SS1 FIRST TERM: E-LEARNING NOTES

SCHEME FIRST TERM

WEEK	TOPIC	CONTENT
1	**INDICES**	(a) Revision of standard form. (b) Relationship between indices and standard form. (c) Laws of indices; (i) $a^x \times a^y = a^{x+y}$ (ii) $a^x \div a^y = a^{x-y}$ (iii) $(a^x)^y = a^{xy}$, etc. (d) Application of indices, simple indicial equation.
2	**LOGARITHMS 1**	(a) Deducing logarithm from indices and standard form. (b) Definition of Logarithms. (c) Graph of $y = 10^x$. (d) Reading of logarithm and the antilogarithm tables.
3	**LOGARITHMS 2**	(a) Use of logarithm table and antilogarithm table in calculation involving (multiplication, division, powers and roots. (b) Application of logarithm in capital market and other real life problems.
4	**SETS 1**	(a) Definition of set. (b) Set Notation – (i) listing or roster method (ii) rule method (iii) set builder notation. (c) Types of sets.
5	**SETS 2**	(a) Set operations (i) Union (ii) Intersection (iii) Complement. (b) Venn diagram. (f) Application of Venn diagram up to 3 set problem.
6	**NUMBER BASE SYSTEM**	(a) Conversion from other bases to base 10 and vice versa. (b) Conversion of decimal fraction in other bases to base 10 and vice versa.
7	**MID-TERM BREAK**	
8	**NUMBER BASE SYSTEM**	(a) Conversion of number from one base to another base. (b) Addition, subtraction, multiplication and division of number bases. (c) Application to computer programming.
9	**SIMPLE EQUATIONS AND VARIATIONS**	(a) Formulae, substitution and simple binary operations. (b) Change of subject of formulae. (c) Variations (i) Direct and inverse, (ii) joint and partial. (d) Application of variation.
10	**REVISION**	
11	**EXAMINATION**	

WEEK 1:
DATE……………………….
Subject: Mathematics
Class: SS 1
TOPIC: Indices
Content:

- Revision of standard form
- Relationship between indices and standard form.
- Laws of indices:(i) $a^x \times a^y = a^{x+y}$ (ii) $a^x \div a^y = a^{x-y}$ (iii) $(a^x)^y = a^{xy}$, etc.
- Application of indices, simple indicial equation.

Revision of standard form

Example 1: Express the following in standard form

(a) 5.37 (b) 53.7 (c) 537 (d) 35.65 (e) 7500 (f) 1403420

Solution:

(a) $5.37 = 5.37 \times 1$
$= 5.37 \times 10^0$

(b) $53.7 = 5.37 \times 10^1$

(c) $537 = 5.37 \times 100$
$= 5.37 \times 10 \times 10$
$= 5.37 \times 10^2$

(d) $35.65 = 3.565 \times 10$
$= 3.565 \times 10^1$

(e) $7500 = 7.5 \times 1000$
$= 7.5 \times 10^3$

(f) $1403420 = 1.403420 \times 1000000$
$= 1.403420 \times 10^6$

Example 2; Express the following in standard form

(a) 0.037 (b) 0.00065 (c) 0.0058 (d) 0.61

Solution:

Method 1:

(a) $0.037 = 3.7 \times 0.01$
$= 3.7 \times 10^{-2}$

(b) $0.00065 = 6.5 \times 0.0001$
$= 6.5 \times 10^{-4}$

(c) $0.0058 = 5.8 \times 0.001$
$= 5.8 \times 10^{-3}$

(d) $0.61 = 6.1 \times 0.1$
$= 6.1 \times 10^{-1}$

Method 2:

(a) $0.037 = \frac{0.037}{1}$

$= \frac{3.7}{100}$

$= \frac{3.7}{10^2}$

$= 3.7 \times 10^{-2}$

(b) $0.00065 = \frac{0.00065}{1}$

$= \frac{6.5}{10000}$

$= \frac{6.5}{10^4}$

$= 6.5 \times 10^{-4}$

(c) $0.0058 = \frac{0.0058}{1}$

$= \frac{5.8}{1000}$

$= \frac{5.8}{10^3}$

$= 5.8 \times 10^{-3}$

(d) $0.61 = \frac{0.61}{1}$

$= \frac{6.1}{10}$

$= 6.1 \times 10^{-1}$

Class Activity:

Express the following in standard form

1. (a) 2037000 (b) 0.00469 (c) 0.000513 (d) 146200

Write each of the following numbers in full

2. (a) 6×10^4 (b) 5.142×10^7 (c) 0.7883×10^{-5} (d) 2.095×10^9

Laws Of Indices and Application of indices

Thefollowing are the laws governing the mathematical operations involving index numbers. These laws are true for all values of m, n and $x \neq 0$.

(1) $a^x \times a^y = a^{x+y}$

(2) $a^x \div a^y = a^{x-y}$

(3) $a^{-x} = \frac{1}{a^x}$

(4) $a^0 = 1$

(5) $(a^x)^y = a^{xy}$

(6) $a^{\frac{1}{n}} = \sqrt[n]{a}$

(7) $a^{\frac{x}{y}} = \sqrt[y]{a^x}$

Example 1: Simplify $\left(\frac{1}{81}\right)^{-\frac{1}{4}}$

$$\left(\frac{81}{1}\right)^{\frac{1}{4}}$$

$$(3^4)^{\frac{1}{4}}$$

$$3^{\frac{4}{4}}$$

$$= 3$$

Example 2: Simplify $125^{\frac{2}{3}}$

$$\left(\sqrt[3]{125}\right)^2$$

$$\left(\sqrt[3]{5^3}\right)^2$$

$$5^2$$

$$= 25$$

Class Activity:

Simplify the following;

1. $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$
2. *$3a^2b$ x $(2ab)^{-3}$*

Simple indicial equation

Indicial equations are equations which have the variable or unknown quantity as an index or exponent.

Examples: Solve for x in the following

1. $4^{-3x} = \frac{1}{64}$

$$4^{-3x} = \frac{1}{4^3}$$

$$4^{-3x} = 4^{-3}$$

$-3x = -3$

$\therefore x = 1$

2. $(x + 7)^3 = 27$

$(x + 7)^3 = 3^3$

$x + 7 = 3$

$x = 3 - 7$

$x = -4$

PRACTICE EXERCISE:

1. Write 3.654×10^{-10} in full
2. Simplify $\frac{32 \times 10^4}{8 \times 10^7}$
3. Simplify $\sqrt{\frac{196\, m^3 n^7}{4m^5 n}}$
4. Simplify $\frac{2^{-n} \times 8^{2n+1} \times 16^{2n}}{4^{3n}}$
5. Solve the equation, $9^x = \frac{1}{3}(27^x)$
6. Solve the equation, $3^{x-4} = 234^2$

ASSIGNMENT:

1. Given that: $2 \times 4^{1-x} = 8^{-x}$, find x
2. Solve for the value of n in the equation, $9^n \times \frac{1}{3^{n-1}} = 27^{n+3}$
3. Without using calculator, work out $\frac{0.0125 \times 0.00576}{0.0015 \times 0.32}$ leaving your answer in standard form
4. Solve for x if $4^x \times 8^{2x-1} = \frac{1}{16^{1-x}}$
5. Given that $\frac{m^8 \times n^3}{m^4 \times n^5} = m^x \times n^y$, calculate the value of 2x – y
6. If $8^{x/2} = 2^{3/8} \times 4^{3/4}$, find x ***[WAEC]***

KEYWORDS: Number, Index, Power, Roots, Base etc

WEEK 2
DATE……………………….
TOPIC: LOGARITHMS
CONTENT:

- Deducing logarithm from indices and standard form
- Definition of Logarithms
- Definition of Antilogarithms
- The graph of $y = 10^x$
- Reading logarithm and Antilogarithm tables

Deducing logarithm from indices and standard form

There is a close link between indices and logarithms
$100 = 10^2$. This can be written in logarithmic notation as $\log_{10}100 = 2$.
Similarly $8 = 2^3$ and it can be written as $\log_2 8 = 3$.
In general, $N = b^x$ in logarithmic notation is $\text{Log}_b N = x$.

We say the logarithms of N in base b is x. When the base is ten, the logarithms is known as common logarithms.

The logarithms of a number N in base b is the power to which b must be raised to get N.

Re-write using logarithmic notation (i) $1000 = 10^3$ (ii)$0.01 = 10^{-2}$ (iii) $2^4 = 16$ (iv)$\frac{1}{8} = 2^{-3}$

Change the following to index form

(i) $\text{Log}_4 16 = 2$ (ii) $\log_3 (\frac{1}{27}) = -3$

The logarithm of a number has two parts and integer (whole number) then the decimal point. The integral part is called the characteristics and the decimal part is called mantissa. To find the logarithms of 27.5 form the table, express the number in the standard form as $27.5 = 2.75 \times 10^1$. The power of ten in this standard form is the characteristics of Log 27.5. The decimal part is called mantissa.

Remember a number is in the standard form if written as $A \times 10^n$ where A is a number such that $1 \leq A < 10$ and n is an integer.

$27.5 = 2.75 \times 10^{1,}$ 27.5 when written in the standard form, the power of ten is 1. Hence the characteristic of Log 27.5 is 1. The mantissa can be read from 4-figure table. This 4-figure table is at the back of your New General Mathematics textbook.

Below is a row from the 4-figure table

											Differences								
X	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	1	3	4	6	7	9	11	12	14

To check for Log27.5, look for the first two digits i.e 27 in the first column.

Now look across that row of 27 and stop at the column with 5 at the top. This gives the figure 4393.

Hence Log27.5 = 1.4393

To find Log275.2, 2.752×10^2

The power of 10 is the standard form of the number is 2. Thus, the characteristic is 2.

Log275.2 = 2. 'Something'

For the mantissa, find the figure along the row of 27 with middle column under 5 as before (4393). Now find the number in the differences column headed. This number is 3. Add 3 to 4393 to get 4396. Thus Log275.2 = 2.4396.

Example 1

Use table to find

i. Log 37.1
ii. Log 64.71
iii. Log 7.238

If the logarithm of a number is given, one can determine the number for the antilogarithm table.

Example 2: Find the antilogarithms of the following (a) 0.5670 (b)2.9504

Solution

(a) The first two digits after the decimal point i.e .56 is sought for in the extreme left column of the antilogarithm table then look across towards right till you, get to the column with heading 9. (Read 56 under 9) there you will see 3707. Since the integral part of 0.5690 is 0, it means if the antilog (3707) is written in the standard form the power of 10 is zero.
i.e antilog of 0.5690 = 3.707×10^0
= 3.707

(b) The antilog of 2.9504 is found by checking the decimal part .9504 in the table.

(c) Along the row beginning with 0.95 look forward right and pick the number in the column with 0 at the top. This gives the figures 8913 proceed further to the difference column with heading 4 to get 8. This 8 is added to 8913 to get 8921. The integral part of the initial number (2.9504) is 2. This shows that there are three digits before the decimal point in the antilog of 2.9504 so antilog 2.9504 = 892.1.

The graph of $y = 10^x$ can be used to find antilogarithm (and logarithm). Below is the table of values.

X	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$Y = 10^x$	1	1.3	1.6	2.0	2.5	3.2	4.0	5.0	6.3	7.9	10

For example the broken line shows that the antilog 0.5 is approximately 3.2 or inversely that Log $3.2 \simeq 0.5$

Class Activity:

1. Find the logarithms of

(i) 32.7
(ii) 61.02
(iii) 3.247

2. Use antilog tables to find the numbers whose logarithms are
(i) 1.82 (ii)2.0813 (iii) 0.2108

PRACTICE EXERCISE

Find the results of the following,

1. Log 436.2
2. Log 25.38
3. Log 3.258
4. $10^{0.0148}$
5. Find the number whose log is 2.6021

ASSIGNMENT

Use Logarithm table to evaluate each of the following,

1. $10^{1.1844}$
2. $10^{3.0631}$
3. $10^{5.1047}$

Use antilogarithm tables to find the numbers whose common logarithms are:

4. 0.0254
5. 1.4662
6. 6.0129

KEYWORDS: character/Integer, Mantissa/Decimal fraction, Logarithm, Antilogarithm etc

WEEK 3

DATE……………………….

TOPIC: LOGARITHMS

CONTENT:

Use Of Logarithm Table And Antilogarithm Table In Calculation Involving

i. Multiplication
ii. Division
iii. Powers
iv. Roots
v. Application of logarithm in capital market and other real life problems

Logarithm and antilogarithm tables are used to perform some arithmetic basic operations namely: multiplication and division. Also, we use logarithm in calculations involving powers and roots.

The basic principles of calculation using logarithm depends strictly on the laws on indices. Recall that.

(a) Log MN = Log M + Log N

(b) Log $\frac{M}{N}$ = Log M - Log N

Hence, we conclude that in logarithm;

1. When numbers are multiplied, we add their logarithms
2. When two numbers are dividing, we subtract their logarithms.

- **Multiplication of numbers using logarithm tables**

Example 1

Evaluate 92.63 x 2.914

Solution

Number	Standard form	Log	operation
92.63	9.263×10^1	1.9667	
2.914	2.914×10^0	0.4645	add
		2.4312	

Antilog of 2.4312 =269.9

Example 2

Evaluate 34.83 x 5.427

Solution

Number	Standard form	Log	operation
34.83	3.483×10^1	1.5420	
5.427	5.427×10^0	0.7346	add
		2.2766	

Antilog of log 2.2766 = 189.1

Class Activity:

Evaluate the following

1. 6.26 x 23.83
2. 409.1 x 3.932
3. 8.31 x 22.45 x 19.64
4. 431.2 x 21.35

- **Division of numbers using logarithm**

Example 1

Evaluate 357.2 ÷ 87.23

Solution

Number	**Standard form**	**Log**	**operation**
357.2	3.572×10^2	**2.5529**	**subtraction**
87.23	8.723×10^1	**1.9406**	
		0.6123	

Antilog of 0.6123 = 4.096

Example 2

Use a logarithm table to evaluate 75.26 ÷ 2.581

Solution

Number	Standard form	Log	operation
75.26	7.526×10^1	1.8765	
2.581	2.581×10^0	0.4118	subtraction
		1.4647	

Antilog of 1.4647 = 29.16

Class Activity:

Use table to evaluate the following

(1) 53.81 ÷ 16.25 (2)632.4 ÷ 34.25 (3) 63.75 ÷ 8.946 (4) 875.2 ÷ 35.81

- **Powers Using Logarithm**

Study these examples.

At times, calculations can involve powers and roots. From the laws of logarithm, we have

(a) $\text{Log } M^n = n\text{LogM}$

(b) $\text{Log } M^{1/n} = \frac{1}{n}\text{Log } M = \text{Log}\sqrt[n]{M}$

(c) $\text{Log } M^{x/n} = \frac{x}{n}\text{Log } M = \frac{x\log M}{n}$

Example 1.

Evaluate the following $(53.75)^3$

Solution

Number	**Standard form**	**Log**	**operation**
$(53.75)^3$	$(5.375 \times 10^1)^3$	**1.7304**	**Multiply log by 3**
		X 3	
		5.1912	

Antilog of 5.1912 = 155300

Example 2: 64.59^2
Solution

Number	Standard form	Log	operation
$(64.59)^2$	$(6.459 \times 10^1)^2$	1.8102	Multiply Log by 2
		X 2	
		3.6204	

Antilog of 3.6204 = 4173

Class Activity:
Evaluate the following
1. 5.632^4
2. $\sqrt{35.81}$
3. 19.18^3
4. $(67.9/5.23)^3$
5. $\sqrt{679.5}$ x 92.6

- **Roots using Logarithms**

Example 1
Use tables of Logarithms and antilog to calculate $\sqrt[5]{27.41}$
Solution

Number	Standard form	Log	operation
$\sqrt[5]{27.41}$	$(2.741 \times 10^1)^{1/5}$	1.4380 ÷ 5	Divide Log by 5
		0.2876	

Antilog of 0.2876 = 1.939

Example 2: use table to find $\sqrt[3]{\frac{218}{3.12}}$

Solution
Hint: Workout 218 ÷ 3.12 before taking the cube root.

Number	Standard form	Log	operation
218	(2.18×10^2)	2.3385	subtraction
3.12	(3.12×10^0)	0.4942	
		1.8443 ÷3	division
		0.6148	

Antilog of 0.6148 = 4.119

Example 3

Evaluate $63.75^2 – 21.39^2$

We can use difference of two squares

i.e $A^2 – B^2 = (A+B)(A-B)$

$63.75^2 – 21.39^2 = (63.75+21.39)(63.75-21.39)$

= (85.14)(42.36)

= 85.14 X 42.36

Number	**Log**	**operation**
85.14	**1-9301**	
42.36	**1.6270**	**Addition**
	3.5571	

Antilog of 3.5571 = 3607

Class Activity:

Evaluate the following

1. $(39.65^2 – 7.43^2)^{1/2}$
2. 84.35^2
3. 36.95^2
4. $64.74^2 – 55.26^2$
5. $94.68^2 – 43.25^2$
6. $25.14^2 – 7.52^2$

Application of logarithm in capital market and other real life problems.

To start a big business or an industry, a large amount of money is needed. It is beyond the capacity of one or two persons to arrange such a huge amount. However, some persons partner together to form a company. They then, draft a proposal, issue a prospectus (in the name of the company), explaining the plan of the project and invite the public to invest money on this project. They then pool up the form from the public, by selling them shares from the company.

Examples:

1. On Wednesday 8th August 2008 an investor bought 6274 383 shares on the floor of a stock market exchange at #92.85 per share. Four years thereafter, he sold them at #134.76 per share. Calculate his profit, correct to three significant figures.
   Solution:
   On 8th August, 2008:
   1 share = #92.85
   Then let, 6274 383 shares = #x

This gives, x = 6274 383 x 92.85
Four years thereafter:
1 share = 134.76
Then let, 6274 383 = #y
This gives, y = 6274 383 x 134.76
Thus, his profit four years thereafter is (y - x) naira,
That is, 6274 383 x 134.76 - 6274 383 x 92.85 = 6247 383 (41.91)
≃ 6247 000 x 41.91

Number	log
6247000	6.7957
41.91	**1.6223**
2.618 x 10^8	**8.4180**
= 261800000	

Hence the investor's profit was #262 million

2. #67,200 are invested in #100 shares which are quoted at #120. Find the income if 12% dividend is declare in the shares.
   Solution:
   Sum invested = #67,200
   And M.V of each share = #120
   Therefore No. of shares bought = #67,200 ÷#120

No.	Log
67200	4.8274
120	2.0792
560.02	2.7482

Given: dividend (income) on 1 share = 12% of N.V = 12% of #100 = #12
Therefore, total income from the shares = #560 x #12 = #6,720

PRACTICE EXERCISE:

Find the values of the following using logarithm tables.

1. $\frac{17.83 \times 24.69}{2.56 \times 32.8}$
2. $\sqrt{43.67 \times 45.86}$
3. $(987.3)^{\frac{1}{3}}$
4. $\sqrt[3]{\frac{218 \times 37.2}{95.43}}$
5. $\frac{943}{11.64 \times 7.189}$

ASSIGNMENT

Use tables to find the values of;

1. $\left(\frac{85.32}{9.82}\right)^2$
2. $\frac{17.4^2 \times 4.42}{\sqrt[3]{858\,000}}$
3. $\sqrt[3]{\left(\frac{38.32 \times 2.964}{8.637 \times 6.285}\right)^2}$
4. In one day a total of 478900 shares were traded on the floor of a stock exchange. If the value of each share was #23.50, use tables to calculate, to three significant figures, the value of the traded shares.
5. An investor buys 2650 shares at #1.44 each. Use logarithm tables to calculate his profit to the nearest naira if he sells them 3 years later at #1.565 each.

KEYWORDS: Character/Integer, Mantissa/Decimal fraction, Logarithm, Antilogarithm etc

WEEK 4
DATE……………………….
Subject: Mathematics
TOPIC: Sets

- Definition of sets
- Set notations
- Types of sets

DEFINITION

A set is a general name for any group or collection of distinct elements. The elements of a set may be objects, names, points, lines, numbers or idea

The elements must have unique characteristics (specification) that can help to distinguish them from any other element outside the group or set. Hence, a set is a collection of well defined objects e.g.

(i) a set of mathematics text books
(ii) a set of cutleries
(iii) a set of drawing materials etc.

Sometimes there may be no obvious connection between the members of a set. Example: {chair, 3, car, orange, book, boy, stone}.

Each item in a given set are normally referred to as member or element of the set.

SET NOTATION

This is a way of representing a set using any of the following.

(i) Listing method
(ii) Rule method or word description
(iii) Set builders notation.

(i) Listing Method

A set is usually denoted by capital letters and the elements in it can be defined either by making a list of its members. Eg A = {2, 3, 5, 7}, B = {a, b, c, d, e, f, g, h, i} etc.

Note that the elements of a set are normally separated by commas and enclosed in curly brackets or braces

(ii) Rule Method. The elements in a set can be defined also by describing the rule or property that connects its members. Eg C = {even number between 7 and 15. D= {set of numbers divisible by 5 between 1 and 52.}, B = {x : x is the factors of 24}etc

(iii) Set–Builders Notations

A set can also be specified using the set – builder notation. **Set – builder notation is an algebraic way of representing sets using a mixture of word, letters , numbers and inequality symbols** e.g. $B = \{x : 6 \leq x < 11, x \in z\}$ or $B = \{x/6 \leq x < 11, x \in I\}$. The expression above is interpreted as "B is a set of values x such that 6 is less than or equal to x and x is less than 11, where x is an integer (z)"

- The stroke (/) or colon (:) can be used interchangeably to mean "such that"
- The letter Z or I if used represents integer or whole numbers.

Hence, the elements of the set $B = \{x : 6 \leq x < 11, x \in z\}$ are B = { 6, 7, 8, 9,10}.

<u>NB</u>:

- The values of x starts at 6 because $6 \leq x$
- The values ends at 10 because $x < 11$ and 10 is the first integer less than 11.

The set builder's notation could be an equation, which has to be solved to obtain the elements of the set. It could also be an inequality, which also has to be solved to get the range of values that forms the set.

Class Activity

(a) Define Set
(b) $C = \{x : 3x - 4 = 1, x \in z\}$
(c) P = {x : x is the prime factor of the LCM of 15 and 24}
(d) Q = {The set of alphabets}

(e) R = {x : $x \geq 5$, x is an odd number}

Set – Builders Notations (conts.)

Examples 1:

List the elements of the following sets

(i) A = {x : $2 < x \leq 7$, x ϵ ƶ}.

(ii) B = {x : $x > 4$, x ϵ ƶ}

(iii) C = {x : $-3 \leq x \leq 18$, x ϵ ƶ}.

(iv) D = {x : $5x - 3 = 2x + 12$, x ϵ Z}.

(v) E = {x : $3x - 2 = x + 3$, x ϵ I}

(vi) F = {x : $6x - 5 \geq 8x + 7$, x ϵ ƶ}

(vii) P = {x : $15 \leq x < 25$, x are numbers divisible by 3}

(viii) Q = {x : x is a factor of 18, }

Solution:

(i) A = {3, 4, 5, 6, 7}

Note that:

- the values of x start at 3, because $2 < x$

-The values of x ends at 7 because $x \leq 7$ i.e. because of the equality sign.

(ii) B = {5, 6, 7, 8, 9, .. .}

Note that:

the values of x start from 5 because 5 is the first number greater than 4 (i.e. we are told that x is greater than 4)

(iii) C = {-3, -2, -1, 0, 1, . . , 15, 16, 17, 18}

Note that:

- The values of x starts from -3 because $-3 \leq x$, and ends at 18 because $x \leq 18$ (there is equality sign at both ends).

(iv) To be able to list the elements of thisset, the equation defined has to be solved

i.e. $5x - 3 = 2x + 12$

$5x - 2x = 12 + 3$

$3x = 15$

$x = {}^{15}/_{3}$

$\therefore x = 5$

$\therefore D = \{5\}$

(v) We also need to solve the equation to get the set values

$3x - 2 = x + 3$

$3x - x = 3 + 2$

$2x = 5$

$\therefore x = {}^{5}/_{2}$

Since ${}^{5}/_{2}$ is not an integer (whole number) therefore the set will contain no element.

$\therefore \epsilon = \{\ \}$ or Ø

(vi) Solving the inequality to get the range of values for the set, we have

$6x - 5 \geq 8x + 7$

$6x - 8x \geq 7 + 5$

$-2x \geq 12$

$x \leq {}^{12}/_{-2}$

$\therefore x \leq -6$

$\therefore F = \{\ldots, -8, -7, -6\}$

(vii) P = {15, 18, 21, 24}

Note that:

The values of x start at 15 because it is the first number divisible by 3 and falls within the range defined.

(viii) Q = {1, 2, 3, 6, 9, 18}

Example 3:

Rewrite the following using set builder notation

(i) A = {8, 9, 10, 11, 12, 13, 14}

(ii) B = {3, 4, 5, 6 . . . }

(iii) C = {. . . 21, 22, 23, 24}

(iv) D = {7, 9, 11, 13, 15, 17 . . .}

(v) P = {1, -2}

(vi) Q = {a, e, i, o, u}

Solution:

(i) $A = \{x : 7 < x < 15, x \in \mathbb{Z}\}$ OR

$A = \{x : 8 \leq x < 15, x \in \mathbb{Z}\}$ OR

$A = \{x : 7 < x \leq 14, x \in \mathbb{Z}\}$ OR

$A = \{x : 8 \leq x \leq 14, x \in \mathbb{Z}\}$

(ii) $B = \{x : x > 2, x \in \mathbb{Z}\}$ OR

$B = \{x : x \geq 3, x \in \mathbb{Z}\}$

(iii) $C = \{x : x < 25, x \in \mathbb{Z}\}$ OR

$C = \{x : x \leq 24, x \in \mathbb{Z}\}$

(iv) $D = \{x : x > 8$ or $x \geq 7$, x is odd, $x \in \mathbb{Z}\}$

(v) P={1,2} suggests the solutions of a quadratic equation. Therefore, the equation or set-builders notation can be obtained from :

$x^2 -$ (sum of roots)x + product of roots $= 0$

$x^2 - (-1)x + (1 \times -2) = 0$

$x^2 + x - 2 = 0$

$P = \{x : x^2 + x - 2 = 0, x \in \mathbb{Z}\}$

(vi) Q = {x : x is a vowel}

Class Activity

1. List the elements in the following Sets
    - (a) A = {x : $-2 \leq x < 4$, x ϵ ƶ}
    - (b) B = {x : $9 < x < 24$, x ϵ N}
    - (c) C = {x : $7 < x \leq 20$, x is a prime number, x ϵ I}
    - (d) D = {x / $2x - 1 = 10$, x ϵ Z}
    - (e) P = {x : x are the prime factor of the LCM of 60 and 42}
2. Rewrite the following using Set – builder notations.
    - (a) Q = {. . . 2, 3, 4, 5}
    - (b) A = {2, 5}
    - (c) B = {2, 4, 6, 8, 10, 12 . . .}
    - (d) A = {-2, -1, 0, 1, 2, 3, 4, 5, 6}
    - (e) C = {1, 3, -2}

TYPES OF SETS

Finite Sets

Refers to any set, in which it is possible to count all the elements that make up the set. These types of sets have end. E.g.

A = {1, 2, 3, . . , 8, 9, 10}

B = {18, 19, 20, 21, 22}

C = {Prime number between 1 and 15} etc.

Infinite Sets

Refers to any set, in which it is impossible to count all the elements that make up the set. In other words, members or elements of these types of set have no end. These types of set, when listed are usually terminated with three dots or three dots before the starting values showing that the values continue in the order listed. E.g.

(i) A = {1, 2, 3, 4, . . }

(ii) B = {…,-4,-3,-2,-1,0,1,2,3,…}

(iii) C = {Real numbers} etc.

Empty or null Set

A set is said to be empty if it contains no element. Eg {the set of whole number that lies between 1 and 2}, {the set of goats that can read and write}, etc Empty sets are usually represented using ø or { }.

It should be noted that {0} is **NOT** an empty set because it contains the element 0, Another name for empty set is null set.

Cardinality of a set/Number of Elements in a Set

Given a set A = {-2, -1, 0, 1, 2, 3, 4, 6} the number of elements in the set A denoted by n(A) is 9; i.e. n (A) = 7

If B = {2,3, 5} then n (B) = 3

If Q = {0} then n (Q) = 1
Other examples are as follows:

***Example 4*:**

Find the number of elements in the set:

P = $\{x : 3x - 5 < x + 1 < 2x + 3, x \in \mathbb{Z}\}$

***Solution*:**

$3x - 5 < x + 1$ and $x + 1 < 2x + 3$

$3x - x < 1 + 5$ and $x - 2x < 3 - 1$

$2x < 6$ $\quad\quad -x < 2$

$x < {}^6/_2$ $\quad\quad x > -2$

$x < 3$ $\quad\quad -2 < x$

$-2 < x < 3$

The integers that form the solution set are

P = {-1, 0, 1, 2}

∴n {P} = 4

***Example 5*:**

Find the number of elements in the set

A = $\{x : 7 < x < 11$, x is a prime number$\}$

Solution:

The set A = { } or Ø since 8, 9, 10 are no prime numbers. ∴n(A) = 0

Example 6:

Find the number of elements in the following sets:

(i) B = $\{x : x \leq 7, x \in \mathbb{Z}\}$

(ii) C = $\{x : 3 < x \leq 8$, x is a number divisible by 2$\}$.

Solution:

(i) B = {. . . 3, 4, 5, 6, 7}.
The values of the set B has no end hence it is an infinite set i.e. n(B) = ∞

(ii) C = {4, 6, 8}. ∴n(C) = 3

The Universal Set

This is the Set that contains all the elements that are used in a given problem. Universal Sets vary from problem to problem. It is usually denoted using the symbols ξ or μ.

Note that when the Universal Set of a given problem is defined, all values outside the universal set cannot be considered i.e. they are invalid.

Equivalent Sets

Two Sets are said to be equivalent if the Sets have equal number of elements. E.g.
If A = {2, 3, 4, 5} and B = {a, b, c, d} then the Sets A ≡ B (A is equivalents to B) since n(A) = n(B).

Equality of Sets

Equal sets are special cases of equivalent sets. They have exactly equal elements on both sets. The order of writing the elements is not important. We use equality sign "=" to indicate equal sets

Given two sets P = {3,4,5} and Q = {4,3,5} . The elements of the two sets are equal. Therefore **P=Q**

Subset and Superset

Suppose that A={1,2,3} and B={1,2,3,4,5}. Notice that every element of set A is also an element of set B . We say that A is a subset of B written as $\subset B$. While B is a superset of A is written as B $\supset A$

Power Sets

Given a set A then the power set of A, denoted by P(A) is the set of all possible subsets of A

The number of possible subsets in P(A) is given by 2^n where n is the number of elements in the set

Class Activity

(1) List the elements in the following Sets

(a) A = {x : -2 ≤ x < 4, x ϵ ƶ}

(b) B = {x : 9 < x < 24, x ϵ N}

(c) C = {x : 7 < x ≤ 20, x is a prime number, x ϵ I}

(d) D = {x / 2x – 1 = 10, x ϵ Z}

(e) P = {x : x are the prime factor of the LCM of 60 and 42}

(2). Find the number of elements in the sets in question (1) above

(3). If

(a) A= {3,5,7,8,9,10,}, Then n(A) =

(b) B= {1, 3, 1, 2, 1, 7}, Then n(B) =

© Q= {a, d, g, a, c, f, h, c,} , Then n(Q) =

(d) P= {4,5,6,7,…,12,13}, Then n(P) =

(e) D ={ days of the week} , then n(D)=

PRACTICE EXERCISE:

(1). State if the following are finite, infinite or null set

(i) Q = {x : x ≥ 7, x Є Z}

(ii) P = {x : -4 ≤ x < 16, x Є I}

(iii) A = {x : 2x – 7 = 2, x Є Z}

(iv) B = { sets of goats that can fly}

(v) D ={sets of students with four legs}

(2) List the following sets in relation to the universal set. Given that the universal set $\xi = \{x: 1 < x < 15, x \varepsilon z\}$

(i) $A = \{x; -3 \leq x \leq 7, x \varepsilon Z\}$

(ii) $B = \{x: 5 < x < 6, x \varepsilon z\}$,

(iii) $C = \{x: X \geq 5, x \varepsilon z\}$

(3) List the elements of the following universal sets.

(i) The set of all positive integers

(ii) The set of all integers

(iii) $\xi = \{x: 1 < x < 30$, x are multiples of 3}

(iv) $\xi = \{x: 7 \leq x < 25$, x are odd numbers}

(v) $\xi = \{x: x \geq 10, x \varepsilon z\}$

(4) Find all the possible subsets of each of the following sets

(i) A={1,2}

(ii) B={ 7,9}

(iii) C={2,4,6}

(5) Find the power set of each of the following set

(a) A={0,5}

(b) B = {7,8,9}

ASSIGNMENT

1. Identify each of the following set as null, finite or infinite set
   a. A={x : 0<x<10,x is an integer}
   b. B= {x: x>2, x is an integer}
2. Calculate the number of possible subsets of each of the following set
   a. {a,b}
   b. {x,y,z}
   c. {m,n,o,p}
3. Write down the possible subsets of each of the following set
   d. {a,b}
   e. {x,y,z}
   f. {m,n,o,p}
4. Given that A ={x;x is an even number}

   and B={2,3,4,6}

Show that B is not a subset of A

5. If $A \subset \{\ \}$, then what is A?

KEYWORDS: finite, infinite, subset, superset, cardinality, power-set etc

WEEK 5

DATE……………………….

Subject: Mathematics

TOPIC: Sets

CONTENT:

- Set operations
- Union
- Intersection
- Complement.
- Venn diagram.
- Application of Venn diagram up to 3 set problem.

Operations on Sets

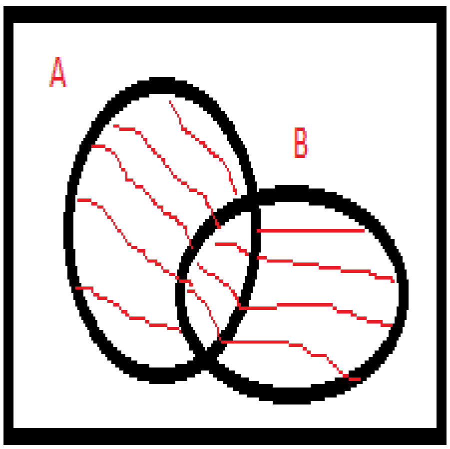
The Union of Sets:

The Union of Sets A and B is the Set that is formed from the elements of the two Sets A and B. This is usually denoted by "A ∪B" meaning A Union B. Thus A ∪B is the Set which consists of elements of A or of B or of both A and B.

When represented using Venn diagram we have

A ∪B

ξ



Using Set notations, the Union of two Sets A and B is solved as follows

Example 1*:*

Given that A = {3, 7, 8, 10}

and B = {3, 5, 6, 8, 9} then

A∪ B= {3, 5, 6,7, 8, 9,10}

Example 2*:*

If A = {a, b, c, d}, B = {1, 2, 3, 4} and C ={a, 3, θ} Then A ∪B ∪C = {a, b, c, d,1, 2, 3, 4, θ}

Class Activity

A = {7, 8, 9, 10}, B = {8, 10, 12, 14} and C = {7, 9, 10, 14. 15}

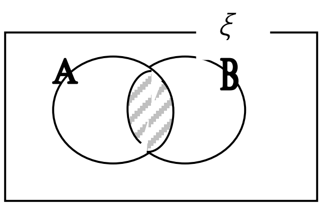
find the following:

1. (a) A ∪B (b) B∪ C

2. A ∪B ∪ C

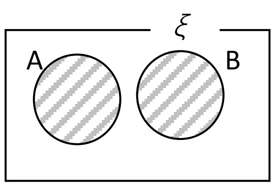
Intersection of Sets

The intersection of Sets A and B is the set of elements that are common to both A and B. This is usually denoted by "A ∩ B" meaning A intersection B. When represented using Venn diagram we have



Disjoint sets

If A ∩ B = Ø, then the Sets A and B are said to be disjoint. Disjoint Sets are Sets that have no element in common.



***Example 1*:**

Given that A = {5, 7, 8, 10} and B = {3, 5, 6, 8, 9}, then A ∩ B = {5, 8}.

Example 2:

If P = {a, b, c, d, e, f, g}, Q = {b, c, e, g} and R = {a, c, d, f, g}
Then, P ∩ Q ∩ R = {c, g}

***Example 3*:**

If A = {1, 2, 3} and B = {6, 8, 10}, then
A ∩ B = { } or Ø. The Set A and B are disjoint.

Class Activity

Given that ξ= {21, 22, 23, 24, . . ., 29, 30},
P = {21, 23, 25, 26, 28},
Q = {22, 24, 26, 27, 28} and
R = {21, 25, 26, 27, 30} are Subsets of ξ Find:

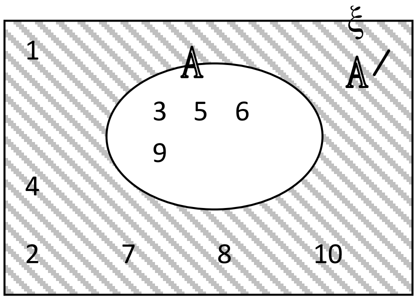
1. (i) P ∩Q (ii) P ∩Q
2. (i) Q ∩R (ii) P ∩Q ∩R

The Complement of Set

If A is a Subset of the Universal Set ξ, then, the complement of the Set A are made up of elements that are not in A, but are found in the Universal Set ξ. This is usually denoted by A^c or A′. For example,

If ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and A = {3, 5, 6, 9} then A^c or A′ = {1, 2, 4, 7, 8, 10}

Using Venn diagram, this is represented by the shaded portion below:



Note that to find the complement of a Set, the Universal Set must be properly defined.

Example 1:

Given that

ξ = {11, 12, 13, 14, 15, 16, 17, 18, 19, 20} A = {11, 13, 16, 18, 20} and
B = {12, 14, 16, 18, 19, 20}.

Find the following:

(i) A′ (ii) B′ (iii) (A ∪B)′
(iv) (A ∩B)′ (v) A′∩B′ (vi) A′∪B′
(vii) (A′)′

Solution:

(i) A′ = {12, 14, 15, 17, 19}
(ii) B′ = {11, 13, 15, 17}
(iii) A ∪B = {11, 12, 13, 14, 16, 18, 19, 20}
(A ∪B)′= {15, 17}
(iv) A ∩B = {16, 18, 20}
(A ∩B)′ = {11, 12, 13, 14, 15, 17, 19}
(v) (A′∩B′) = {15, 17}
(vi) A′∪ B′ = {11, 12, 13, 14, 15, 17, 19}
(vii) A′ = {12, 14, 15, 17, 19}
(A′)′ = {11, 13, 16, 18, 20} = A

NB:

From the example above, observe that from **(iii)** and **(v),** (A ∪B)′ = A′∩B′
Also, from **(iv)** and **(vi)**
(A ∩B)′ = A′∪B′ and from **(vii)**(A′)′ = A

Example 2:

Given that ξ = {a, b, c, d, e, f, g, h, i, j}
A = {a, c, e, g, i} and
B = {b, c, d, f, i, j}.

Find the following:

(i) A' (ii) B'
(iii) $A' \cup B'$ (iv) $A' \cap B'$
(v) $(A \cap B)'$ (vi) $(A \cup B)'$
(vii) $(B')'$

Solution:

(i) $A' = \{b, d, f, h, j\}$
(ii) $B' = \{a, e, g, h\}$
(iii) $A' \cup B' = \{a, b, d, e, f, g, h, j\}$
(iv) $A' \cap B' = \{ h \}$
(v) $A \cap B = \{c, i\}$
$(A \cap B)' = \{a, b, d, e, f, g, h, j\}$
(vi) $A \cup B = \{a, b, c, d, e, f, g, i, j\}$
$(A \cup B)' = \{ h\}$
(vii) $B' = \{a, e, g, h\}$
$(B')' = \{b, c, d, f, i, j\} = B$

From the example above, we also observe that $A' \cup B' = (A \cap B)'$ **---** From **(iii)** and **(v)**
$A' \cap B' = (A \cup B)'$**---** From **(iv)** and **(vi)**
And $(B')' = B$ **---** From **(vii)**From the last two examples we can clearly see that $(A \cup B)' = A' \cap B'$,

$(A \cap B)' = A' \cup B'$

and $(A')' = A$

Generally, for any two Subsets A and B of a Universal Set ξ, the following are true:

(i) $(A \cup B)' = A' \cap B'$
(ii) $(A \cap B)' = A' \cup B'$
(iii) $(A')' = A$ or $(B')' = B$

These are known as **De Morgan's Laws of Complementation.**

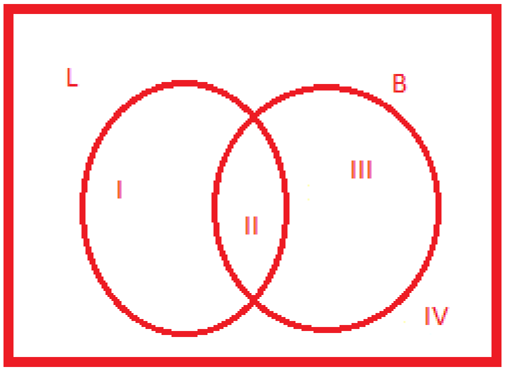
VENN DIAGRAMS

A Mathematician by name John Venn was the man to first represent the relationship between sets with diagrams. Ever since sets may be represented by diagrams called Venn diagrams.

The rectangle is used to represent the Universal set, and Circles for other sets, as we shall see later.

PROBLEMS INVOLVING TWO SETS.

For two intersecting sets, the diagram is given below with the labels of what each compartment represents.



Compartment I:represents the set of
elements in A only. i.e. $A \cap B^{/}$ using set notations.
Compartment II: represents the set of elements common to both A and B i.e. $A \cap B$
Compartment III: represents the set of elements in B only i.e. $A^{/} \cap B$
Compartment IV: represents the set of elements that are neither in A nor B i.e. $(A \cup B)^{/}$ or $A^{/} \cap B^{/}$

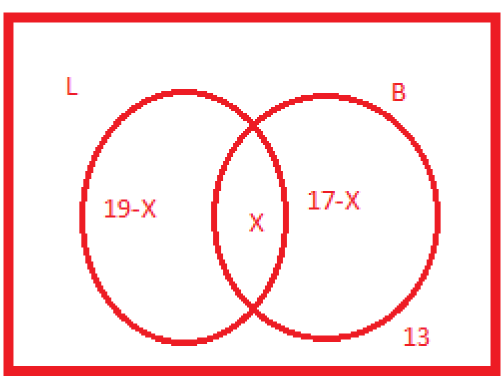
Example 1:

In a Survey of 40 Students in a class, 19 have visited Lagos and 17 have visited Benin City. If 13 have visited neither. How many Students have visited:
(i) Both Cities; (ii) Benin City but not Lagos (i.e. Benin City only)

Solution:

$n(\xi) = 40$
$n(L) = 19$
$n(B) = 17$
$n(L \cup B)^{/} = 13$

Let x represents those that have visited both Cities i.e. $n(L \cap B) = x$



$19 – x + x + 17 – x + 13 = 40$

$$49 - x = 40$$
$$49 - 40 = x$$
$$9 = x$$

$\therefore$ 9 Students have visited both Cities

(b) Those that have visited Benin City only are = 17 – x

= 17 – 9

= 8

$\therefore$ 8 have visited Benin City only.

Example 2:

In a Class of 45 Students, if 21 offer Agricultural Science, 25 offer Biology and 6 offer both subjects. Find

(i) those that offer neither.

(ii)the number that offers Biology but not Agricultural Science (i.e. Biology only)

Solution:

(i) $n(\xi) = 45$

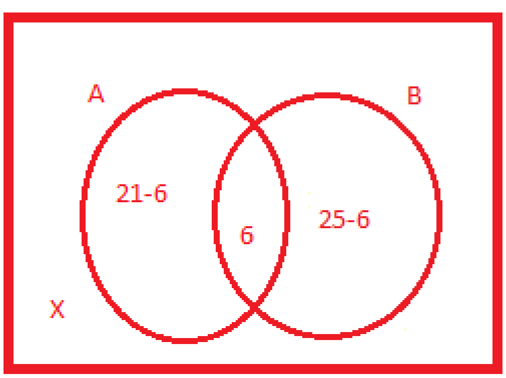
$n(A) = 21$

$n(B) = 25$

$n(A \cap B) = 6$

Let $n(A \cup B)' = x$

i.e. Let those that offer neither be x.



$$21 - 6 + 6 + 25 - 6 + x = 45$$
$$15 + 6 + 19 + x = 45$$

$40 + x = 45$

$x = 45 - 40$

$\therefore \quad x = 5$

$\therefore$ Those that offer neither,

i.e. $n(A \cup B)' = 5$

ii. $n(B \cap A') = 19$

Class Activity

(1). In a gathering of 30 people, x speak Hausa and 15 speak Yoruba. If 5 people speak both languages, find how many people that speak

(i) Hausa.

(ii)Yoruba only

(iii)Hausa only.

(2) In a Birthday party attended by 22 people, 10 ate fried rice and 13 ate salad. If x ate both fried rice and salad and (2x–5) ate none of the two. How many ate

(i) both fried rice and salad?

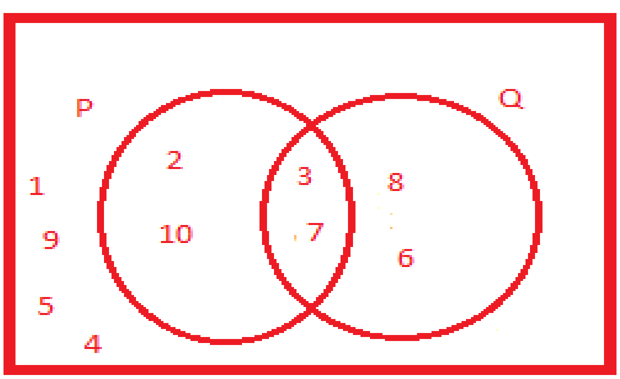
(ii) salad but not fried rice?

(ii) neither fried rice nor salad?

(3) The Venn diagram below represents a universal set ξ of integers and its subsets Pand Q**.** List the elements of the following sets;

(a) $P \cup Q$ (c) $P \cup \xi$ (d) $\xi \cap Q$

(b) $P \cap Q$



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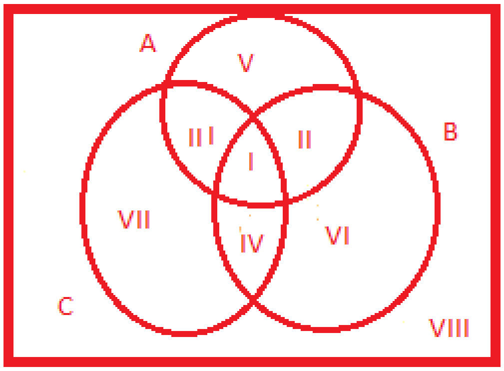
VENN DIAGRAM:PROBLEMS INVOLVING THREE SETS.

The Venn diagram is made up of eight compartments as shown below:



Compartment I: represents $A \cap B \cap C$
(elements common to the three sets A, B and C).
Compartment II: represents $A \cap B \cap C^{/}$
(elements common to both A and B only).
Compartment III: represents $A \cap B^{/} \cap C$
(elements common to both A and C only).
Compartment IV: represents $A^{/} \cap B \cap C$
(elements common to both B and C only).
CompartmentV : represents $A \cap B^{/} \cap C^{/}$
(elements of A only).
CompartmentVI :represents $A^{/} \cap B \cap C^{/}$
(elements of B only).
Compartment VII: represents $A^{/} \cap B^{/} \cap C$
(elements of C only).
Compartment VIII:represents $(A \cup B \cup C)^{/}$ or $A^{/} \cap B^{/} \cap C^{/}$ elements that are not in any of the three sets but are in the Universal set.

Example 1:

There are 80 people in a sports camp. Each play at least one of the following games, volleyball, football and handball. 15 play volleyball only, 18 play football only, and 21 play handball only .If 5 play volleyball and foot ball only, 8 play volleyball and handball only, and 10 play football and Handball only.
(a) Represent the above information in a Venn diagram
(b) How many people play the three games?
(c) How many people play football ?

Solution:

List of information given in the question is as follows

Let V be Volleyball

F be Football

H be Handball

$n(\xi) = 80$

$n(V \cap F' \cap H')$ i.e. Volleyball only = 15

$n(V' \cap F \cap H')$ i.e. Football only = 18

$n(V' \cap F' \cap H)$ i.e. Handball only = 21

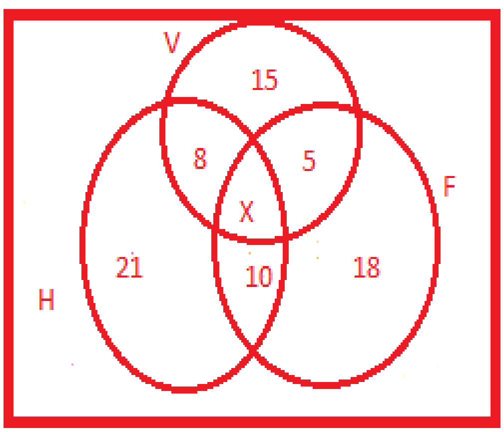
$n(V \cap F \cap H')$ i.e. Volleyball and Football only = 5

$n(V \cap F' \cap H)$ i.e. Volleyball and Handball only = 8

$n(V' \cap F \cap H)$ i.e. Football and Handball only = 10

Let $n(V \cap F \cap H) = x$. i.e. Those that play the three games = x

(a)



(b) $15 + 5 + 18 + 8 + x + 10 + 21 = 80$

$77 + x = 80$

$x = 80 - 77$

$\therefore x = 3$

$\therefore$ **3** people play the three games.

(c) The number that plays football

$n(F) = 18 + 5 + 10 + x$

$= 33 + 3$

$= 36$

Example 2:

There are 80 people in a sports camp and each plays at least one of the following games: volleyball, football and handball. 31 play volleyball, 36 play football and 42 play *handball. If 8 play volleyball and football, 11 play volleyball and handball and 13 play football and handball.*

(a) Draw a Venn diagram to illustrate this information, Using x to represent the number that play the three games.

(b) How many of them play:

(i) All the three games,

(ii) Exactly two of the three games,

(iii) Exactly one of the three games

(iv) handball only?

Solution:

Step 1: list out all information given in the question.

Let **V** be Volley ball

F be Football

H be Hand ball

(a) $n(\xi) = 80$

$n(V) = 31$

$n(F) = 36$

$n(H) = 42$

$n(V \cup F \cup H) = 80$ (Since each play at least one of the games).

$n(V \cap F) = 8$

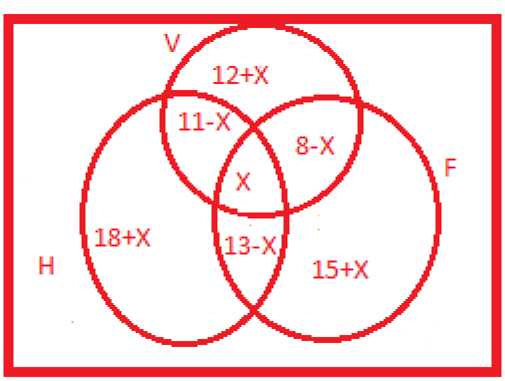
$n(V \cap H) = 11$

$n(F \cap H) = 13$.

Let $n(V \cap F \cap H) = x$.

To fill the Venn diagram we start with the centre Compartment

where $n(V \cap F \cap H) = x$.



How we obtained the value for each of the other compartments is shown below.

For Volleyball and football only

i.e. $n(V \cap F \cap H')$

(Since x is already in the circle of $V \cap F$)

$= n(V \cap F) - x$

$= 8 - x$

For Volleyball and Handball only

i.e.$n(V \cap F' \cap H)$

(Since x is already in the circle of $V \cap H$)

$= n(V \cap H) - x$

$= 11 - x$

For Football and Handball only

i.e. $n(V' \cap F \cap H)$

(Since x is already in the circle of $F \cap H$)

$= n(F \cap H) - x$

$= 13 - x$

For Handball only

i.e. $n(V' \cap F' \cap H)$

n(H) – (All values already written in the circle of Handball)

$= 42 - [(11 - x) + x + (13 - x)]$

$= 42 - [24 - x]$

$= 42 - 24 + x$

$= 18 + x$

For Football only i.e. $n(V' \cap F \cap H')$

n(F) – (All values already written in the circle of Football)

$= 36 - [(8 - x) + x + (13 - x)]$

$= 36 - [21 - x]$

$= 36 - 21 + x$

$= 15 + x$

For Volleyball only i.e.n $(V \cap F' \cap H')$

n(V) – (All values already written in the circle of Volley ball).

$= 31 - [(8 - x) + x + (11 - x)]$

$= 31 - [19 - x]$

= 31 – 19 + x

= 12 + x

(b) *To get the value of x, which represent those that play all three games, we add all the Compartments of the Venn diagram together and equate it to the total value in the Universal set and solve for x.*

i.e. 12 + x + 8 – x + 15 + x + x + 11 – x + 13 – x + 18 + x = 80

77 + x = 80

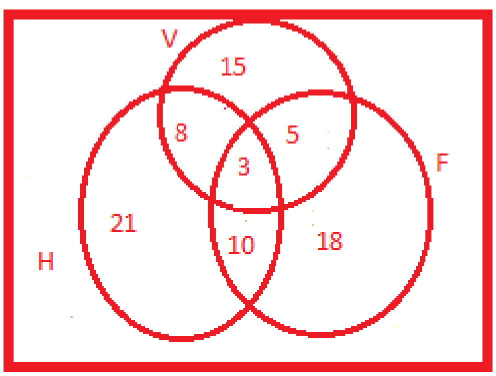
x = 80 - 77

x = 3

∴ 3 people play all three games

NOTE THAT

If this value, x =3, is substituted into the Venn diagram, the answer obtained in the previous example would be got.



b (ii) **Exactly two of the three games**

$= n(V \cap F \cap H^{/}) + n(V \cap F^{/} \cap H) + n(V^{/} \cap F \cap H)$

= 8 – x + 11 – x + 13 – x

= 32 – 3x

= 32 – 3(3)

= 32 – 9

= 23

∴ 23 of them play exactly two of the three games.

b (iii) **Exactly one of the three games**

$= n(V \cap F^{/} \cap H^{/}) + n(V^{/} \cap F \cap H^{/}) + n(V^{/} \cap F^{/} \cap H)$

= 12 + x + 15 + x + 18 + x

= 45 + 3x

= 45 + 3(3)

$= 45 + 9$

$= 54$

$\therefore$ 54 of them play exactly one of the three games.

b (iv) F**or Handball only**

$n(V' \cap F' \cap H) = 18 + x$

$= 18 + 3$

$= 21$

$\therefore$ 21 of them play Handball only.

Class activity

In a Class of 80 undergraduate Students, 21 took elective Courses from Botany only, 16 took from Zoology only, 13 took from Chemistry only. If each of the Students took elective from at least one of the above-mentioned Courses, 7 took Botany and Zoology only, 3 took Zoology and Chemistry only and 8 took Botany and Chemistry only.

(1) Draw a Venn diagram to illustrate the information above using x to represent those that took the three.

(2) Find the:

(i) Value of x

(ii) Number that took Botany

(iii) Number that took Zoology and Chemistry.

PRACTICE EXERCISE:

(1) In a Group of 120 Students, 72 of them play Football, 65 play Table Tennis and 53 Play Hockey. If 35 of the Students play both Football and Table Tennis, 30 play both Football and Hockey, 21 play both Table Tennis and Hockey and each of the Students play at least one of the three games,

(a) Draw a Venn diagram to illustrate this information.

(b) How many of them play:

(i) All the three games;

(ii) Exactly two of the three games;

(iii) Exactly one of the three games

(iv) Football alone?

SSCE, NOV. 1996, № 6 (WAEC).

(2) The set A = {1,3,5,7,9,11}, B = {2,3,5,7,11,15} and C={3,6,9,12,15} are subsets of ε = {1,2,3,…,14,15} (a) draw a Venn diagram to illustrate the given information.
(b) use your Venn diagram to find

(i) $C \cap A'$

(II) $A' \cap (B \cup C)$

WASSCE, *June* 2002, *No. 2*

(3) In a class of 80 students, every student had to study Economics or Geography or both Economics and Geography. If 65 students studied Economics and 50 studied Geography; how many studied both subjects? (1991)

(4) In a certain class, 22 pupils take one or more of Chemistry, Economics and Government. 12 take Economics (E), 8 take Government (G) and 7 take Chemistry (C). Nobody takes Economics and Chemistry and 4 pupils take Economics and Government.

(a) (i) Using set notation and the letters indicated above, write down the two statements in the last sentence.

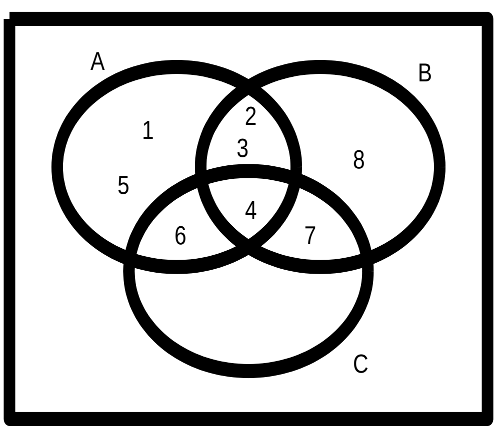
(i) Draw a Venn Diagram to illustrate the information

(b) How many pupils take

(i) both Chemistry and Government

(ii) Government? (1991)

(5) What is $A \cap B$ in the diagram below? (1993)



ASSIGNMENT

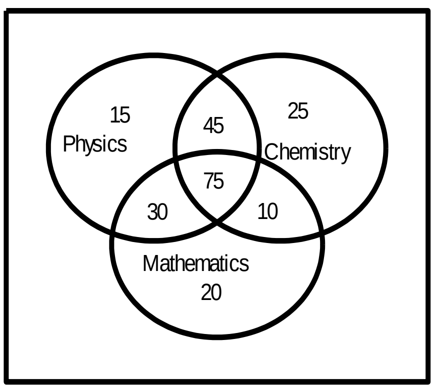
1. In a class of 40 students, 15 like mangoes, 21 like Pineapples and 6 like the two fruits.
   (i) Represent the information in a Venn diagram
   (ii) How many do not like mangoes and pineapples?
   (iii) What percentage of the class like mangoes only? NECO2014

2. Out of the 400 students in the final year in Secondary School, 300 are offering Biology and 190 offering Chemistry.

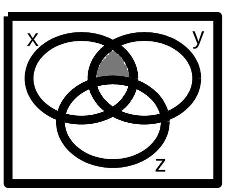
(i) How many students are offering both Biology and Chemistry, if only 70 students are offering neither Biology nor Chemistry?

(ii) How many students are offering at least one of Biology or Chemistry? (1988)

3. The Venn diagram shows the number of students who studied Physics, Chemistry and Mathematics in a certain school. How many students took at least two of the three subjects? (1989)



4. a. What portion is shaded in the Venn diagram below? (1990)



b. The subsets A, B and C of a universal set are defined as follows:

A = {m, a, p, e} B = {a, e, I, o, u} C = {l, m, n, o, p, q, r, s, t, u}.

List all the elements of the following sets and present the information in a Venn diagram.

(ii) $A \cap B$ (ii) $A \cup B$ (iii) $A \cup (B \cap C)$

5. In a certain class, 22 pupils take one or more of Chemistry, Economics and Government. 12 take Economics(E),8 take Government(G) and 7 take Chemistry(C). Nobody takes Economics and Chemistry and 4 pupils take Economics and Government.

a. i. using set notation and letters indicated above, write down the two statements in the last sentence.

ii. draw a Venn diagram to illustrate the information.

b. How many pupils take?

i. Both Chemistry and Government

ii. Government only

KEYWORDS: union, intersection, complement, Venn diagram, subsets etc.

WEEK 6

DATE……………………….

Subject: Mathematics

TOPIC: Number Base System

CONTENT

- Conversion from other bases to base 10 and vice versa.
- Conversion of decimal fraction in other bases to base 10 and vice versa.

Concept of expanded notation: Every decimal number X can be expressed uniquely in the form:

$$X = I_n \times 10^n + I_{n-1} \times 10^{n-1} + I_{n-2} \times 10^{n-2} + \cdots + I_{n-n} \times 10^{n-n}$$

This is known as the expanded notation

EXAMPLE 1

Express the following in expanded notation form

(a) 45078 (b) 0.0235 (c) 930.133

(a) $45078 = 4 \times 10^4 + 5 \times 10^3 + 0 \times 10^2 + 7 \times 10^1 + 8 \times 10^0$

$= 4 \times 10000 + 5 \times 1000 + 0 \times 100 + 7 \times 10 + 8 \times 1$

$(b) 0.0235 = 0 \times 10^0 + 0 \times 10^{-1} + 2 \times 10^{-2} + 3 \times 10^{-3} + 5 \times 10^{-4}$

$= 0 \times 1 + 0 \times \frac{1}{10} + 2 \times \frac{1}{10^2} + 3 \times \frac{1}{10^3} + 5 \times \frac{1}{10^4}$

$(c) 930.133 = 9 \times 10^2 + 3 \times 10^1 + 0 \times 10^0 + 1 \times 10^{-1} + 3 \times 10^{-2} + 3 \times 10^{-3}$

$$= 9 \times 10^2 + 3 \times 10^1 + 0 \times 1 + 1 \times \frac{1}{10^1} + 3 \times \frac{1}{10^2} + 3 \times \frac{1}{10^3}$$

EXAMPLE 2

Write the following in expanded notation form

(a) 32.51_6

(b) 0.1001_2

(a) $32.51_6 = 3 \times 6^1 + 2 \times 6^0 + 5 \times 6^{-1} + 1 \times 6^{-2}$

$$= 3 \times 6 + 2 \times 1 + 5 \times \frac{1}{6} + 1 \times \frac{1}{6^2}$$

$(b) 0.1001_2 = 0 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4}$

$$= 0 \times 1 + 1 \times \frac{1}{2^1} + 0 \times \frac{1}{2^2} + 0 \times \frac{1}{2^3} + 1 \times \frac{1}{2^4}$$

$$= 0 + \frac{1}{2} + \frac{0}{4} + \frac{0}{8} + \frac{1}{16}$$

CLASS ACTIVITY

1. Write the following decimal numbers in an expanded notation form

(a) 402

(b) 60.008

(c) 0.0153

2. Write the following numbers in their ordinary form

(a) $6 \times 10^3 + 0 \times 10^2 + 5 \times 10^1 + 8 \times 10^0$

(b) $4 \times 10^1 + 3 \times 10^0 + 0 \times 10^{-1} + 2 \times 10^{-2}$

(c) $5 \times 7^0 + 8 \times 7^{-1} + 9 \times 7^{-2}$

CONCEPT OF NUMBER BASE SYSTEM

A number system is defined by the base it uses, the base being the number of different symbols required by the system to represent any of the infinite series of numbers. A base is also a number that, when raised to a particular power (that is, when multiplied by itself a particular number of times, as in $10^2 = 10 \times 10 = 100$), has a logarithm equal to the power. For example, the logarithm of 100 to the base 10 is 2.

CONVERSION FROM ANY BASE TO BASE 10

Two digits—0, 1—suffice to represent a number in the binary system; 6 digits—0, 1, 2, 3, 4, 5—are needed to represent a number in the sexagesimal system; and 12 digits—0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A(ten), B (eleven)—are needed to represent a number in the duodecimal system. The number 30155 in the sexagesimal system is the number $(3 \times 6^4) + (0 \times 6^3) + (1 \times 6^2) + (5 \times 6^1) + (5 \times 6^0) = 3959$ in the decimal system; the number 2BA in the duodecimal system is the number $(2 \times 12^2) + (11 \times 12^1) + (10 \times 12^0) = 430$ in the decimal system.

To convert from any base to base ten, expand the given number(s) in the powers of their bases and simplify.
Examples:

1. Convert 1243_{five} to base ten

Solution:

$$1243_{five} = (1 \times 5^3) + (2 \times 5^2) + (4 \times 5^1) + (3 \times 5^0)$$
$$= 125 + 50 + 20 + 3$$
$$= 198_{ten}$$

2. Convert 1111110_{two} to a number in base ten.

Solution:

$$1111110_{two} = (1 \times 2^6) + (1 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2)$$
$$(1 \times 2^1) + (0 \times 2^0)$$
$$= 64 + 32 + 16 + 8 + 4 + 2 + 0$$
$$= 126_{ten}$$

Thus, the decimal system in universal use today (except for computer application) requires ten different symbols, or digits, to represent numbers and is therefore a base-10 system.

CONVERSION FROM OTHER BASE GREATER THAN TEN TO BASE TEN

Expansion method can be used to convert numbers in base say base thirteen to base ten. Remember in base thirteen the digits we have are 0, 1, 2, 3,4,5, 6,7,8,9, A, B, C. where A represents ten B represents eleven and C represents twelve. Letters are used for two- digits numbers less than the base thirteen.

Example 1

Convert $1B9_{thirteen}$ to denary number

Solution

$1B9_{thirteen} = 1x13^2+Bx13^1+9x13^0$
$= 1x169+11x13+9x1$
$= 169+143+9$
$= 321_{ten}$

Example 2: Convert $20C_{fifteen}$ to a denary number

Solution

$20C_{fifteen} = 2x15^2+0x15^1+12x15^0$
$= 2x225+0x15+12x1$
$= 450+0x15+12x1$
$= 462_{ten}$

CLASS ACTIVITY

A. Convert the following to denary numbers

1. 1024_{eleven}
2. 2059_{twelve}

3. $51C_{fourteen}$

B. Convert the following numbers to denary numbers

(i) 10011_{two}

(ii) 768_{nine}

(iii) $10A_{eleven}$

(iv) $B12_{twelve}$

CONVERSION OF DECIMAL FRACTIONS IN ONE BASE TO BASE TEN

Sometimes we are faced with numbers which are not whole numbers. Hence it is very necessary to study also the conversion of fractional parts of numbers. The following examples can be used in our study of the conversion of fractional parts of other bases to decimal system.

Example 1:

Convert 6.4_7 to denary number

Solution

$$6.4_7 = 6 \times 7^0 + 4 \times 7^{-1}$$
$$= 6 \times 1 + 4 \times \frac{1}{7}$$
$$= 6 + \frac{4}{7}$$
$$(6\frac{4}{7})_{10}$$

Example 2:

Convert 101.011_{two} to base ten.

Solution

$$101.011_{two}t = 1 \times 2^2 + 0 \times 2^1 \times 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}$$
$$= 1 \times 4 + 0 \times 2 + 1 \times 1 + 0 \times \frac{1}{2} 1 \times \frac{1}{2^2} + 1 \times \frac{1}{2^3}$$
$$= 4 + 0 + 1 + 0 + \frac{1}{4} + \frac{1}{8}$$
$$= 5 + \frac{1}{4} + \frac{1}{8}$$
$$= 5\frac{3}{8_{ten}} \quad or\ 5.375_{ten}$$

CLASS ACTIVITY

1. Convert 11.011_{two} to a given number in base ten.
2. Convert the binary number11011.11 to base 10.

CONVERSION OF FRACTIONS IN BASE TEN TO ANY OTHER BASE

Fraction in base ten can be converted to other bases using various methods.

Example 1:

Express $\frac{7}{8}$ to bicimals.

Solution

Change $\frac{7}{8}$ to decimal fraction i.e$\frac{7}{8} = 0.875$ and multiply by 2.

0.875
X 2
1750
X 2
1500
X 2
1000

As we multiply 2×0.875, we get 1.750. Keep the 1 and multiply 750 by 2, and get 1.500. Keep the 1 and multiply 500 by 2 and get 1.000. Stop when all is zero. The value of $\frac{7}{8} = 0.111_{two}$ or convert 7 and 8 to base two and then divide.

2	7
2	3 R 1
2	1 R 1
	0 R 1

$7_{ten} = 111_{two}$

2	8
2	4 R 0
2	2 R 0
2	1 R 0
	0 R 1

$8_{ten} = 1000_{two}$

$$\therefore \left[\frac{111}{1000}\right] = 0.111_{two}$$

Example 2:

Express $(19\frac{13}{25})_{ten}$ to base five.

Solution

5	19	
5	3 R 4	↑
	0 R 3	

$= 34_5$

5	13	
5	2 R 3	↑
	0 R 2	

$= 23_{five}$

5	25	
5	5 R 0	↑
5	1 R 0	
	0 R 1	

$= 100_{five}$

$$= \left[34 + \frac{23}{100}\right]_{five}$$

$$= [34 + 0.23]_{five}$$

$$= 34.23_{five}$$

CLASS ACTIVITY

1. Express $\frac{5}{16}$ to base two
2. Express (11011.1001_2) to decimal base.

SIMPLE EQUATIONS IN NUMBER SYSTEM

Simple linear equations and simultaneous equation can be solved using the knowledge of expansion to base 10.

Example 1:

If $x_{10}=1214_{5}$, find x.

$x \times 10^0 = 1 \times 5^3 + 2 \times 5^2 + 1 \times 5^1 + 4 \times 5^0$

$x = 125 + 50 + 5 + 4$

$x = 184$

Example 2:

If $55_x + 52_x = 77_{10}$, find x.

$5 \times x^1 + 5 \times x^0 + 5 \times x^1 + 2 \times x^0 = 77$

$5x + 5 + 5x + 2 = 77$

$10x + 7 = 77$

$10x = 70$

$x = 7$

PRACTICE EXERCISE:

1. Convert the following numbers in denary to the base indicated.

$(a) 37.31_{ten}$ to base 6

(b) 10.8_{ten} to base 3.

2. Find the value of x in each of the following equations.
   - (a) $23_x + 14_x = 42_x$
   - (b) $53_x - 24_x = 25_x$
   - (c) $113_x + 121_x = 300_x$
   - (d) $562_x - 153_x = 407_x$

3. Find the value of x and y in the following pairs of equation.

$32_x + 53_y = 63$

$24_x + 35_y = 45$

4. Find the value of x and y in the following pairs of equation.

$54_x - 11_y = 36$

$25_x + 10_y = 21$

5. Find the value of x and y in the following pairs of equation.

$34_x + 21_y = 26$

$42_x - 12_y = 17$

ASSIGNMENT

1. Find the value of x and y in the following pairs of equation.

(a) $64_x - 53_y = 25$

$47_x - 34_y = 21$

2. Given that $124_x = 7(14_x)$, find the value of x. WAEC 2011
3. *Given R = 343_{five} and S = 14_{five}, calculate*

   a. *R + S*

   b. *R – S*

   c. *R × S*

   d. R ÷ S
4. x and y are non-zero digits such that $xxx_{three} = yy_{eight}$. Find x and y.

   Given that $4P4_5 = 119_{10}$, find the value of P.

   A. 1 B. 2 C. 3 D. 4 SSCE 2003

5. Evaluate $(111_{two})^2 - (101_{two})^2$

   A. 10_{two} B. 100_{two} C. 1100_{two} D. 11000_{two} SSCE 2003

6. If $M5_{ten} = 1001011_{two}$, find the value of M.

   A. 5 B. 6 C. 7 D. 8 SSCE 2002

7. Evaluate $(20_{three})^2 - (11_{three})^2$ in base three.

   A. 101 B. 121 C. 202 D. 2020

   SSCE 2001
8. Find if $200_x + 144_{nine} = 14B_{twelve}$
9. Solve for x and y if $32_x + 53_y = 61_{nine}$

   $24_x + 35_y = 45_{ten}$

KEYWORDS*: *Binary, denary, bicimal, base, decimal etc.

WEEK 7

MID TERM BREAK

WEEK 8
DATE: ………………….
SUBJECT: **MATHEMATICS**
CLASS: **SS 1**
TOPIC: Number Base System
CONTENT:

- Conversion of number from one base to another base.
- Addition, subtraction, multiplication and division of number bases.
- Application to computer programming.

CONVERSION OF NUMBERS FROM ONE BASE TO ANOTHER BASE.
To convert from a base to another you may have to pass through base ten.
Example 1: Convert 301_{four} to a base six number.
Solution
First 301_{four} will be converted to a base ten number
$301_{four} = 3x4^2 + 0x4^1 + 1x4^0$
$= 48 + 0 + 1$
$= 49_{ten}$
49_{ten} will now be converted to a base six number by repeated division

6	49
6	8 r 1 ↑

6 1 r 2
0 r 1

$301_{four} = 121_{six}$

Example 2: convert 110111_2 to base 5.

Solution

$110111_2 = 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 1 \times 32 + 1 \times 16 + 0 \times 8 + 1 \times 4 + 1 \times 2 + 1 \times 1$

$= 32 + 16 + 0 + 4 + 2 + 1$

$= 55_{10}$

Then we convert 55_{10} to a number in base 5

5	55
5	11r0
5	2 r 1
5	0 r 2

Hence, $55_{10} = 210_5$

CLASS ACTIVITY

1. Convert 2210_{three} to a base five number
2. Convert 5201_{seven} to a binary (base two) number

ADDITION, SUBTRACTION AND MULTIPLICATION OF NUMBER BASES

Operation in other bases other than base ten are carried out in a manner similar to what is obtained in base ten. We can illustrate the procedure as shown in the example below.

Example 1: 167eight + 125eight

Solution

167eight
\+ 145eight

7+5 = 12. This exceeds the value of the base. 12 contain a bundle of 8 and 4 units. That one bundle of 8 is carried to the next column as 1

1 + 6 + 4 = 11

11 is another single bundle of 8 and three, Hence we write 3 and carry the bundle to the next column as 1

167eight……………………………………
\+ 145eight
334eight

Example 2: $501_{twelve} - 3B_{twelve}$

$$
\begin{array}{r}
501_{eight} \\
-\ \underline{3B_{eight}} \\
\underline{\qquad}
\end{array}
$$

Recall B in base twelve is eleven.
If 1 is 'borrowed from 5 in the third column, getting to the next column on the right becomes a twelve. From it we can take one to the next column to the right again. To get 12+1 = 13 from which we finally subtract B (i.e eleven)

$$
\begin{array}{r}
501_{twelve} \\
-\ \underline{3B_{twelve}} \\
\underline{482_{twelve}}
\end{array}
$$

Notice that after borrowing 1 from the middle column, eleven was left. If is out of this eleven that 3 is subtracted to get 8 in the second column of the answer.

Example1: Simply 134_{six} x 5_{six}

$$
\begin{array}{r}
154_{six} \\
+\ \underline{5_{six}} \\
\underline{\qquad}
\end{array}
$$

5 x 4 = 20 i.e 3 bundles of 6 plus 2 units. Write 2 add 3 to the product of 5 x 5 of second column to get 28. 28 = 4(sixes) plus 4. Take the 4 bundles to next column. 4 + 5x 1 = 9 which is 13_{six}. So 154_{six} x 5_{six} = 1342_{six}

Example 2: Simplify 134_{five} x 24_{five}

$$
\begin{array}{r}
134_{five} \\
\times\ \underline{24_{five}} \\
\underline{\qquad}
\end{array}
$$

4 x 4 = 16 i.e 3(fives) and 1 unit.
These 3 bundle of 5 is added to the product of (3x4) of the second column. 3x4+3 = 15.
15 = 3(fives) and zero
This new 3 bundles of 3 is to be added to the product 1x 4 of the third column
1 x 4 + 3 = 7 which will written as 12_{five} similar thing is done with 134_{five} times the distance 2, thus

$$
\begin{array}{r}
134_{five} \\
\times\ \underline{24_{five}} \\
1201 \\
\underline{323\ } \\
\underline{4431_{five}}
\end{array}
$$

CLASS ACTIVITY

1. 1205_{six} x 3_{six}
2. 143_{five} + 24_{five}
3. 211_{four} - 32_{four}
4. 103_{four} x 32_{four}

Division of numbers bases

Since in binary (base two) system, the digits we have are 0 and 1. Each digit of the quotient 110111 ÷ 101 must be either 1 or 0.Therefore, $110111_{two} \div 101_{two}$ is done as follows:

```
         1011
101 | 110111
       101
         111
        101
          101
          101
```

Once you start the division, the digits are brought down one after the other.

Example $240_{six} \div 20_{six}$

```
      12
20 | 240
      20
      40
      40
```

So $240_{six} \div 20_{six} = 12_{six}$

CLASS ACTIVITY

1. $477_{eight} + 367_{eight}$
2. $BB_{twelve} + A1_{twelve}$
3. $1011_{tw0} \times 111_{two}$

APPLICATION TO COMPUTER PROGRAMMING

Binary system is very important because of its use in most electronic devices.

Digital computers perform their two-way functions because only two digits -0 and 1, called bits are coded into them as a programming language. For instance, if a device is ON, it is represented by a 1 and if it is OFF, a 0 is represented.

Other situations with only two possibilities include: UP or DOWN, TRUE or FALSE, MAGNETIZE or DEMAGNETIZE.

PRACTICE EXERCISE

1. Which is bigger $E5A_{sixteen}$ or $1271_{fifteen}$
2. $387_{nine} \div 25_{nine}$
3. Evaluate $(20_{three})^2 - (11_{three})^2$ in base three.

   A. 101 B. 121 C. 202 D. 2020

   SSCE 2001

4. Find the missing numbers in the addition of the following numbers in base seven.

```
  4 3 2 1
  1 2 3 4
  * * * *
 ________
 12 3 4 1
```

5. Find the missing number in the addition if the addition is in base eight.

```
  1 2 6 0
  * * * *
    2 3 5
    1 2 4
  _______
  3 0 1 1
```

ASSIGNMENT

1. Evaluate $(1010_{two})^2$ and leave your answer in base 2.

2. The subtraction below is in base seven. Find the missing number.

```
  5 1 6 2
  2 6 4 4
  _______
  2 * 1 5
```

   A. 2 B. 3 C. 4 D. 5 SSCE 2010

3. Simplify $11011_{two} - 1101_{two}$

A. 101000_{two} B. 1100_{two} C. 1110_{two} D. 1011_{two} SSCE 2006

4. Evaluate $202^2_{three} - 112^2_{three}$

A. 21120 B. 21121 C. 21112 D. 21011 SSCE 2004

5. If $y = 23_{five} + 101_{three}$ find y, leaving your answer in base two.

A. 1110 B. 10111 C. 11101 D. 111100 SSCE 2004

WEEK 9
DATE: ………………….
SUBJECT: MATHEMATICS
CLASS: SS 1
TOPIC: Simple Equations and variations:

CONTENT:

- Formulae, substitution and Change of subject of formulae.
- simple binary operations.
- Variations (i) Direct and inverse, (ii) joint and partial.
- Application of variation.

SUBJECT OF A FORMULA

The subject of a formula is the variable that is expressed in term of the other variables. In the relation y=x+4, y is called the subject of the formula. To make x the subject means rewriting this relation in an equivalent form, where x will be alone on one side of the equality sign. The relation is normally written with the subject on the left-hand side of the formula. For example, y=2x-3 , the x can be made subject of formula as follows, $x = \frac{y+3}{2}$.

SUBSTITUTION

A formula is an equation in which letters represent quantities. The value of one variable in a formula or algebraic equation may be found by substituting (i.e replacing) known values in the same formula

Examples 1: The sum of the squares of the first n integers is given by $s_n = \frac{n(n+1)(2n+1)}{6}$

Calculate (a) s_{20} (b) the sum of the squares from 21 to 40 inclusive.

Solution: (a) s_{20} means the values of s_n when $n = 20$

$$s_{20} = \frac{20(20+1)(2\times20+1)}{6}$$

$$s_{20} = \frac{20(21)(41)}{6}$$

$$= 2870$$

(b) Sum of squares from 21 to 40 means = $s_{40} - s_{20}$

But, s_{20} is already known to be 2870,

$$s_{40} = \frac{40(40+1)(2\times40+1)}{6}$$

$$= \frac{40(41)(81)}{6}$$

$$= 22140$$

$$\therefore s_{40} - s_{20} = 22140 - 2870$$

$$= 19270$$

Example 2: Find the value of $2\pi\sqrt{\frac{l}{g}}$, when $\pi = 3\frac{1}{7}$, $l = 98$ *and* $g = 32$

Solution: $2\pi\sqrt{\frac{l}{g}} = 2 \times \frac{22}{7}\sqrt{\frac{98}{32}}$

$$= \frac{44}{7}\sqrt{\frac{49}{16}}$$

$= \frac{44}{7} \times \frac{7}{4}$

$= 11$

Class Activity

(1) Make the letters that appear as the subject of the formula

$(a)\ T = \frac{E}{\sqrt{R^2 - W^2 L^2}}\ , \quad (R, L)$

$(b)\ V = \pi h^2 \left(r + \frac{h}{3}\right), \quad (r, h)$

(2) (a) If $v = kl(D^2 - d^2)$, find d when $v = 1250, D = 1000, k = 5\ and\ l = 4$

CHANGE OF SUBJECT OF FORMULAE

A literal equation is a simple equation that involves more than one variable (unknown). The procedure of solving such an equation is usually to find one of the variables (unknowns) in terms of the other(s).

Example 1: Given $A = \frac{1}{2}h(a + b)$, make a the subject of the formula

Solution: $A = \frac{1}{2}h(a + b)$

Cross-multiply

$$2A = h(a + b)$$

$$2A = ah + bh$$

$$2A - bh = ah$$

Also, $ah = 2A - bh$

$$a = \frac{2A - bh}{h}$$

$$a = \frac{2A}{h} - \frac{bh}{h}$$

$$\therefore a = \frac{2A}{h} - b$$

Example 2: Make v the subject of the formula;

$$H = \frac{m(v^2 - u^2)}{2gx}$$

Solution: $H = \frac{m(v^2 - u^2)}{2gx}$

Cross-multiply

$$2gxH = m(v^2 - u^2)$$

$$2gxH = mv^2 - mu^2$$

$$2gxH + mu^2 = mv^2$$

$$\frac{2gxH + mu^2}{m} = v^2$$

Take square root of both sides;

$$v = \pm\sqrt{\frac{2gxH + mu^2}{m}} \quad \text{or} \quad \pm\sqrt{\frac{2gxH}{m} + u^2}$$

Class Activity

(1) The period of a compound pendulum is given by $T = 2\pi\sqrt{\left(\frac{h^2 + k^2}{gh}\right)}$ express k in terms of $T, h\ and\ g$.

(2) Make the letters that appear as the subject of the formula

(a) $I = \frac{nE}{R + nr}$, (n, R)

BINARY OPERATION

A binary operation is any rule of combination of any two elements of a given non-empty set. Asterisk symbol $(*)$ is used to denote binary operation. Some authors uses degree symbol (°) or zero symbol(o) to denote binary operation. However, the most commonly use is Asterisk symbol $(*)$.

In binary operation, the most common operations includes:

Addition of real numbers $(+)$

Subtraction of real numbers $(-)$

Multiplication of real numbers $(\times)$

Division of real numbers $(\div)$.

Closure Property:

Given a non-empty set $\boldsymbol{S}$, **S** is said to be closed under a binary operation $*$ if for all $\boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{S}, \boldsymbol{a} * \boldsymbol{b} \in \boldsymbol{S}$.

Where $\boldsymbol{a}$ and $\boldsymbol{b}$ are elements in (belonging to) set $\boldsymbol{S}$ and $\in$ means belong.

For example, the set $\boldsymbol{Z}$ of all integers is closed under addition$(+)$, subtraction$(-)$ and multiplication $(\times)$ **except** for division$(\div)$.

To illustrate non-closure of real numbers under division operation$(\div)$, lets consider this example: Given $2, 4 \in \boldsymbol{Z}$, then $2\div 4 = \frac{1}{2}$, but $\frac{1}{2}$ does not belong to $\boldsymbol{Z}$. Hence, the $\boldsymbol{Z}$ is not closed under the division operation$(\div)$.

Example1:

Let the operation $*$ be defined $on\ \boldsymbol{R}$, the set of real numbers,if $\boldsymbol{a} * \boldsymbol{b} = \boldsymbol{a} + \boldsymbol{b} + \boldsymbol{2ab}$, evaluate:

(a) $1 * 2$
(b) $3 * 4$
(c) $(3 * 4) * 5$
(d) $3 * (4 * 5)$

Solutions:

(a) $1 * 2 = 1 + 2 + 2(1)(2)$
$= 7$
(b) $3 * 4 = 3 + 4 + 2(3)(4)$
$= 31$
(c) $(3 * 4) * 5 = 3 + 4 + 2(3)(4) * 5$
$= 31 * 5$
$= 31 + 5 + 2(31)(5)$
$= 36 + 310$

$$= 346$$

(e) $3 * (4 * 5) = 3 * \{4 + 5 + 2(4)(5)\}$

$$= 3 * \{9 + 40\}$$
$$= 3 * 49$$
$$= 3 + 49 + 2(3)(49)$$
$$= 52 + 294$$
$$= 346$$

Example 2:

Suppose $D = \{odd\ integers\}$ and $*$ is defined on such that for every $a, b \in D,\ a * b = a + b$. Is D closed under $*$?

Solution:

For every $a, b \in D$

$a * b = a + b \in D$ since when two odd integers are added the result is an even number. Hence D is not closed under. In other words D is not closed under addition.

CLASS ACTIVITY

1. The operation $*$ on the set Q of rational numbers is defined by:
   $\boldsymbol{p} * \boldsymbol{q} = (\boldsymbol{p}^2 + q - 3pq)\frac{1}{2};\ p, q \in Q.$
   Determine: (a) $2 * \mathbf{1}$ (b) $-4 * \mathbf{5}$ (c) $\frac{1}{2} * \frac{\mathbf{4}}{\mathbf{3}}$
2. The operation ∇ on the set R of real numbers is defined by:
   $x\nabla$y = 3xy, for $x, y \in R$. Find
   a) $-2\nabla 5$
   b) $3\nabla - 2$

VARIATIONS

Variation is a connection of sets of numerical values by an equation which indicates some kind of proportionality. We have four major types of variations which are direct, inverse, joint and partial variations.

Direct variations

Considering two quantities x and y. If when y increases x also increases and when y decreases x also decreases in a constant proportion, then x and y are said to be in direct variation. i.e. x varies directly as y or x is directly proportional to y. Written as $x \propto y \Rightarrow x = ky$ where k is the constant of proportionality or variation.

Other examples are as follows:

(i) A varies directly as the square of B

i.e. $A \propto B^2$

$\therefore$ $A = KB^2$ *(where K is the constant of variation)*

(ii) P varies directly as the square root of q.

i.e. $P \propto \sqrt{q}$

$\therefore$ $P = K\sqrt{q}$ *(where k is the constant of variation).*

Example 1: If P varies directly with q and q = 2, p = 10. Find p when q is 5.

Solution: $p \propto q$

$$p = kq$$

$$10 = 2k$$

$$k = 5$$

Formula connecting p and q is p = 5q,

Then, p = 5 x 5 = 25

Example 2: If $x - 3$ is directly proportional to the square of y and $x = 5\ when\ y = 2, find\ x\ when\ y = 6$

Solution; $x - 3 \propto y^2$

$$x - 3 = ky^2$$

$$5 - 3 = k \times 2^2$$

$$2 = 4k$$

$$k = \frac{1}{2}$$

Formula connecting $x - 3\ and\ y\ is\ \ x - 3 = \frac{1}{2}y^2$.

Then, $x - 3 = \frac{1}{2} \times 6 \times 6$

$x - 3 = 18$

$\therefore x = 21$

CLASS ACTIVITY

1. x is directly proportional to y. If $x = 5\ when\ y = 3, find\ y\ when\ x = \frac{2}{7}$
2. The wages of a labourer varies directly as the number of hours worked by the labourer. The labourer earned ₦500 when he worked for 2 hours. Find

   (i) The amount he would earn if he works for 7 hours.

   (ii) The number of hours he would work if he is paid ₦800.

Inverse variation

Considering two quantities x and y. If when y increases, x decreases and when x increases y decreases in a constant ratio, then we say x and y varies inversely. It is normally written as $x \propto {}^{1}/_{y}$ or $y \propto {}^{1}/_{x}$.

Note that if $x \propto {}^{1}/_{y}$, then $x = {}^{k}/_{y}$ where K is the constant of variation or proportionality.

Other examples are as follows:

Example 1: A varies inversely as the cube root of B

i.e. $A \propto \frac{1}{3\sqrt{B}}$

$\therefore$ A = K x $\frac{1}{3\sqrt{B}}$ *(where k is constant)*

(ii) P varies inversely as the square of q

I.e. $P \propto \frac{1}{q^2}$

$\therefore$ P = $\frac{k}{q^2}$ *where k is constant*

Example 2:

(i) $t \propto \frac{1}{d}$ $and\ t = 0.15\ when\ d = 120$

(a) find t when d = 45

(b) find d when t = 0.12

Solution: $t \propto \frac{1}{d}$

$t = \frac{k}{d}$

$0.15 \times 120 = k$

$k = 18$

Formula connecting t and d, $t = \frac{18}{d}$

(a) $t = \frac{18}{45}$

$= 0.4$

(b) $t = \frac{18}{d}$

$0.12d = 18$

$d = \frac{18}{0.12}$

$\therefore d = 150$

*(*ii) If P is inversely proportional to the square root of q, when P = 3, q = 25. Find

(a) P when q = 49

(b) q when p = 13

Solution:

$P \propto \frac{1}{\sqrt{q}}$

$P = \frac{k}{\sqrt{q}}$

--------------------(1) (where k is constant)

When P = 3, q = 25

$3 = \frac{k}{\sqrt{25}}$

$3 \times 5 = k$

$\therefore k = 15$

Substitute in (1) to get the law of variation

$P = \frac{15}{\sqrt{q}}$

(a) To find p when q = 49

Substitute in the law of variation to have

$P = \frac{15}{\sqrt{49}}$

$p = \frac{15}{7}$

$\therefore p = 2^1/_7$

(b) To find q when p = 13, substitute in the law of variation to have

$13 = \frac{15}{\sqrt{q}}$

$13\sqrt{p} = 15$

$\sqrt{p} = \frac{15}{13}$

$p = \left(\frac{15}{13}\right)^2$

$p = \frac{225}{169}$

$\therefore p = 1^{56}/_{169}$

CLASS ACTIVITY

(1) $If\ x\ varies\ inversely\ as\ the\ square\ of\ y\ and\ x = 8\ when\ y = 4, find\ y\ when\ x = 32$

(2) If y varies inversely as x and y = 6 when x = 2. Find
(i) The law of variation
(ii) x when y = 10
(iii) y when x = 7

Joint variation

This type of variation involves three or more quantities joined together with a combination of two direct variations or one direct and one inverse or two inverse variations.

Example 1*:*
(i) x varies directly as y and jointly as z
I.e. $x \propto yz$
x = kyz *(where k is constant)*
(ii) x varies directly as y and inversely as the square of z.
i.e. $x \propto \frac{ky}{z^2}$

$\therefore\ x = \frac{ky}{z^2}$ *(where k is constant)*

(iii) P varies inversely as q and inversely as the square root of r.
I.e. $p \propto \frac{1}{q\sqrt{r}}$

$\therefore\ p = \frac{1}{q\sqrt{r}}$ *(where k is constant)*

Example 2:
If x varies directly as y and inversely as the square of z, when y = 5 and z = 3, x = 20. Find
(a) z when x = $21^3/_5$ and y = 15
(b) x when y = 6 and z = 4.

Solution:
$X \propto \frac{y}{z^2}$

$x = \frac{ky}{z^2}$ -------- (1) *(where k is constant)* when y = 5 and z = 3, x = 20

$$20 = \frac{k \times 5}{3^2}$$

$$20 \times 9 = k5$$

$$k = \frac{20 \times 9}{5}$$

$$k = 4 \times 9$$

$$\therefore\ k = 36$$

Substitute in (1) above to get the law of variation.

$$X = \frac{36y}{z^2}$$

(a) To find z when $x = 21\frac{3}{5}$ and y = 15

$$\frac{108}{5} = \frac{36 \times 15}{z^2}$$

$$108z^2 = 36 \times 15 \times 5$$

$$z^2 = \frac{36 \times 15 \times 5}{108}$$

$$z = \sqrt{25}$$

$$\therefore\ z = 5.$$

(b) To find x when y = 6 and z = 4.

$$x = \frac{36 \times 6}{4^2}$$

$$x = \frac{36 \times 6}{16}$$

$$\therefore\ x = 13.5$$

Example

1: *x varies directly as the product of u and v and inversely as their sum, if* $x = 3$ *when* $u = 3$ *and* $v = 1$, *what is the value of x if* $u = 3$ *and* $v = 3$?

Solution: $x \propto \frac{uv}{u+v}$

$$x = \frac{kuv}{u+v}$$

$$3 = \frac{k \times 3 \times 1}{3+1}$$

$$3 \times 4 = 3k$$

$$k = 4$$

Law connecting x, u and v is $x = \frac{4uv}{u+v}$

Then, $x = \frac{4\times3\times3}{3+3}$

$$\therefore x = 6$$

Example 2: $A \propto BC, when\ B = 4\ and\ C = 9, A = 6$

(a) find the formula that connects A,B & C

(b) find A when B = 3 and C = 10

(c) find C if A = 20 and B = 15

Solution: $A \propto BC$

$$A = kBC$$

$$6 = k \times 4 \times 9$$

$$k = \frac{1}{6}$$

(a) $A = \frac{1}{6}BC$

(b) $A = \frac{1}{6}BC$

A= $\frac{1}{6} \times 3 \times 10$

A = 5

(c) $A = \frac{1}{6}BC$

$20 = \frac{1}{6} \times 15 \times C \qquad \therefore C = 8$

Class Activity

(1) A varies directly as B and inversely as C. When B = 3, C = 5, A = 40. Find

(i) The law of variation

(ii) A when B = 8 and C = 12.

(2) $x \propto \underline{y}$. when y = 36 and z = 16, x = 81.

$\sqrt{z}$

Find (i) The law of variation

(ii) y when x = 56 and z = 25

(iii) x when y = 27 and z = 9

Partial variation

This consists of two or more parts or quantities added together. Both parts may be made of variables or one part may be constant, while the other can either vary directly or inversely. The values of the constants of variation are usually found out by solving simultaneous equations.

Types of Partial Variation

(i) x is partly constant, and partly varies as y

x = a+by *(where a and b are constant)*

(ii) P varies partly as q and partly as r.

p = aq + br *(where a and b are constants)*

(iii) v varies partly as u and partly as the reciprocal of w^2.

Example 1: P is partly constant and partly varies as Q, when Q is 5, P is 20 and when Q is 8, P is 26. Find P when Q is 4.

Solution: P = c + kQ

Note: c & k are constants which must be obtained through simultaneous equation.

$$20 = c + 5k \quad \ldots\ldots\ldots (i)$$

$$26 = \mathrm{c} + 8\mathrm{k} \ldots\ldots\ldots. (\mathrm{ii})$$

-6 = -3k

K = 2

substituting the value of k in equation (i)

$$20 = c + 5(2)$$

$$20 - 10 = c$$

$c = 10$

Thus, P = 10 + 2Q   (formula connecting P & Q)

Then, P = 10 + 2(4)

P = 10 + 8

∴ P = 18

Example2:
X is partly constant and partly varies as y. when y = 7, x = 15; and when y = 5, x = 7. Find

(a) The law of variation

(b) x when y = 2

(c) y when x = 11

Solution
(a) x = a + by --- (1) *where a and b are constants of variation.*

When y = 7, x = 15 and when y = 5, x = 7

15 = a + 7y ----------------- (2)

7 = a + 5y ----------------- (3)

Solve equation (2) and (3) simultaneously

Eqn (2): 15 = a + 7b _

Eqn (3):   7 = a + 5b

8 =   2b

$\frac{8}{2} = b$

∴ b = 4

Put in eqn (2) to have

15 = a + 7 x 4

$15 = a + 28$

$a = 15 – 28$

$a = -13$

Substitute $a = -13$ and $b = 4$ in eqn (1) to get the law of variation.

$x = -13 + 4y$ -------------*(law of variation)*

(b) To find x when y = 2, put in the law of variation

$x = -13 + 4 \times 2$

$x = -13 + 8$

$\therefore x = -5$

(c) To find y when x = 11, put in the law of variation

$11 = -13 + 4y$

$11 + 13 = 4y$

$24 = 4y$

$y = \frac{24}{4}$

$\therefore y = 6.$

CLASS ACTIVITY

(1) M is partly constant and partly varies with N, when N = 40, M = 150 and when N = 54, M = 192

(a) Find the formula connecting M and N, (b) Hence find M when N = 73

(2) Two quantities P and Q are connected by a linear relation of the form $P = aQ + b$, where a & b are constants. If Q = 80 when P = 12 and Q = 300 when P = 50, find the equation connecting P and Q

APPLICATION OF VARIATION

Example 1: A particle moves in such a way that its displacement S metres, at time t seconds is given by the relation S = at^2, where ‘a’ is constant. Calculate ‘a’ if S = 32 when t = 4

Solution; S = at^2

32 = a x 4^2

$\frac{32}{16} = a$

a = 2

Example 2: The cost of feeding a number of students of Deeper Life High School in a regional excursion is partly constant and partly varies directly as the number of students. The cost of feeding 75 students during the excursion is $875 and the cost of feeding 100 students during the same period of time is $1000. Find the cost of feeding 220 students over the same period of time.

Solution: The equation is given as; C = a + kN where C is the cost and N is the number of students

875 = a + 75k ……….(i)

1000 = a + 100k ………. (ii)

Solving both equations simultaneously, we obtain k = 5 & a = 500.

Formula connecting C and N implies; C = 500 + 5N

Then, C = 500 + 5(220)

C = 500 + 1100

∴ C = $1600

CLASS ACTIVITY

(1) The electrical resistance R of a wire varies inversely as the square of the radius r. use a constant k to show the law between R and r

(2) Given that the energy E, varies directly as the resistance R and inversely as the square if the distance d, (i) obtain an equation connecting E, R and d given that E = $\frac{32}{25}$ when R =

8 and d = 5. (ii) calculate (a) the value of R when E = 16 and d = 3 (b) the value of d when R = 5 and d = $\frac{5}{6}$ (c) find the percentage increase in the value of R when each of E and d increases by 3%

PRACTICE EXERCISE

1. Make W the subject of the formula; $R - d = \sqrt{R^2 - W^2}$. Given that R = 1.25 and d = 0.25. calculate W
2. Given that $T = 2\pi\sqrt{\frac{l^2+k^2}{2gl}}$
   (a) Make k the subject of the formula
   (b) Find the value of k when $l = 1$ and $T^2 = 4\pi^2 g$
3. The resistance R to the motion of a car is partly constant and partly proportional to the square of the speed v. when the speed is 30km/h, the resistance is 190Ω and when the speed is 50km/h, the resistance is 350Ω. find for what speed the resistance is 302.5Ω
4. Simplify $p(q - 2p) - (p + q)(q - 2p)$ find the value of the expression when $p = -2$ and $q = 1$
5. Evaluate $\sqrt{\frac{s^2-(b+c)s+bc}{bc}}$ $when\ b = 25, c = 7\ and\ s = 28$

ASSIGNMENT

1. The area of a box varies directly as the square of its length. When the area is 175cm^2, the length is 5cm. Find the;
   (i) Equation connecting the quantities.
   (ii) Area when the length is 8cm
   (iii) Length when the Area is 100cm^2
2. y varies directly as the square of x. If y = 98 when x = 7, calculate y when x = 5.

[WAEC]

3. If $D \propto S$ and $D = 140$ $when$ $S = 35, find$

   (I) The relationship between D and S.

   (II) The value of S when D = 176

4. If y varies inversely as the square of x and y = 8 when x = 3. Find

   (i) The relationship between x and y.
   (ii) x when y = 5
   (iv) y when x = 9
5. If P varies inversely as the square root of q and p = 12 when q = 4. Find
   (i) p when q = 25
   (ii) q when p = 8.
6. U varies directly as V and inversely as the square of W. When V = 3 and W = 4, U = 24. Find (i) U when V = 5 and W = 8
   (ii) W when U = 30 and V = 8
7. The electrical resistance of a copper wire varies directly as its length and inversely as the square of its radius. If a copper wire 500 meters long and radius 0.2cm has a resistance of 30 ohms, calculate the resistance of the same type of copper wire 750 meters long and radius 0.25cm. ***(WAEC).***
8. If $P \propto Q\sqrt{R}$. When R = 16 and Q = 3, P = 48. Find (i) The law of variation
   (ii) P when R = 25 and Q = 7
   (v) R when P = 36 and Q = 9
   (vi) The cost of producing a wooden frame varies directly as the width of the frame and partly as the square root of its length. When the width is 10cm and the length is 25cm, the cost is ₦115.00 and when the width is 18cm and the length is 36cm, the cost is ₦240. Find the
   (a) Law of variation
   (b) Cost of a frame of width 12cm and the length 49cm.

9. The cost of producing a textbook is partly constant and partly varies as the number of books produced. It cost ₦4000 to produce 20 books and ₦6000 to produce 70 books. Find the

   (a) Cost of producing 120 books

   (b) Number of books produced at ₦10, 000.

10. The cost of sinking a well varies partly as the depth of the well and partly as the number of laborers used for the job. If it cost ₦1500 to sink a well of 6 metres deep with 2 laborers and ₦2500 to sink a well of 9 metres deep with 5 laborers. Find the

   (a) Law of variation

(b) Cost of sinking a well 15 metres deep with 6 laborers

(c) Number of laborers that would sink a well of 12 metres deep at ₦3000.

KEYWORDS: Subject of formula, joint variation, direct variation, inverse variation, partial variation etc.

WEEK 10 REVISION

WEEK 11 EXAMINATION