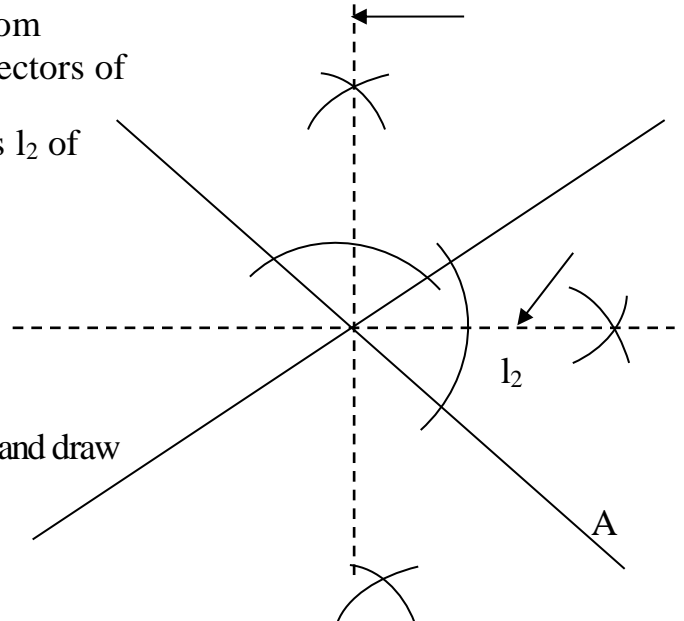


the perpendicular bisector of the line
 joining the two points A and B.

l_1

A
 B

2. The locus of point equidistant from
 two intersecting lines is the pair of bisectors of
 the angles between the lines e.g. locus l_2 of
 point equidistant from AB and CD.



Step 1: Fix pin at the point of intersection O, and draw
 an arc to cut the two lines
 C

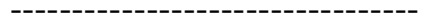
Step 2: Fix pin at points where the arc cut the two lines
 and draw arcs to intersect

Step 3: Join the point of intersection of the
 l_2

arc to the point of intersection
 of the line, O and produce through.

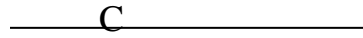
STEP 4: carry out the same steps above for D
 B

lines AB and CD is a line parallel to AB and CD



- ← locus

at equal distance from each



D

5. Locus of points at a given distance x cm

from a straight line AB

Step 1: Draw the line AB

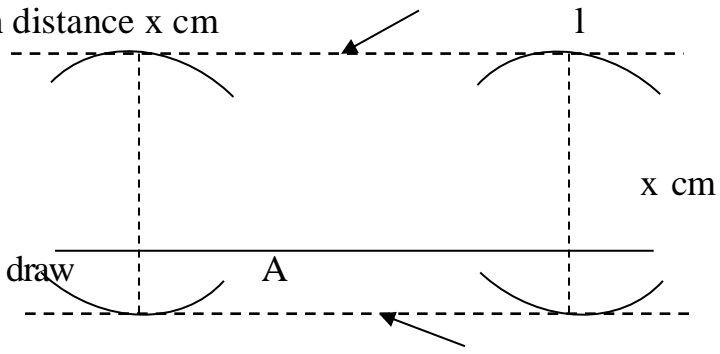
x cm

Step 2: Measure x cm, fix pin at A and draw

B

an arc above and below the line

x cm



x cm

l

Step 3: With the same x cm, fix pin at B and draw an arc above and below the line.

Step 4: Join the arcs on top with a straight line parallel to AB .

Step 5: Join the arc below AB,

with a straight line parallel to AB.

Class Activity

(1) Using a ruler and a pair of compasses only, construct

- (i) A triangle XYZ in which $|YZ| = 8\text{cm}$, $\angle XYZ = 60^\circ$ and $\angle XZY = 75^\circ$. Measure $|XY|$.
- (ii) The locus l_1 of points equidistant from Y and Z.
- (iii) The locus l_2 of points equidistant from YX and YZ.
- (iv) The locus l_3 , 4cm from X.
- (b) Measure $|QY|$ where Q is the point of intersection of l_1 and l_2 .

SSCE, June 1994, No 8 (WAEC).

- (2) Using a ruler and a pair of compasses only
- (a) Construct a triangle ABC such that $|AB| = 6\text{cm}$, $|AC| = 8.8\text{cm}$ and $\angle BAC = 120^\circ$.
- (b) Construct a locus l_1 of points equidistant from point A and B.
- (c) Construct the locus l_2 of points equidistant from AB and AC.
- (d) Find the points of intersection P_1 and P_2 of l_1 and l_2 and measure $|P_1 P_2|$.

G.C.E, Nov 1990, No 10 (WAEC).

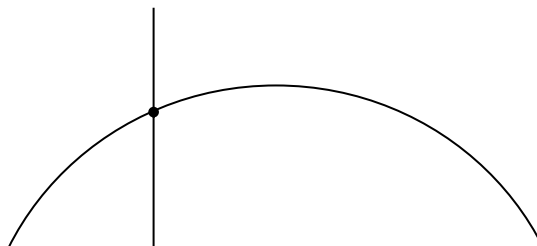
Example 7:

Using a ruler and a pair of compasses only construct

a triangle ABC such that $AB = 7.5\text{cm}$

$\angle ABC = 75^\circ$, $BC = 6.5\text{cm}$.

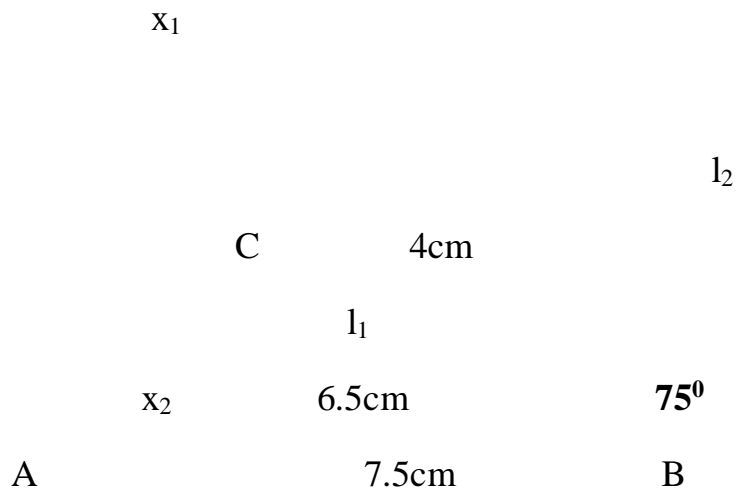
- (a) *Find the locus l_1 of points equidistant from A and B.*
- (b) *The locus l_2 of points 4cm from C*
- (c) *Locate the points of intersection x_1 and x_2 of l_1 and l_2 .*



Measure $|x_1 x_2|$

Solution:

$|x_1 x_2| = 6.9\text{cm}$



Class Activity:

(1) Using a ruler and a pair of compasses only (a) Construct

(i) a ΔABC such that $|AB| = 5\text{cm}$, $|AC| = 7.5\text{cm}$ and $\angle CAB = 120^\circ$.

(ii) The locus l_1 of points equidistant from A and B.

(iii) The locus l_2 of points equidistant from AB and AC which passes through triangle ABC.

(b) Label the point P where l_1 and l_2 intersect.

(c) Measure $|CP|$

SSCE, June 1988, No 11 and SSCE, June 1992, No 7 (WAEC).

(2) (a) Using a ruler and a pair of compasses only, construct a triangle ABC such that $|AB| = 9\text{cm}$, $|BC| = 7\text{cm}$ and $|AC| = 6\text{cm}$.

(b) Construct the locus l_1 equidistant from AB and BC.

(c) Construct the locus l_2 , 4cm from A.

(d) Locate the points of intersection P_1 and P_2 of l_1 and l_2 . Measure $|P_1 P_2|$.

(3) Using a ruler and a pair of compasses only, construct the following

(a) A trapezium ABCD such that $AB = 7\text{cm}$, $\angle DAB = 60^\circ$, $AD = 5\text{cm}$ and $DC = 4\text{cm}$. $DC \parallel AB$.

(b) Locus l_1 equidistant from A and B

(c) Locus l_2 , 4cm from B.

(d) Locate the points of intersection X_1 and X_2 of l_1 and l_2 . Measure $|X_1 X_2|$.

SUB-TOPIC

DIVIDING A LINE SEGMENT INTO N EQUAL PARTS

Example 13:

Divide the line $AB = 10\text{cm}$ in the ratio 5:2

Solution:

The line would be divided into $5 + 2 = 7$ parts.

Step 1: – Draw the line AB to be divided in the ratio 5:2

Step 2: – Draw any other line AP through A.

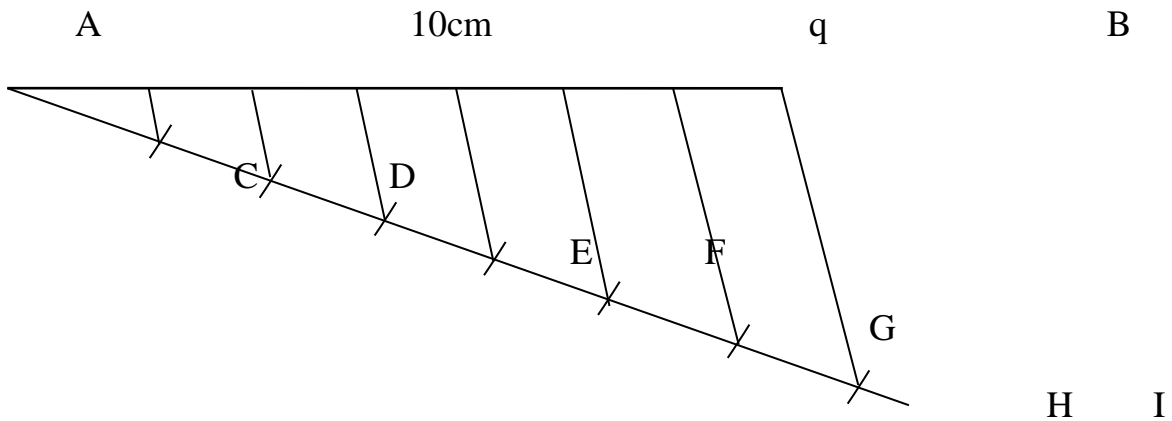
Step 3: – Set your compasses at any convenient radius, divide the line drawn in step 2 into 7

equal parts AC, CD, DE, EF, FG, GH and HI.

Step 4: – Join BI.

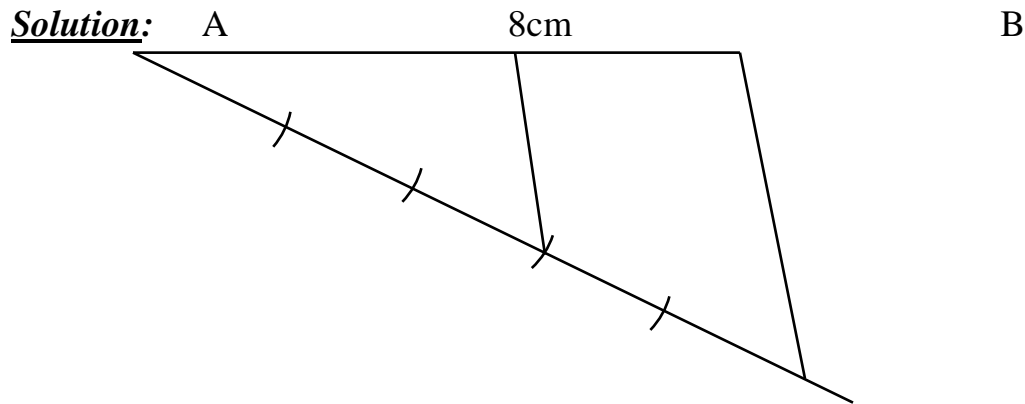
Step 5: – Construct lines parallel to BI at the points H, G, F, E, D, C using your setsquare and a ruler. (The assistance of a teacher is needed for detailed explanations).

Step 6: – The line is in the ratio 5:2 at the point q on AB.



Example 14:

Divide the line AB = 8cm in the ratio 3:2



Example 15:

(a) Using a ruler and a pair of compasses only,

construct a triangle ABC with AB = 7.5cm,

$BC = 8.1\text{cm}$ and $\angle ABC = 105^\circ$.

(b) Locate the point D on BC such that

$$|BD| : |DC| \text{ is } 3:2$$

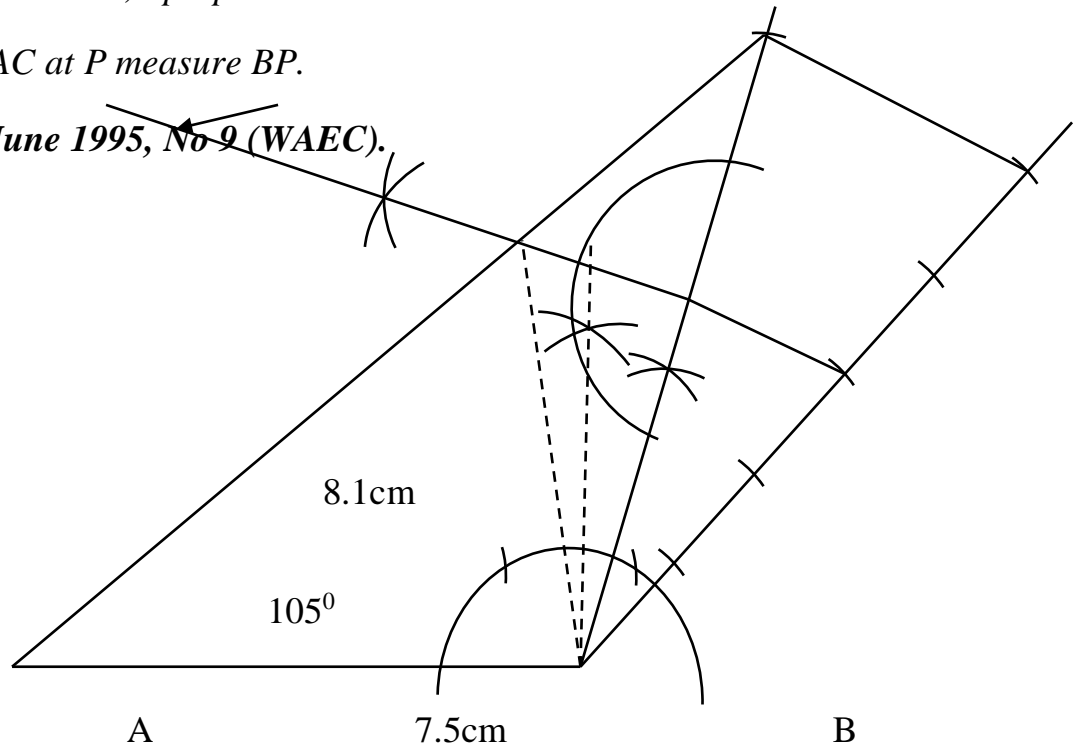
(c) Through D construct a line, l perpendicular to BC .

(d) If the line l meets AC at P measure BP .

SSCE, June 1995, No 9 (WAEC).

Solution:

C



$|BP| = 5.3\text{cm}$

PRACTICE EXERCISE:

(1) (a) Using a ruler and a pair of compasses only, construct

(i) A triangle QRT with $|QR| = 8\text{cm}$, $|RT| = 6\text{cm}$ and $|QT| = 3\text{cm}$.

(ii) A trapezium $PQRS$, which has a common side QR with ΔQRT , given that PQ is parallel to SR , $|PQ| = 7\text{cm}$, $|QR| = 8\text{cm}$, $|RS| = 4\text{cm}$ and PTQ is a straight line.

(iii) The locus l_1 of points equidistant from PQ and PS .

- (iv) The locus l_2 of points equidistant from T and R.
- (b) Measure $|TX|$, where X is the point of intersection of l_1 and l_2 .

G.C.E, Nov 1985, No 8 (WAEC).

- (2) (a) Using a ruler and compasses only, construct
- (i) A triangle PQR such that $|PQ| = 6\text{cm}$, $|QR| = 7\text{cm}$ and $\angle PQR = 135^\circ$.
- (ii) The locus l_1 of points equidistant from P and Q.
- (iii) The locus l_2 of points equidistant from PQ and QR.
- (iv) The locus l_3 of point at which QR subtends an angle of 90° .
- (b) Locate;
- (i) The point of intersection X of l_1 and l_2 .
- (ii) The point of intersection Y of l_2 and l_3 .
- (c) Measure $|XY|$.

G.C.E, Nov 1985, No 21 (WAEC).

- (3) Using a ruler and a pair of compasses only construct a triangle ABC in which $|AB| = 8\text{cm}$, $|AC| = 5\text{cm}$ and $\angle BAC = 45^\circ$. Measure $|BC|$. Construct a circle with center P on BC such that AB and AC produce are tangent of circle. Measure the radius of the circle.

KEYWORD: BISECT, LOCUS, PARALLEL, PERPENDICULAR, EQUIDISTANT ETC

SSCE, Nov 1992, No 11 (WAEC).

WEEK 7

MID-TERM BREAK

WEEK 8

Date.....

SUBJECT: MATHEMATICS

CLASS: SS 1

TOPIC: PROOFS OF SOME BASIC THEOREMS

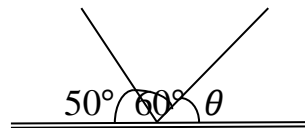
CONTENT:

- Proof: The sum of the angles in a triangle
- The exterior angle of a triangle is equal to the sum of the opposite interior angles
- Congruency and similarity of triangles.

INTRODUCTION

Geometry is the study of the properties of shapes. In theoretical or formal geometry the facts are proved for general cases by a method of argument or reasoning rather than by measurement. Geometrical basic facts are called theorems. Theorems are the foundations upon which geometry is built. Interior and exterior angles of triangle.

Recall: Angles on a straight line is 180° . Thus,

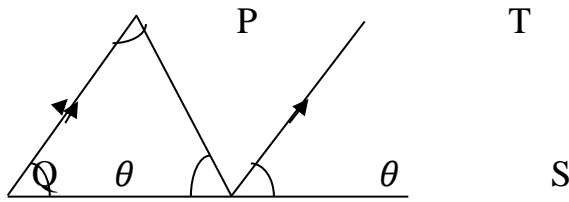


$$\theta + 50 + 60 = 180$$

$$\theta = 180 - 110$$

$$= 70^\circ$$

Using the diagram below;



- i. Use the angle properties related to parallel lines to explain why;
 - (a) Angle TRS = angle PQR corresponding or 'F' angles
 - (b) Angle TRP = angle QPR. Alternate or 'z' angles
- ii. Explain why the sum of the three angles at R is 180° . Angles on a straight line

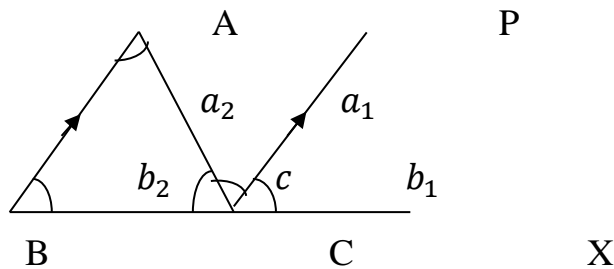
Theorem : The sum of the angles of a triangle is 180°

Given: any $\triangle ABC$

To prove: $\hat{A} + \hat{B} + \hat{C} = 180^\circ$

Construction: produce \overline{BC} to a point X.

Draw \overline{CP} parallel to \overline{BA}



Proof:

With the lettering of the diagram

$$a_1 = a_2 \text{ (alternate angles)}$$

$$b_1 = b_2 \text{ (corresponding angles)}$$

$$C + a_1 + b_1 = 180^\circ \text{ (angles on straight line)}$$

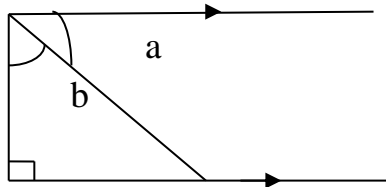
$$C + a_2 + b_2 = 180^\circ$$

$$\hat{A} + \hat{B} + \hat{C} = 180^\circ$$

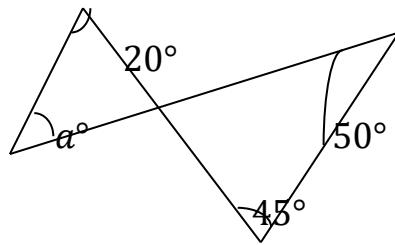
$$\therefore \hat{A} + \hat{B} + \hat{C} = 180^\circ$$

Class Activity:

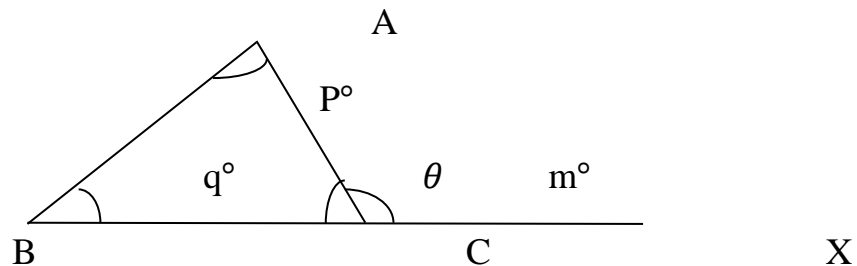
- (i) Prove that $a = 90^\circ - b$ in the figure below



- (ii) Find the value of angle 'a' below and state clearly any theorem applied



Theorem : The exterior angle of a triangle is equal to the sum of the opposite interior angles.



Given: any $\triangle ABC$ with \overline{BC} produce to X

Proof: With the lettering of the diagram,

$$\hat{A}CX + \hat{A}CB = 180^\circ \text{ (angles on a straight line)}$$

$$\therefore \hat{A}CX = \hat{A} + \hat{B} (= 180^\circ - \hat{A}CB) \quad \text{or}$$

The diagram shows that $m = p + q$ as stated above,

$$\text{Since } p + q + \theta = 180^\circ \quad \text{(sum of angles in a } \Delta \text{)}$$

$$m + \theta = 180^\circ \quad \text{(angles on a straight line)}$$

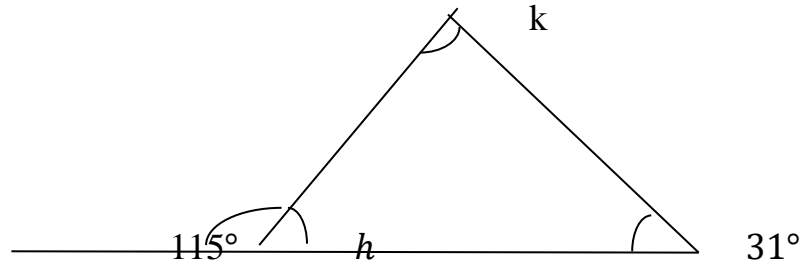
$$m + \theta = p + q + \theta$$

$$\text{i.e. } m = p + q$$

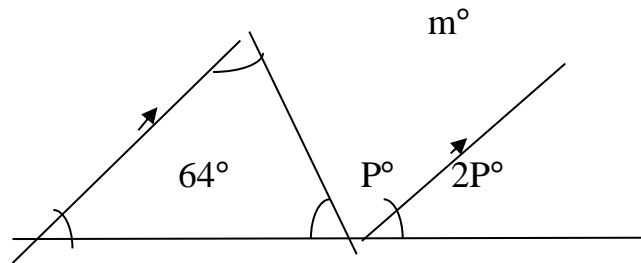
Class Activity:

Find the sizes of the lettered angles in the diagrams below. State clearly the reason for each statement

(a) i.

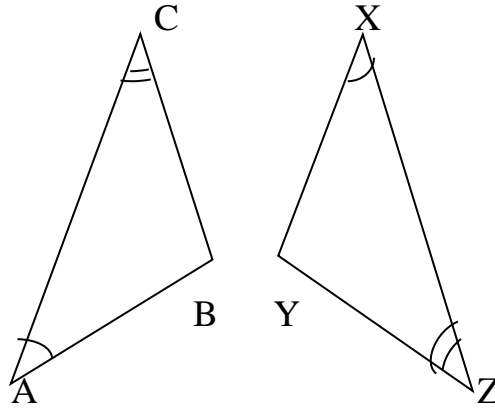


II.



Congruency and similarity of triangles.

Similar figures have the same shape but not necessarily the same size. The condition for two triangles to be similar is that the three angles of one triangle are respectively equal to the three angles of the other. While if two shapes are congruent, it means they are equal in every way – all their corresponding sides and angles are equal.

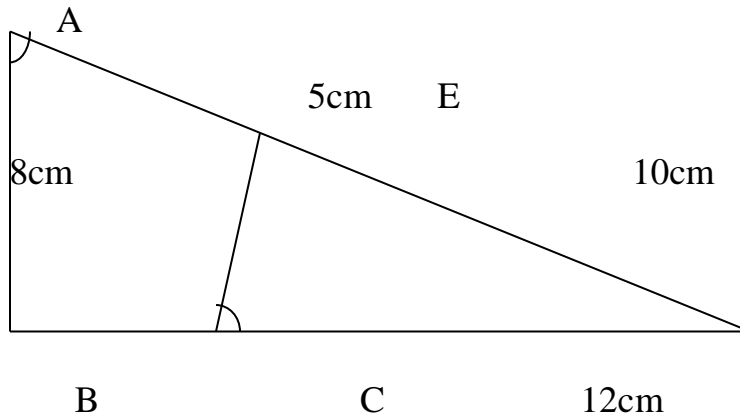


In the diagrams above $\angle A = \angle X$ and $\angle C = \angle Z$. This follows that $\angle B = \angle Y$ and triangle ABC is similar to triangle XYZ. Since the two triangles are similar, their corresponding sides must be in same ratio. That is

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$$

Examples

1. In the diagram below . Calculate /BC/ and /EC?



D

In the triangles ABC and CED

$\angle BAD = \angle ECD$ (given)

$\angle D$ is common

$\angle ABD = \angle CED$ (3^{RD} \angle of triangle)

Triangles ABD and CED are similar

$$\frac{AB}{CE} = \frac{AD}{CD} = \frac{BC}{ED}$$

Therefore

$$\frac{8}{CE} = \frac{15}{CD} = \frac{BD}{ED}$$

If $\frac{8}{CE} = \frac{15}{12}$, then

$$EC = \frac{8 \times 10}{5} = \frac{32}{5}$$

$$/EC/ = 6\frac{2}{5}cm$$

If $\frac{15}{12} = \frac{BD}{10}$

Then $/BD/ = \frac{15 \times 10}{12} = \frac{25}{2} = 12\frac{1}{2}cm$

$$/BD/ = 12.5cm$$

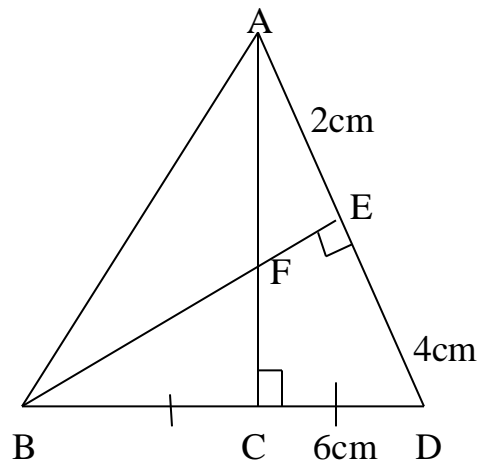
$$/BC/ = /BD/ - /CD/$$

$$= 12.5cm - 12cm$$

$$= 0.5cm$$

It also follows that when triangles are similar; their areas are proportional to the squares of corresponding sides.

2. In the diagram below, find the ratio of the area of triangle AFE and BFC



Given: the diagram as shown above

To prove: the ratio of triangle AFE: triangle BFC = 4:9

Proof: in the triangle $\triangle AFE$ and $\triangle BFC$

$$\angle AEF = \angle BCF \text{ (each a rt. } \angle)$$

$$\angle AFE = \angle BFC \text{ (vert. opp } \angle\text{s)}$$

$$\angle FAE = \angle FBC \text{ (3rd } \angle\text{s in the triangle)}$$

Triangles AFE and BFC are similar, hence their areas are proportional to the squares of corresponding sides. i.e triangle AFE: triangle BFC = $\frac{AE^2}{BC^2}$

$$\frac{BC}{CD}$$

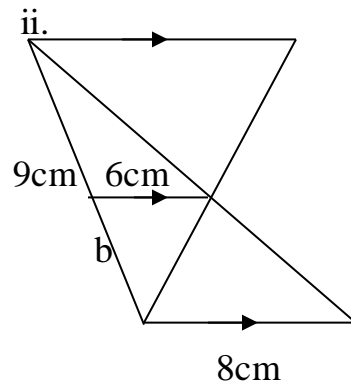
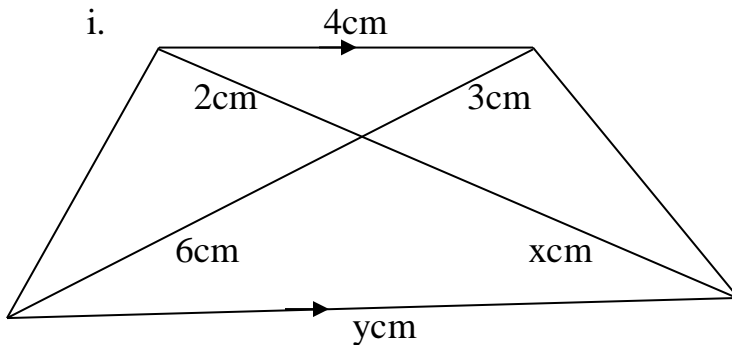
$$\frac{BC}{BC} = 3\text{cm}$$

$$\frac{AE^2}{BC^2} = 2^2 : 3^2 = 4 : 9$$

Therefore triangle AFE: triangle BFC = 4:9 Q.E.D

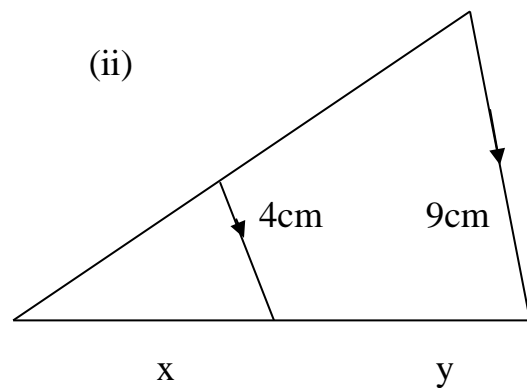
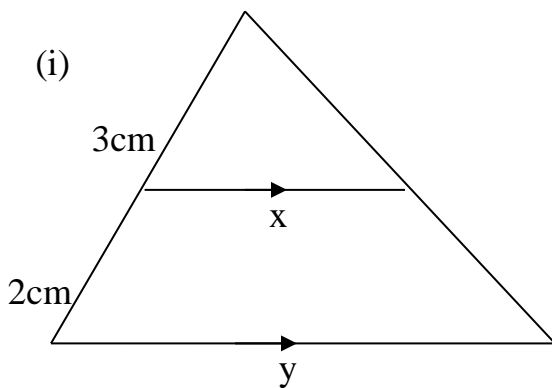
Class Activity:

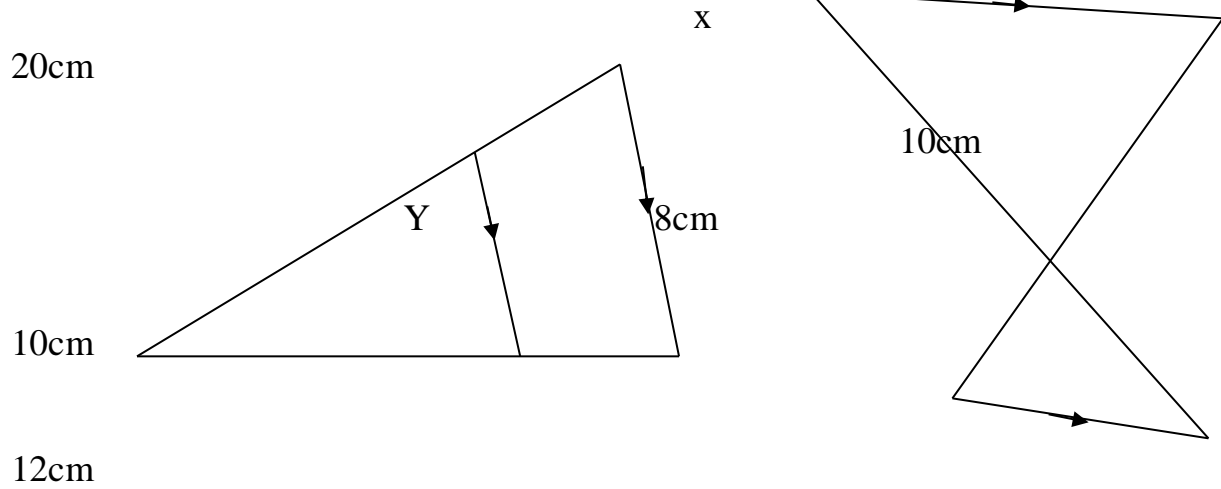
1. Find the unknown marked length of following figures:



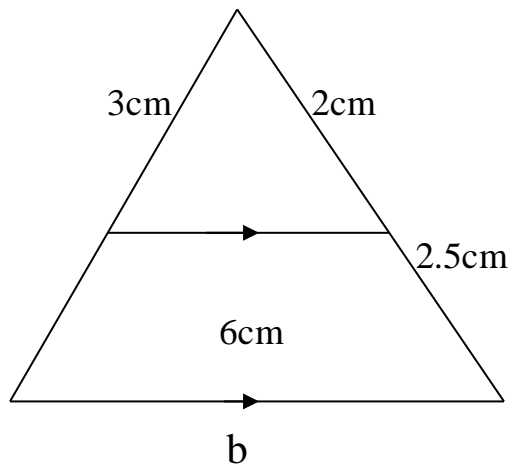
PRACTICE EXERCISE

1. In the diagrams below, find the ratio x:y





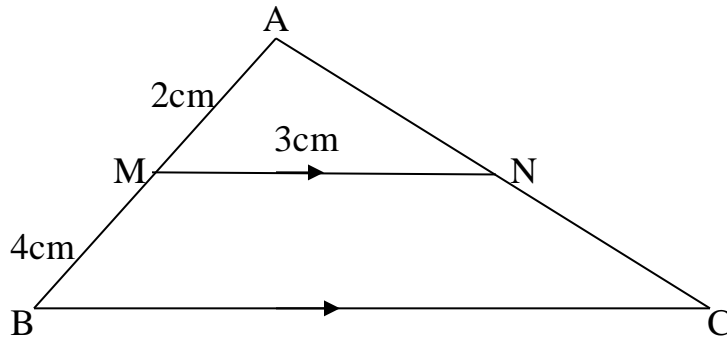
2. Find the unknown marked length of the diagram



ASSIGNMENT

1. A vertical pole, 4m high, has shadow of 2.5m. At the same time, the shadow of tower is 21m. Find the height of the tower.
2. ABC is triangle in which $/BC/ = 18\text{cm}$. $/DE/$ is a line parallel to $/BC/$ cutting $/AB/$ at D $/AC/$ at E. $/AD/ = 12\text{cm}$ and $/DB/ = 18\text{cm}$, calculate $/DE/$ and the ratio of $/AE/$ and $/AC/$

3. A conical lampshade has radii 27cm, 42cm at its two ends which are 30cm apart. If the cone were completed, find what its height would be.
4. A pole of height 2m casts a shadow of 1.5m long. Find the height of a tree whose shadow is 25m long.
5. In the triangle ABC, MN is parallel to BC . $AM=2\text{cm}$, $MB=4\text{cm}$ and $MN=3\text{cm}$. Calculate BC . (WAEC)



KEYWORDS: GEOMETRY, THEORY, PROVE, CONGRUENT, SIMILAR ETC

WEEK 9

Date.....

SUBJECT: MATHEMATICS

CLASS: SS 1

TOPIC: PROOFS OF SOME BASIC THEOREMS

CONTENT:

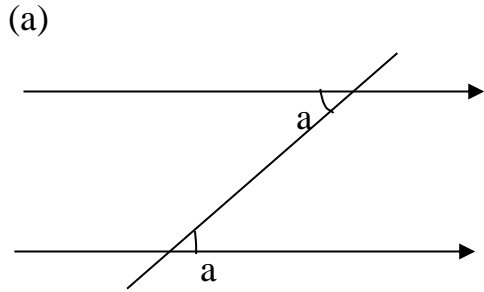
- (i) Riders including angles of parallel lines
- (ii) Angles in a polygon
- (iii) Congruent triangles
- (iv) Properties of Parallelogram
- (v) Intercept theorem

ANGLES OF PARALLEL LINE

Recall Basic geometrical facts are called theorems. The first is the sum of the angles of a triangle is 180^0 . Many other theorems depend on it. For this reason they are often called Riders. (Since they ride on theorem 1)

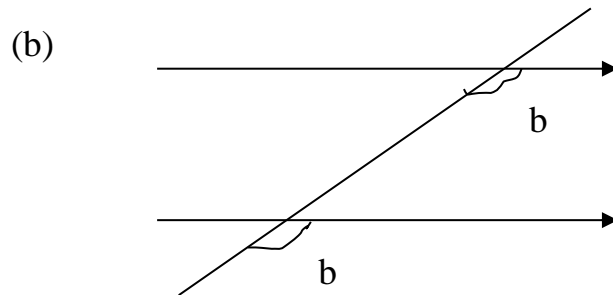
If two parallel lines are intersected by a Transversal.

- (i) The alternate angles are equal
- (ii) The corresponding angles are equal
- (iii) The interior angles on the same side of the transversal are supplementary
viz:

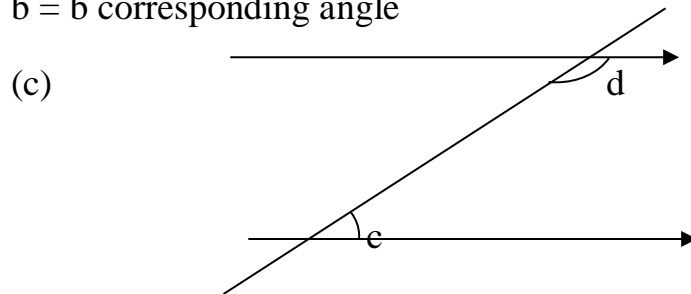


Transversal

$a = a$ alternate angle



$b = b$ corresponding angle



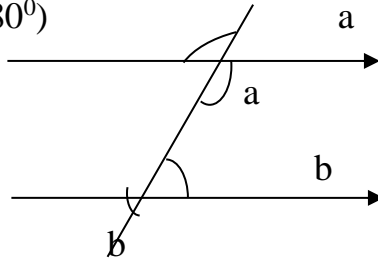
co-interior / allied

$c + d = 180^\circ$. Supplementary angle

Other angles formed are: vertically opposite angles, they are equal.

Angle at a point (360°) and on

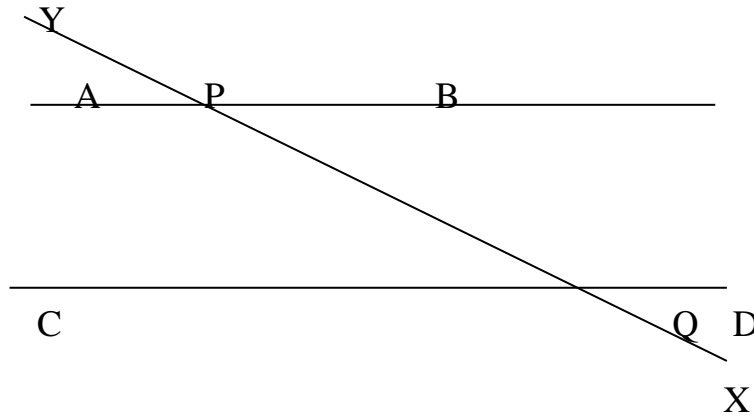
Straight line (180°)



$a = a$ vertically opposite angle

Examples:

- (1) In the diagram below $\angle YPA = (3X + 10)$, $\angle APQ = (4x - 20)$ and $\angle PQD = x + 70^\circ$.
Prove that $\overline{AB} \parallel \overline{CD}$



Given: as above

Proof: $\overline{AB} \parallel \overline{CD}$

$$3x - 10 + 4x - 20 = 180^\circ \text{ (angle on straight line)}$$

$$3x + 4x - 10 - 20 = 180^\circ.$$

$$7x - 30^\circ = 180^\circ.$$

$$7x = 180^\circ + 30^\circ$$

$$7x = 210^\circ$$

$$X = 30^\circ.$$

$$\text{If } 4x - 20^\circ = x + 70^\circ \text{ (alternate angle)}$$

Substituting for x

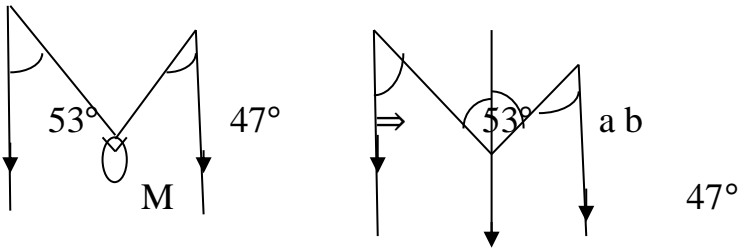
$$4(30) - 20 = 30 + 70$$

$$120 - 20 = 30 + 70$$

$$100 = 100$$

$\therefore \overline{AB} \parallel \overline{CD}$ where \overline{YX} is transversal.

2. Calculate the size of the marked angle in the diagram below.



$$a = 53^\circ \text{ (alternate angle)}$$

$$b = 47^\circ \text{ (alternate angle)}$$

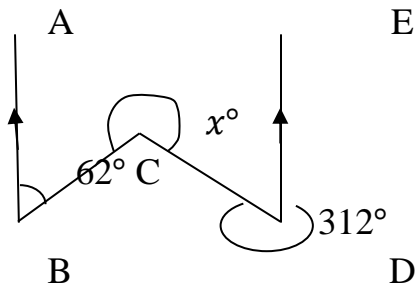
$$M = 360^\circ - (a + b) \text{ (angle at a point)}$$

$$M = 360^\circ - 100^\circ$$

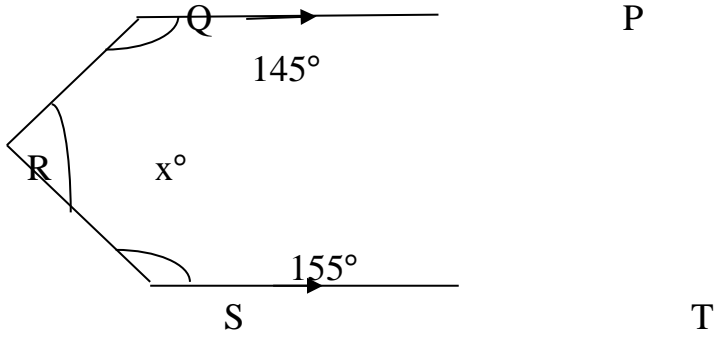
$$M = 260^\circ.$$

Class Activity

1. In the diagram below, \overline{BA} is parallel to \overline{DE} . Find the value of x



2. In the diagram below: \overline{QP} and \overline{ST} are parallel. If $\angle PQR = 145^\circ$ and $\angle RST = 155^\circ$. Find the value of x



3. In the diagram below: $\overline{ST} \parallel \overline{QR}$, $\widehat{PST} = 52^\circ$, $\widehat{QTS} = 26^\circ$. and $\widehat{PQ} \cong \widehat{QT}$.
Calculate PRQ.

Congruent Triangles

When there exists equality relationship in terms of corresponding sides and angles of triangles.

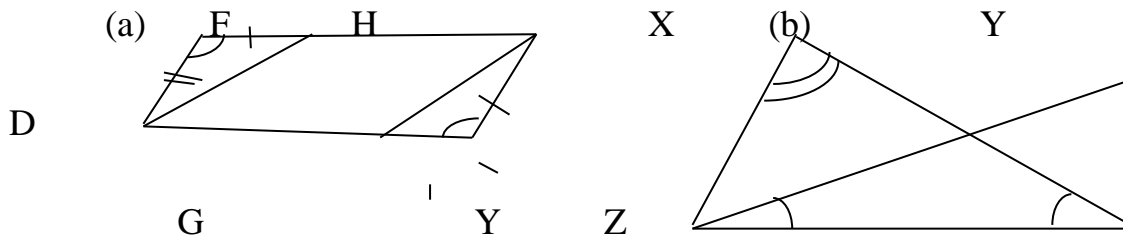
We say that the triangles are congruent

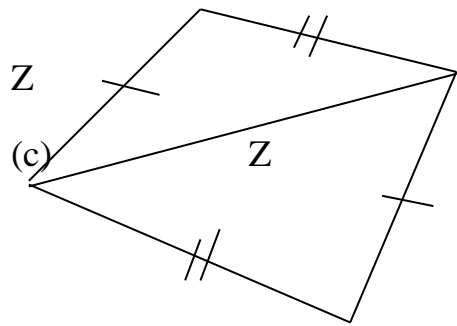
Two triangles are congruent if proved that

- I. Two sides and the included angle of one are equal to the corresponding two sides and the included angle of the other (SAS)
- II. Two angles and a side of one are equal to the corresponding two angles and a side of the other. ASA or AAS
- III. Three sides of one are respectively equal to three sides of the other (SSS)
- IV. In right – angled triangles, the hypotenuse and another side are equal to the hypotenuse and a side of the other (RHS)

EXAMPLES

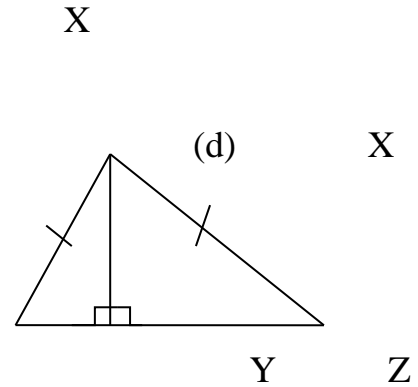
1. Name the triangle which is congruent to $\triangle XYZ$, giving the letters in the correct order. State the condition of Congruency.





Y
A

X



C

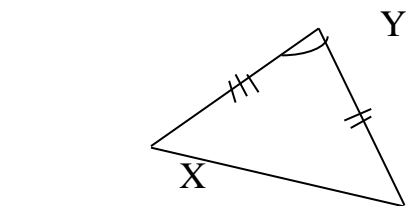
The congruency is as follows:

- (a) $XYZ \equiv GHF$ (SAS)
- (b) $XYZ \equiv ZDX$ (AAS) (ASA)
- (c) $XYZ \equiv YXC$ (SSS)
- (d) $XYZ \equiv XAZ$ (RHS)

2. In each of the following, the statements refers to Δs XYZ and MNO. In each case, sketch the triangles and mark in what is given. If the triangles are congruent, state three other pairs of equal elements and give the condition of congruency.

$$|XY|=|MN|, |YZ|=|NO|, \hat{Y} = \hat{N}$$

Solution:

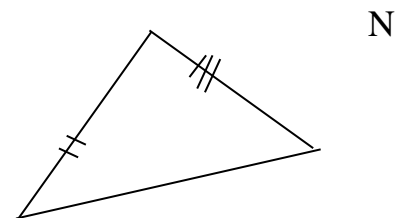


M

Z

$$\hat{X} = \hat{M}$$

(SAS

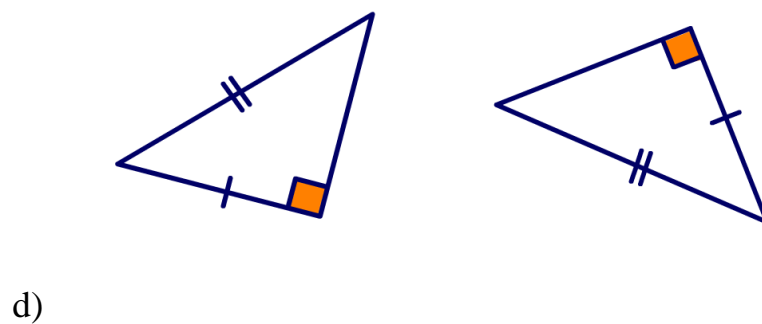
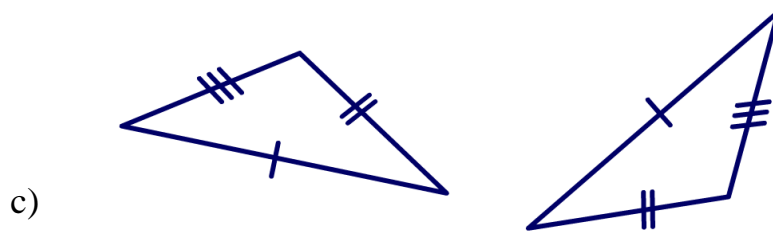
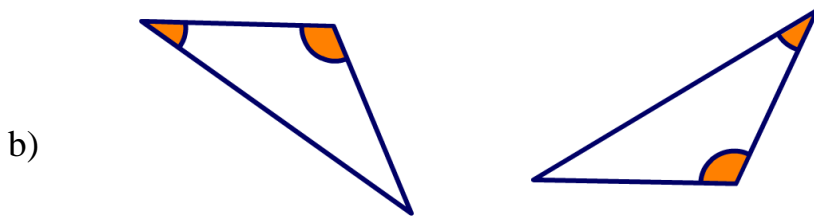
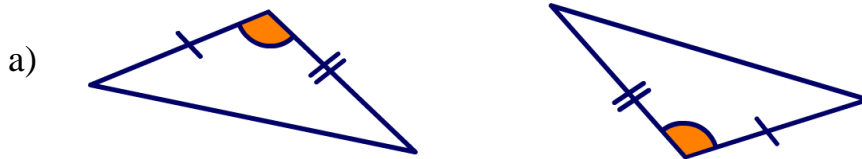


O

$$\hat{Z} = \hat{O}$$

Class Activity

1. Decide whether the following pairs of triangles are congruent or not. If they are, what condition do they satisfy?



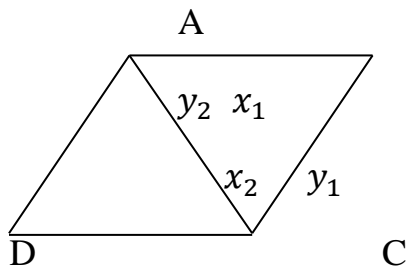
PROPERTIES OF PARALLELOGRAM

Definition: A parallelogram is a quadrilateral which has both pairs of opposite sides parallel.

The properties of parallelogram are:

- i. The opposite sides are parallel
- ii. The opposite angles are equal
- iii. The opposite sides are equal
- iv. The diagonal bisects one another
- v. The diagonals make pairs of alternate angles which are equal
- vi. The allied or co-interior forming adjacent vertices of a parallelogram are supplementary

Theorem : In a parallelogram, (a) the opposite sides are equal (b) the opposite angles are equal



Given: Parallelogram ABCD

To prove: (i) $|AB|=|CD|$, $|BC|=|AD|$

$$(ii) \hat{B} = \hat{D}, \hat{A} = \hat{C}$$

Construction: Draw the diagonal AC

Proof: In Δ s ABC and CDA, $x_1 = x_2$ (*alt. angles, AB||DC*)

$$y_1 = y_2 \text{ (*alt. angles, AD||BC*)}$$

AC is common

$$\therefore \Delta ABC \equiv \Delta CDA \text{ (ASA)}$$

\therefore (i) $|AB| = |CD|$, $|BC| = |DA|$ *corresponding sides*

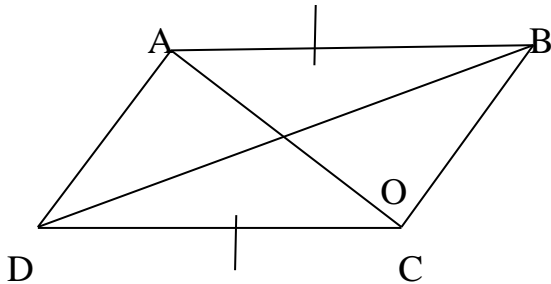
(ii) $\hat{B} = \hat{D}$ corresponding angles

$$\hat{A} = x_1 + y_2$$

$$= x_2 + y_1$$

$$= \hat{C}$$

Theorem : The diagonals of a parallelogram bisect one another



Given: Parallelogram ABCD with diagonals AC and BD intersecting at O.

To prove: $|AO|=|OC|$, $|BO|=|OD|$

Proof: In $\Delta s AOB$ and COD ,

$$x_1 = x_2 \quad (\text{alt. Angles, } AB \parallel CD)$$

$$y_1 = y_2 \quad (\text{alt. Angles, } AB \parallel CD)$$

$$|AB|=|CD| \quad (\text{opp sides of llgm})$$

$$\therefore \Delta AOB \equiv \Delta COD \quad (\text{ASA})$$

$$\therefore |AO| = |CO| \quad (\text{corresponding sides})$$

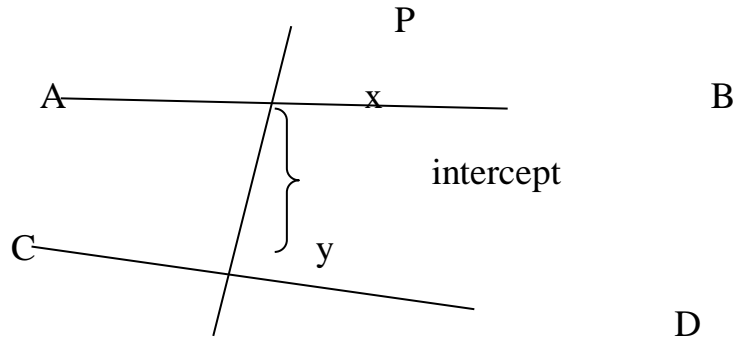
$$\text{and } |BO|=|DO| \quad (\text{corresponding sides})$$

Class Activity

Page 39 NGM book 1 EX. 2e, nos 1, 2, 3, 6

Intercept Theorem

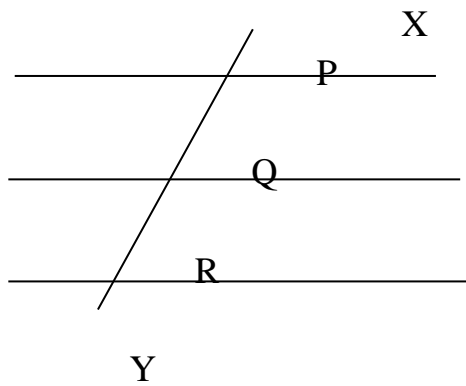
Intercept: This is to stop something that is going from one place to another. Mathematically from the diagram below; a transversal line is stopped by two lines there by forming an intercept between the lines.



If a transversal cut two or more parallel lines, the parallel lines cut the transversal into line segments known as *intercept*

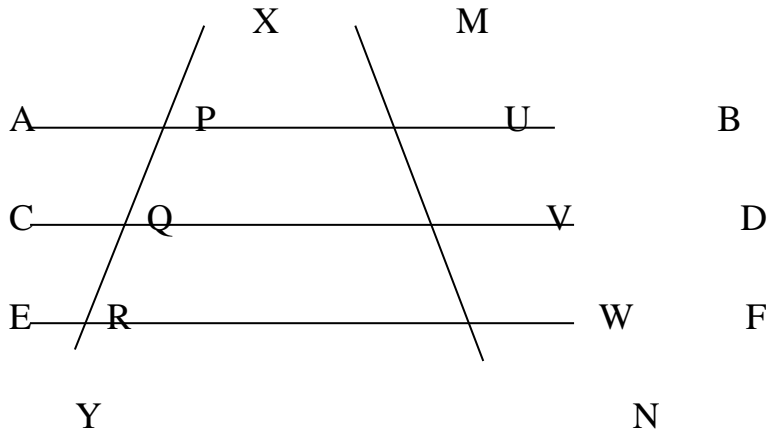
Examples:

i.



\overline{PQ} and \overline{QR} are intercept of \overline{XY}

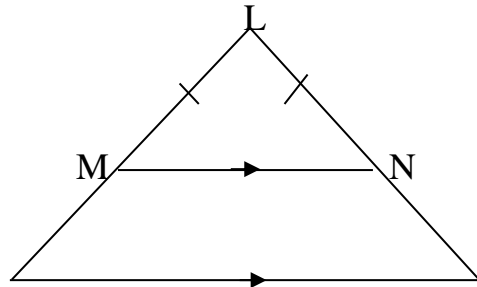
ii.



\overline{PQ} and \overline{QR} are the intercept of \overline{XY} while \overline{UV} and \overline{VW} are the intercept of \overline{MN}

Class Activity:

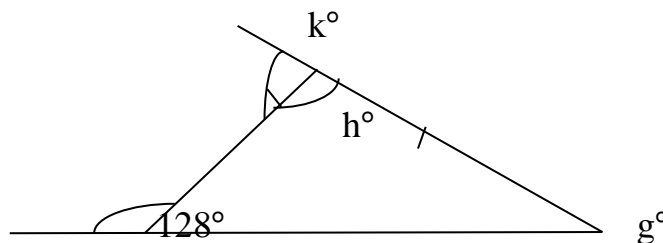
1. In the diagram below, $|LM|=|LN|$, $\overline{MN} \parallel \overline{OP}$ and $\angle OPN = 45^\circ$. Find $\angle PLM$



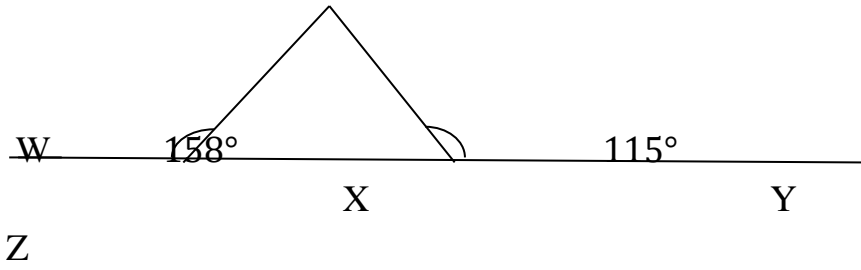
Class Activity: Try to memorize these theorems as riders

PRACTICE EXERCISE

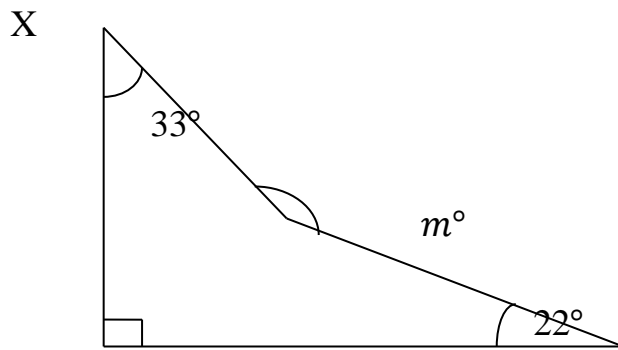
1. Find the sizes of the lettered angles in the diagrams below. State clearly the reason for each statement



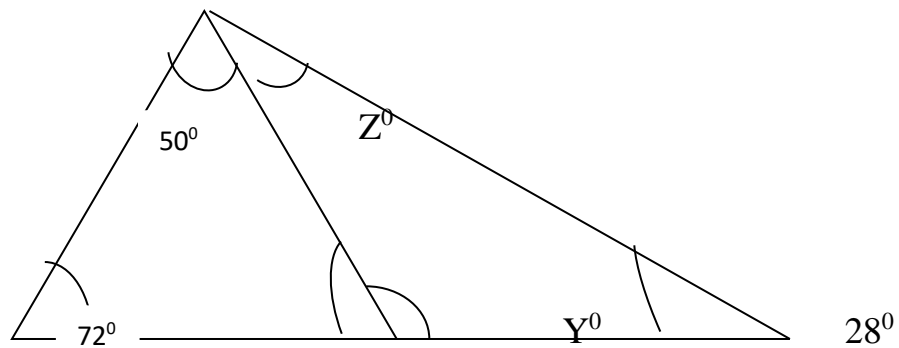
2. The angles of a triangle are $\frac{5a}{2}$, $\frac{3a}{4}$ and $\frac{7a}{4}$. Find the value of the largest angle
3. In the figure below, $\angle WXP = 115^\circ$ and $\angle ZYP = 158^\circ$, calculate $\angle XPY$



4. What is the value of the angle marked m in the figure below?



5.



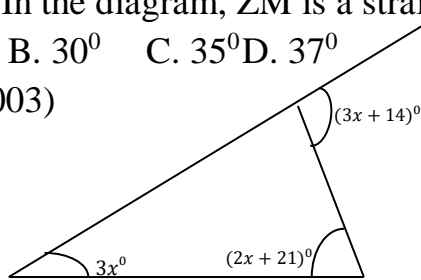
Find the size of each angle marked with letters in the above diagram

ASSIGNMENT

1. In the diagram, ZM is a straight line. Calculate the value of x .

A. 27° B. 30° C. 35° D. 37°

(SSCE 2003)

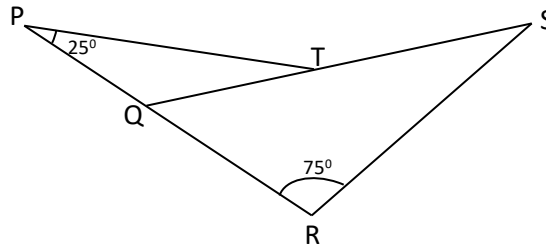


2. In the diagram above, PQT is an isosceles triangle. $|PQ| = |QT|$, $\angle SRQ = 75^\circ$, $\angle QPT = 25^\circ$ and PQR is a straight line. Find $\angle RST$.

A. 20° B. 50° C. 55° D. 70° E. 75°

1992)

(SSCE

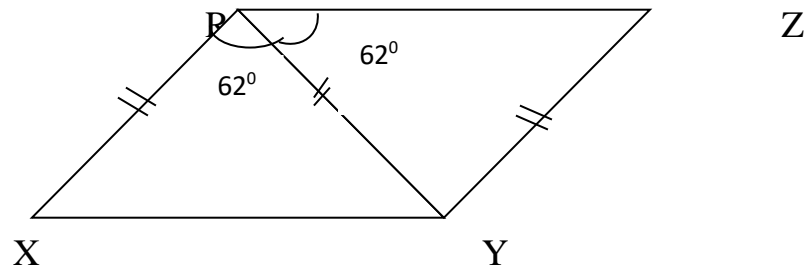


3. If the exterior angles of a quadrilateral are y° , $(2y + 5)^\circ$, and $(3y - 10)^\circ$, find y° . (SSCE 1994)

4. Triangles PQR and XYZ are similar. If the sides of triangles are 6cm, 7cm and 8cm and the shortest side of the triangle XYZ is 2cm. Find the length of the longest side of triangle XYZ. A. 4cm B. 3.5cm C. $2\frac{2}{3}$ cm

D. 2.5cm

5. In the diagram below $|XR| = |RY| = |YZ|$ and $\angle XRY = \angle YRZ = 62^\circ$. Calculate $\angle XYZ$



Having known what and how to prove a given theorem, there is need to identify the **'riders'**.

- i. The sum of the angles of a triangle is 180°
- ii. The exterior angle of a triangle is equal to the sum of the opposite interior angles
- iii. The sum of the interior angles of any n-sided convex polygon is $(2n-4)$ right angles
- iv. The sum of the exterior angles of any convex polygon is 4 right angles
- v. The base angles of an isosceles triangle are equal
- vi. In a parallelogram, (a) the opposite sides are equal (b) the opposite angles are equal
- vii. The diagonals of a parallelogram bisect one another
- viii. If three or more parallel lines cut off equal intercepts on a transversal, then they cut off equal intercepts on any other transversal

KEYWORDS: GEOMETRY, THEORY, PROVE, CONGUENT, SIMILARITY ETC